# A SIMPLIFIED METHOD OF GRILLAGE ANALYSIS

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## ABSTRACT

A new method of analysis is presented for laterally loaded "discrete" grillages, that is, grillages in which the beams cannot be "smeared" into the plating because the major beams are either too few in number or are of different sizes. The GRIDFORM (GRIllage Design FORMulae) method gives explicit formulae and tables for the edge moments and interaction forces, the knowledge of which allows the various beams to be analysed independently by simple beam theory. The method has the same accuracy but requires much less computation than other "discrete grillage" methods. It is therefore suitable for design office (desk calculator) applications while, at the same time, when implemented on a computer it is significantly faster than these other computer-based methods.

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## 1. INTRODUCTION

Cross-stiffened panels of plating constitute the major portion of a ship's structure, and also of other types of structures such as box girder bridges. These panels generally carry both lateral and in-plane loads and the lateral load may usually be idealised as uniform over the panel. For ordinary or "service" structural analysis (as opposed to ultimate failure analysis) these two types of loads may be considered separately. The in-plane load is carried mainly by the plating, whereas the uniform lateral load is carried by the gridwork of beams, with the plating acting merely as a flange for each beam. Thus, in regard to lateral load response (that is, lateral deflection and bending) a stiffened panel acts as a grillage, and grillage analysis is therefore an important and commonly occurring task in ship structural analysis. However, the analysis of grillages is usually difficult because they are very much "statically indeterminate", and most existing methods of grillage analysis are either very approximate or require a great deal of calculation. In this regard, we may distinguish two basic types of grillages:

(a) "continuous" grillages, in which the beams are numerous, uniform and closely spaced in both directions;

(b) "discrete" grillages, in which there are only a few beams in one direction, (say, less than four) and which may differ markedly from one another.

For continuous grillages the beams may be "smeared" into the plating and orthotropic plate formulae and design charts, as pioneered by Schade [ 4 ], may be used. These are simple to use and, in the case of continuous grillages, they give accurate results.

However, for discrete grillages the orthotropic plate approach is not sufficiently accurate and other more exact methods such as those presented by Clarkson [1] must be used. However, these methods are numerical (rather than explicit formulae) and they generally involve a great deal of computation. They are therefore too arduous for anything but a computer solution (whereas an explicit method, or at least a simple "desk calculator" method, would be more suitable for a design office). Also, even as computer methods they require a significant amount of computer time and are therefore not suited for structural optimization, which requires a large number of repeated analyses. Thus, there is a need for a rapid, simple (preferably explicit) and yet accurate method for the analysis of discrete grillages.

The method presented herein fills all of the above needs. It provides explicit formulae for the forces and moments in a uniformly loaded grillage and has the same accuracy as more complicated numerical and/or computer-based methods. For simplicity, this new method will be referred to as the GRIDFORM (GRIllage Design FORMulae) method. The formulae can be rapidly evaluated on a modern desk calculator and do not require a computer, thus making the method well-suited for design office applications. Moreover, when implemented on a computer, the GRIDFORM method is again found to be more efficient than the usual computer methods (that is, faster, for a given accuracy) due to its simple and explicit nature.

In fact, the method was originally developed as part of a computer program for structural optimization. The particular application was to medium size naval vessels, such as that shown in Figure 1, and the GRIDFORM method was used for the analysis of the strength deck. For convenience, this same application will be used as an example in the present work, both to explain the GRIDFORM method and also to show how it can be applied to large portions of a ship's structure.

In the work which is to follow, Sections 2 and 3 present some basic concepts and assumptions. In Section 4, analytical expressions are derived for (a) the end moments and (b) the spring forces for a single uniformly loaded stiffener supported by either one, two or three springs located along its length. In Section 5, it is shown that the expresssions derived from this beam-on-spring model can be extended to apply to a stiffener in a grillage by replacing the spring stiffness by an analogous parameter, the "pseudo-spring stiffness". The expressions then give (a) stiffener end moments, as before, and (b) the girder-stiffener interaction forces, which are analogous to the spring forces. Empirical expressions for the pseudo-spring stiffness are derived in Section 6, using data obtained from a large number of grillage analyses. In Section 7, some examples indicate the accuracy of this method by comparing the results with those obtained using finite element frame analysis and the Distributed Reaction Method.

## 2. DEFINITIONS AND ASSUMPTIONS

For clarity, the set of heavier beams will hereafter be referred to as "girders", $n$ in number, and the set of lighter (and usually more numerous) cross-beams will be referred to as "stiffeners" ( $m$ in number). A typical grillage is shown in Figure 2. Both girders and stiffeners are assumed to be equally spaced. The stiffeners are further assumed to be identical, while the girders are only assumed to be symmetric (that is, for the case of $n = 3$ the centre girder may differ from the side girders). Both girders and stiffeners are assumed to be of negligible torsional stiffness. Loads are assumed to be borne by the stiffeners and transferred to the girders by means of the interaction forces $R_{ij}$ , where $R_{ij}$ represents the interaction force between the $i^{th}$ girder and the $j^{th}$ stiffener. Similarly, $w_{ij}$ represents the deflection at this intersection.

The grillage in Figure 2 is assumed to form part of a larger structure. Provided that all of the boundary conditions (both load and support) can be specified, then this grillage may be isolated from the rest of the structure and treated separately. The adjacent structure provides a rotational restraint for the stiffeners, typified by a rotational stiffness parameter $C$ . In Figure 2, this rotational restraint is represented schematically be a series of identical leaf springs attached to the stiffener ends. A method for the calculation of C will be presented in Section 3.

The girders are always fewer in number, and hence for simplicity the girder end restraint is idealised as either simply supported ($C_g = 0$) or clamped ($C_g = \infty$). Interpolation can be used for intermediate cases.

## 3. ISOLATION AND ANALYSIS OF SHIP GRILLAGES

It is the usual practice (and still the most efficient in most cases) to divide the overall structural analysis of a ship into three levels: longitudinal, transverse and local. In the example of Figure 1, the analysis of the deck grillage forms part of the "level 2" analysis. At this level, only the major longitudinal and transverse members are included; the smaller longitudinals are dealt with at the local level, using the forces and deflections from the grillage analysis as boundary conditions. (The only exception is the total area of these longitudinals, which is determined in the "level 1" analysis.) Also, in the "level 2" analysis of the deck only the lateral load need be considered; the in-plane aspects are dealt with as part of the "level 1" analysis.

One of the most important requirements in the above approach is the proper treatment of the boundary conditions which act on the grillage (both loads and restraints). In regard to the restraint at the ends of the girders, the method presented herein gives solutions for the two idealised cases of simply supported and clamped, so that interpolation may be used for intermediate cases. However, this approach cannot also be used for the stiffeners because when both C and $C_g$ are varied, the basic distribution of the interaction forces changes. Therefore, in the GRIDFORM method, the rotational restraint at the end of the stiffeners is included as a variable. This rotational restraint is due to the rotational stiffness of the adjoining structure, $k_1$, which is defined as the moment required to produce a unit rotation (1 radian) of the adjacent structure when applied to that structure at its point of connection to the stiffener. For the deck grillage of Figure 1, this rotational restraint is provided by the side frames (and ultimately by the rest of the hull) and, as shown in Figure 3(a), $k_1$ is the ratio of an applied moment $M_1$ and the resulting rotation $\theta_1$ . The non-dimensional "rotational restraint parameter" $C$ is then defined as

$$C = \frac{k_1}{EI_s / \ell_s} \quad . \tag{3.1}$$

Because of the importance of correctly estimating $C$ , this estimation will be discussed briefly here for the case of the deck grillage in Figure 1. In principle, the entire hull structure contributes to the stiffness $k_1$ and it is difficult to lay down hard and fast rules for assigning a value to it (and hence to $C$ ) since this will depend on the particular hull structure involved. Faulkner and Snyder [ 2 ] deal at some length with the problem of stiffness estimation in plane frames. They conclude that "for many problems the marginal improvement in accuracy achieved" by accurately assessing the stiffness of adjacent structure (and hence of $C$ ) "does not justify the extra effort, and a reasonable value for most design or analysis purposes would be $C = 6$ ". In fact, the value will depend primarily on the length and moment of inertia of the immediately adjacent member - the first segment of the vertical side frame. The properties are denoted by $\ell_a$ and $I_a$ respectively. If the remainder of the hull structure were totally ignored, then $M_1 / \theta_1 = 3\, EI_a / \ell_a$ and the value of $C$ would depend on the relative values of the ratios $I_a / \ell_a$ and $I_s / \ell_s$ . For ship structures of the type under consideration, the first ratio is often about twice as large as the second, giving the value $C = 6$ as recommended by Faulkner and Snyder. Certainly $C$ would almost always lie within the range $1 < C < 20$ .

More accurate values of $M_1 / \theta_1$ can be obtained by taking more of the hull structure into account, but this greatly increases the task of analysis. Instead of adopting either of these two extreme methods for calculating $C$ (on the one hand considering all of the hull structure and going through an arduous calculation, or on the other hand, simply adopting a fixed value of $C$ ) the best procedure would seem to be a compromise between the two. As shown in Figure 3, only the immediately adjacent member - the side frame - is explicitly included, but the remainder of the hull structure is accounted for by adopting the recommended value of $C_a = 6$ for the dimensionless rotational restraint at the remote end of the side frame. It can easily be shown that

$$k_1 = 3.6 \frac{EI_a}{\ell_a} \tag{3.2}$$

and hence from Eqn. (3.1)

$$C = 3.6 \left(\frac{I_a}{I_s}\right) \left(\frac{\ell_s}{\ell_a}\right) \quad . \tag{3.3}$$

## 4. BEAM ON SPRING SUPPORTS

In this section, expressions are derived for (a) the end moments and (b) girder-stiffener interaction forces occurring when a single stiffener is supported by one or more girders and subjected to a uniform load, $q$ . It will be shown in the following section that these expressions may be adapted to give (a) and (b) for a stiffener which is part of a grillage. The single stiffener shown in Figure 4a is supported by one girder which can be treated as a spring (Figure 4c) of stiffness

$$k_f = \frac{R}{w} \tag{4.1}$$

where $R$ is the girder-stiffener interaction force and $w$ is the final deflection at the point of intersection. It may be shown from elementary beam theory that

$$k_f = \frac{3EI_g}{f^2 g^2 \ell^3} \qquad \text{for } C_g = 0 \qquad (a)$$

$$k_f = \frac{3EI_g}{f^2 g^2 \left(1 - f^2 - g^2 - fg\right) \ell^3} \qquad \text{for } C_g = \infty \qquad (b) \qquad (4.2)$$

The case of a grillage of m stiffeners subjected to a uniform pressure load p from which all stiffeners but the $j^{th}$ have been removed is similar (Figure 4b). Here the $i^{th}$ girder presents a spring stiffness $k_{ij}$ to the $j^{th}$ stiffener

$$k_{ij} = \frac{R_{ij}}{w_{ij}} \qquad (4.3)$$

where $R_{ij}$ and $w_{ij}$ are respectively the interaction force and the deflection, as defined in Section 2. In this case, expressions may be written for the spring stiffness in terms of j and m ,

$$k_{ij} = \frac{3EI_i}{\ell_g^3} \frac{(m+1)^6}{j^2 (m+1-j)^2} \qquad \text{for } C_g = 0 \qquad (a)$$

$$k_{ij} = \frac{3EI_i}{\ell_g^3} \frac{(m+1)^6}{j^3 (m+1-j)^3} \qquad \text{for } C_g = \infty \qquad (b) \qquad (4.4)$$

Expressions for the moment at the end of the stiffener, $M_{Bj}$ , and the force in the spring, $R_{ij}$ (that is, the girder-stiffener interaction force) will now be derived. The rotational restraint at the stiffener end is modelled as a pinned-end beam of the same cross section as the stiffener and equivalent length $3\ell_s / C$ . The first case to be examined is that corresponding to a one-girder grillage, as illustrated in Figure 5.

(Since both the load and the structure are symmetric, only the left half is drawn.) In terms of the slopes $\theta_{Aj}$ and $\theta_{Bj}$ and the girder deflection $w_{ij}$, which are defined in Figure 5b, the equilibrium equations for this structure are

$$\left.\begin{aligned} 0 &= 4\phi_{Aj} + 2\phi_{Bj} && \text{(a)} \\ \frac{3q\ell_s}{16Ck_s} &= 2C\phi_{Aj} + 4(C+6)\,\phi_{Bj} + \frac{216}{C}\,w_{1j} && \text{(b)} \\ \frac{-\,q\ell_s}{32k_s} &= C\phi_{Bj} + \left(12 + \frac{B_{1j}}{16}\right) w_{1j} && \text{(c)} \end{aligned}\right\} \qquad (4.5)$$

where $\phi_{xj} = \frac{3\ell_s}{C}\cdot\theta_{xj}$ , $x = A,B$ ; $q = \frac{p\ell_g}{m+1}$ ; and the non-dimensionalised girder-spring stiffness is given by

$$B_{ij} = \frac{k_{ij}}{k_s} \quad . \qquad (4.6)$$

These equations are solved for $\phi_{Aj}$ , $\phi_{Bj}$ and $w_{1j}$ which are then substituted into the moment-slope equation

$$M_{Bj} = k_s\ell_s\,\frac{C^2}{9}\left(2\phi_A + 4\phi_B\right) \qquad (4.7)$$

to give the stiffener end moment, and into Eqn. (4.3) to give the girder-stiffener interaction force.

Defining the quantities $M'_j$ and $R'_{ij}$ (respectively the non-dimensionalised end moment and interaction force) as follows

$$\left.\begin{aligned} M'_j &= \frac{M_{Bj}(m+1)(n+1)^2}{\ell_g\ell_s^2 p} && \text{(a)} \\ R'_{ij} &= \frac{R_{ij}(m+1)(n+1)}{\ell_g\ell_s p} && \text{(b)} \end{aligned}\right\} \qquad (4.8)$$

the results of the above calculations are

$$M'_j = \frac{C}{12} T_j \quad \text{(a)}$$

$$R'_{1j} = \frac{B_{1j}}{96 + \frac{B_{1j}}{2}} \left(\frac{1}{2} + T_j\right) \quad \text{(b)} \qquad (4.9)$$

where $$T_j = \frac{4 + \frac{B_{1j}}{192}}{2 + \frac{B_{1j}}{24} + C\left(1 + \frac{B_{1j}}{192}\right)} \; .$$

For the case of $n = 2$ (that is, a two-girder grillage) five equilibrium equations are solved to obtain

$$M'_j = \frac{C}{12} T_j \quad \text{(a)}$$

$$R'_{1j} = \frac{B_{1j}}{B_{1j} + 162} \left(1 + T_j\right) \quad \text{(b)} \qquad (4.10)$$

where $$T_j = \frac{\frac{B_{1j}}{6} + 243}{C\left(\frac{B_{1j}}{6} + 27\right) + \frac{5}{3} B_{1j} + 54} \; .$$

For the case of $n = 3$ , five equations are solved to yield

$$M'_j = \frac{C}{4} \phi^*_{Bj} \quad \text{(a)}$$

$$R'_{1j} = \frac{B_{1j} w^*_{1j}}{64} \quad \text{(b)} \qquad (4.11)$$

$$R'_{2j} = \frac{B_{2j} w^*_{2j}}{64} \quad \text{(c)}$$

where

$$\phi^*_{Bj} = \frac{16384 - \frac{2}{3} B_{1j} + \frac{64}{3} B_{2j} + \frac{B_{1j}B_{2j}}{288}}{6144 + 128B_{1j} + 128B_{2j} + \frac{7}{48} B_{1j}B_{2j} + \frac{C}{4}\left(12288 + 40B_{1j} + 64B_{2j} + \frac{B_{1j}B_{2j}}{24}\right)}$$

$$w^*_{1j} = \frac{\frac{40}{3} - 16\phi^*_{Bj}\;(C + 11)}{192 - \frac{B_{1j}}{8}}$$

$$w^*_{2j} = \frac{1}{12} + \frac{1}{2}\phi^*_{Bj} + \left(2 + \frac{B_{1j}}{768}\right) w^*_{1j} \;.$$

## 5. USE OF BEAM-ON-SPRING FORMULAE FOR GRILLAGES

Equations (4.9), (4.10) and (4.11) provide a means of evaluating stiffener end moments and girder-stiffener interaction force for a single uniformly loaded stiffener supported by either one, two or three equally spaced girders. The isolated uniformly loaded stiffener considered above can be contrasted with a stiffener in a grillage in which there are other identical uniformly loaded stiffeners on either one or both sides of it. In the latter case, there will be additional interactions between the girders and the stiffeners, and hence the spring stiffness which the girders provide to each of the stiffeners will no longer be equal to the value $k_{ij}$ given by Eqn. (4.4) or the corresponding non-dimensional stiffness parameter $B_{ij}$ of Eqn. (4.6).

However, for any uniformly loaded grillage it is possible to define the dimensionless grillage response parameter

$$Q_{ij} = \frac{R_{ij}}{w_{ij}k_s} \qquad (i = 1\ ,\ \ldots\ ,\ n; \quad j = 1\ ,\ \ldots\ ,\ m) \tag{5.1}$$

which relates the force of interaction at the intersection between the $i^{th}$ girder and $j^{th}$ stiffener to the final deflection at that point.

The n x m matrix $[Q_{ij}]$ is a grillage property and is analogous to $[B_{ij}]$, the dimensionless spring stiffness used in the simple single stiffener case treated above. For this reason $[Q_{ij}]$ will be referred to as the "pseudo-spring" stiffness. The important difference (between $B_{ij}$ and $Q_{ij}$) is that $[Q_{ij}]$ takes into account the interactions caused by the extra stiffeners and their loads. Hence, if $Q_{ij}$ is used in place of $B_{ij}$ in the formulae derived for an isolated single stiffener (that is, Eqns. (4.9), (4.10) and (4.11)) then these formulae can be applied to a stiffener which is part of a grillage. Making this substitution, the corresponding grillage analysis equations are:

for $n = 1$

$$\left.\begin{aligned} M'_j &= \frac{C}{12} T_j && \text{(a)} \\ R'_{1j} &= \frac{Q_{1j}}{96 + \frac{Q_{1j}}{2}} \left(\frac{1}{2} + T_j\right) && \text{(b)} \end{aligned}\right\} \quad (5.2)$$

where

$$T_j = \frac{4 + \frac{Q_{1j}}{192}}{2 + \frac{Q_{1j}}{24} + C\left(1 + \frac{Q_{1j}}{192}\right)} \quad ;$$

for $n = 2$

$$\left.\begin{aligned} M'_j &= \frac{C}{12} T_j && \text{(a)} \\ R'_{1j} &= \frac{Q_{1j}}{Q_{1j} + 162} \left(1 + T_j\right) && \text{(b)} \end{aligned}\right\} \quad (5.3)$$

where

$$T_j = \frac{\frac{Q_{1j}}{6} + 243}{C\left(\frac{Q_{1j}}{6} + 27\right) + \frac{5}{3} Q_{1j} + 54} \quad ;$$

and for $n = 3$

$$
\left.
\begin{aligned}
M'_j &= \frac{C}{4}\,\phi^*_{Bj} && \text{(a)} \\
R'_{1j} &= \frac{Q_{1j} w^*_{1j}}{64} && \text{(b)} \\
R'_{2j} &= \frac{Q_{2j} w^*_{2j}}{64} && \text{(c)}
\end{aligned}
\right\} \qquad (5.4)
$$

where

$$\phi^*_{Bj} = \frac{16384 - \frac{2}{3} Q_{1j} + \frac{64}{3} Q_{2j} + \frac{Q_{1j} Q_{2j}}{288}}{6144 + 128Q_{1j} + 128Q_{2j} + \frac{7}{48} Q_{1j} Q_{2j} + \frac{C}{4}\left(12288 + 40Q_{1j} + 64Q_{2j} + \frac{Q_{1j} Q_{2j}}{24}\right)}$$

$$w_{1j} = \frac{\frac{40}{3} - 16\phi^*_{Bj}\ (C + 11)}{192 - \frac{Q_{1j}}{8}}$$

$$w_{2j} = \frac{1}{12} + \frac{1}{2}\,\phi^*_{Bj} + \left(2 + \frac{Q_{1j}}{768}\right) w^*_{1j} \quad .$$

From what has been said above, it is clear that if a general expression could be developed for the pseudo-spring stiffness [Q] of a grillage then the above formulae could be used for a grillage, thus providing a rapid and simplified method of grillage analysis. Such an expression has been developed and is presented in the next section.

## 6. ESTIMATION OF PSEUDO-SPRING STIFFNESS

Note: The "ij" subscripts on $B_{ij}$, $Q_{ij}$, etc., indicate location within the grillage. For the most part they are not required in this section and hence for simplicity they will be omitted where not required. They should, however, be remembered whenever comparing or referring to other sections.

### 6.1 Independent Variables

The pseudo-spring stiffness $Q$ is to be used in place of the ordinary girder spring stiffness $B$, which applies only to the case of a single stiffener, in order to correct for the interaction which occurs between multiple stiffeners. This interaction will depend on the relative stiffness of the girders and stiffeners, which is measured by $B$, and also on the number of stiffeners and girders ( $m$ and $n$ ), the location under consideration, ( $i$ and $j$ ), and the boundary conditions ( $C$ and $C_g$ ). Also, in the case of three girders there is the relative stiffness of the central and side girders. This will be specified by a parameter $G_f$ which is defined as the ratio of the moment of inertia of the central girder to that of the side girders; that is, $G_f = I_2 / I_1$. For $n < 3$ $G_f$ is arbitrarily assigned the value zero.

The dependency of $Q$ on all of the above variables may be summarised as

$$Q = f(B, C, G_f, i, j, n, m, C_g) \quad . \tag{6.1}$$

Thus, there are a total of eight independent variables. The first three may be termed "continuous" variables because they have an infinite number of possible values. The last five, on the other hand, are "discrete" variables; that is, they have only integer values and these values are within a definite and quite limited range. Therefore, the total number of combinations of these variables is limited, and it is possible to tabulate them specifically. As will be shown subsequently, this tabular approach has been adopted for these variables.

The total practical range of values of the above variables is as follows:

(a) Continuous Variables

$$20 \leq B \leq \infty$$

$$0 \leq C \leq \infty \quad \text{(with special attention to } 0.2 \leq C \leq 20)$$

$$0 \leq G_f \leq 8 .$$

(b) Discrete Variables

$$1 \leq n \leq 3$$

$$i = \begin{cases} 1 & \text{for } n = 1 \text{ and } 2 \\ 1 \text{ or } 2 & \text{for } n = 3 \end{cases}$$

$$3 \leq m \leq 9$$

$$1 \leq j \leq m_h \quad \text{where} \quad m_h = \begin{cases} \frac{m}{2} & \text{for } m \text{ even} \\ \frac{m+1}{2} & \text{for } m \text{ odd} \end{cases}$$

$$C_g = 0 \text{ or } \infty .$$

The five discrete variables may be reduced to three ( $j$ , $\ell$ , $m$ ) by combining $n$ , $i$ and $C_g$ to form the composite subscript $\ell$ as follows

$$\begin{aligned} \ell &= n + n_\ell \quad \text{for } G_f = 0 \quad \text{(that is, for } n < 3) \\ &= i + n_\ell \quad \text{for } G_f \neq 0 \quad \text{(that is, for } n = 3) \end{aligned} \tag{6.2}$$

where

$$n_\ell = 0 \quad \text{for} \quad C_g = 0$$

$$n_\ell = 2 \quad \text{for} \quad C_g = \infty$$

and $i$ is the number of the girder under consideration. For example, for a $7 \times 1$ grillage with a simply supported girder (that is, $n = 1$ , $m = 7$ , $C_g = 0$ , $G_f = 0$ ) the above rule gives $n = 0$ and hence $\ell = n + n_\ell = 1$ . To take another example: for the side girder of a $7 \times 3$ grillage with identical clamped girders (that is, $i = 1$ , $n = 3$ , $m = 7$ , $C_g = \infty$ , $G_f = 1$ ) the rule gives $n_\ell = 2$ and therefore $\ell = i + n_\ell = 3$ .

## 6.2 Limit Value of Q

This section summarises the theoretical basis and the method of approach for the derivation of an expression for the pseudo-spring stiffness $Q$ . We begin by considering the case in which the girders are much heavier than the stiffeners; that is, very large $B$ . In the limiting case, as $B \rightarrow \infty$ , the stiffeners are fully supported at each girder and are independent of each other (as are the girders, also). In this limiting case (which will be denoted by a subscript $L$ ) the stiffener reaction forces along each girder all have the same value, say $R_L$ , regardless of the number of stiffeners. For the case of a single stiffener we may write the (ordinary) girder-spring stiffness from Eqns. (4.3) and (4.6)

$$B = \frac{R_L / w_1}{k_s} \tag{6.3}$$

where $w_1$ is the deflection at the girder-stiffener intersection (that is, at the centre of the girder).

For multiple stiffeners, the corresponding value is the pseudo-spring stiffness $Q_L$ at the centre of the girder. Since the reaction force at the centre is still $R_L$ the value of $Q_L$ is, from Eqn. (5.1)

$$Q_L = \frac{R_L / w_c}{k_s} \tag{6.4}$$

where $w_c$ is the deflection at the centre of the girder. Now, it may be shown that for a beam having m equal and equi-spaced loads

$$w_c = \frac{m + 1}{2} w_1 \quad . \tag{6.5}$$

It therefore follows from Eqns. (6.3), (6.4) and (6.5) that for this limiting case $(B \rightarrow \infty)$ the central value of Q is

$$Q_L = 2\left(\frac{B}{m + 1}\right) \quad . \tag{6.6}$$

For other stiffener locations, the coefficient in Eqn. (6.6) departs from 2 but the general form of the equation continues to hold. That is, the general equation for $Q_L$ (the "limit value" of Q ) is

$$Q_L = L\left(\frac{B}{m + 1}\right) \tag{6.7}$$

in which the coefficient L will be referred to as the "limit coefficient".

Figure 6 shows a log-log plot of Q versus $B / (m + 1)$ for a 7 x 1 grillage with a simply supported girder (that is, $m = 7$ , $n = 1$ , $i = 1$ , $G_f = 0$ and $C_g = 0$ ) . Because of symmetry it is only necessary to plot curves for the first four stiffeners $(j = 1 , \ldots 4)$ . For large values of $B / (m + 1)$ there are only four curves. As $B / (m + 1)$ decreases, these curves split up according to the stiffener end restraint, C . For clarity this low range behaviour is shown only for the outer and central stiffeners, and for values of $C = 1$ and 10 . The curves were obtained from analyses of a series of uniformly loaded 7 x 1 grillages, with Q being calculated from w and R as defined in Eqn. (5.1). Similar plots may be obtained for other grillages, and Figure 7 is such a plot for the first girder of a 7 x 3 grillage with clamped girders in which the central and outer girders are the same size. In this case, curves are plotted for $C = 1$ , 10 and 100 .

The figures show clearly that for large values of $B$ (that is, very stiff girders) the degree of stiffener interaction decreases and the pseudo-spring stiffness of the girder, $Q$, approaches the limit value $Q_L$.

The limit coefficient $L$ depends only on $j$, $m$ and $C_g$ since the girders act independently of each other and of the relatively light stiffeners. It may be shown that for the case of clamped girders ($C_g = \infty$)

$$L = 2(1 - s^2) \qquad (C_g = \infty) \tag{6.8}$$

where

$$s = 1 - \frac{2j}{m + 1} \quad . \tag{6.9}$$

The case of simply supported girders is more complicated and the expression for this case was found by empirical means.

$$\left.\begin{aligned} L &= -1.595\, s^2 + 0.075s + A(m) \qquad (C_g = 0) \\ &\text{where} \\ A(m) &= 0.0034\, m^2 - 0.051\, m + 1.81 \quad . \end{aligned}\right\} \tag{6.10}$$

In order to indicate the dependency of $L$ on $s$ and $C_g$ we can rewrite Eqn. (6.7) as

$$Q_L = L(s, C_g)\left(\frac{B}{m + 1}\right) \quad . \tag{6.11}$$

6.3 <u>Departure of Q from Limit Value</u>

Referring to Figures 6 and 7, it is seen that as $B/(m + 1)$ decreases in magnitude (that is, as the girder strength decreases relative to the strength of the stiffeners) the value of $Q$ departs from the limit value $Q_L$, and the manner of the departure depends on the stiffener end fixity $C$ . In the figures, for clarity, the departure from the asymptotic behaviour is shown only for the central and outer stiffeners (that is, $j = 4$ and $1$ respectively). It will be noted that in a grillage of highly restrained stiffeners (say $C = 10$) the elastic foundation provided to the girders by the stiffeners is stronger than would be the case if the stiffener ends were more lightly restrained (say $C = 1$) and the "departure" from asymptotic behaviour is correspondingly greater. The "departure parameter" $D$ is therefore defined such that

$$Q = Q_L(1 + D) \quad . \tag{6.12}$$

That is, $D$ is a measure of the departure of the value of $Q$ from the limiting value $Q_L$ . It will be noted from Figures 6 and 7 that for the outer stiffeners $D$ is positive, while for more central stiffeners it is negative. This can be explained by the fact that as the stiffeners become heavier relative to the girders (that is, decreasing $B$ ) two opposing factors begin to operate, causing the $Q$ versus $B/(m + 1)$ curve to depart from the asymptote. The first and most obvious factor is that the stronger stiffeners provide a foundation to the girders and made them stiffer, causing the $Q$ curve to rise (that is, positive $D$ ) . The second factor is that, due to the loadings on the other stiffeners, a girder at its intersection with the $j^{th}$ stiffener will have an initial deflection even before the load on the $j^{th}$ stiffener is transmitted to it. This effectively weakens the pseudo-spring, causing the $Q$ curve to droop (that is, negative $D$ ) . Stiffeners near the end support (that is outer stiffeners) are more affected by the first factor and their $Q$ curves rise, whereas for central stiffeners the second factor dominates and the $Q$ curves droop away from the asymptote. In intermediate stiffeners the two factors contend, and in cases where they are nearly balanced there is negligible departure and $Q$ is given by the asymptotic value over the whole range of $B/(m + 1)$ .

In Figure 8 two values of $D_{ij}$ (for $i = 1$ , $j = 1$ and $i = 1$ , $j = 4$ ) are plotted on log-log scales against $B / (m + 1)$ for various values of $C$ for the same case given in Figure 6 (that is, $1 \times 7$ grillages with simply supported girders). In Figure 9 the same parameters are plotted for the case shown in Figure 7 (that is, $3 \times 7$ grillages with clamped girder ends). From these figures it can be seen that for a given $G_f$ , $\ell$ , $m$ , and $j$ the behaviour of the departure parameter $D$ can be represented on log-log scales by families of straight lines for varying values of $C$ . That is, the equation for $D$ is of the form

$$D = D^{*}\left(\frac{B}{m + 1}\right)^{h} \qquad (6.13)$$

where the exponent, $h$ , corresponds to the slope of the lines and the coefficient $D^{*}$ , which depends on $C$ , corresponds to their intercept with the vertical axis (that is, the axis $B / (m + 1) = 1$ ) . For every class of grillages defined by a particular set of the variables ($G_f$ , $\ell$ , $m$ , $j$ ) there is a characteristic coefficient $D^{*}$ .

In the work which is to follow, this set of variables will be included as subscripts on $D^{*}$ and $h$ whenever it is necessary to indicate particular values. Thus, for example, in Figure 8 the uppermost value of $D^{*}$ is written $D^{*}_{0171}$ to indicate $G_f = 0$ , $\ell = 1$ , $m = 7$ , $j = 1$ .

## 6.4 Calculation of Departure Parameter

In order to find the dependency of $D^{*}$ and $h$ on the remaining variables ($G_f$ , $\ell$ , $m$ , $j$ and $C$) these quantities were calculated for a large number of different grillages, using an accurate, computer-based method. For the most part, the method used was the Distributed Reaction Method (D.R.M.) developed by Chang [3].

(a) Variation of h

As can be seen from Figures 8 and 9, the value of h varies only slightly with C , and therefore the value corresponding to $C = 10$ was used throughout. It has subsequently been verified that the error involved in this approximation is negligible.

Since the discrete variables $j$ , $\ell$ and $m$ have only a few specific values, it is a simple matter to calculate h for each individual combination of these three and to list these values of h in a table. Therefore this approach has been adopted rather than attempting to get expressions for h . The variation of h with respect to $G_f$ (which only arises for three-girder grillages) was found to be approximately linear. Therefore, a simple linear interpolation approach was adopted. Values of h were calculated for $G_f = 0$ (that is, $n = 1$ and $2$ ) ; $G_f = 1$ ($n = 3$ and $I_2 = I_1$) ; and $G_f = 4$ ($n = 3$ and $I_2 = 4I_1$ ) . These values are presented in Tables 1a, 2a and 3a . The latter two tables are used for three-girder grillages $(G_f \neq 0)$ and the value of h is obtained by interpolation from the tabulated values.

(b) Variation of $D^*$

Values of $D^*$ are plotted against C on log-log scales in Figures 10 and 11 for the central and outermost stiffeners of the two example grillage types (viz. $n = 1$ , $m = 7$ , $C_g = 0$ ; and $G_f = 1$ , $n = 3$ , $i = 1$ , $m = 7$ , $C_g = \infty$ ) . On these scales the straight line behaviour indicates that $D^*$ varies exponentially with C between the limits $0.2 \le C \le 20$ . Outside of these limits $D^*$ can be considered constant, as the stiffener ends are for all practical purposes either simply supported $(C < 0.2)$ or clamped $(C > 20)$ . Thus the variation of $D^*$ may be expressed as

$$D^* = rW^t \qquad (6.14)$$

$$\left.\begin{array}{llll} \text{where} & W = C & \text{for} & 0.2 \le C \le 20 \\ & W = 20 & \text{for} & C > 20 \\ & W = 0.2 & \text{for} & C < 0.2 \end{array}\right\} \qquad (6.15)$$

Substituting Eqn. (6.14) into Eqn. (6.13)

$$D = rW^t \left(\frac{B_{ij}}{m + 1}\right)^h \quad . \qquad (6.16)$$

The numerical values of $r$ and $t$ have been calculated for all of the combinations of the discrete variables $j$ , $\ell$ and $m$ . As was the case for $h$ , the variation of $r$ and $t$ with $G_f$ was found to be linear and therefore three sets of values of each were calculated, corresponding to $G_f = 0$ , $G_f = 1$ and $G_f = 4$ . These three sets are given in Tables 1b and 1c, 2b and 2c, and 3b and 3c, respectively. Because they are given in a tabular form, the parameters $h$ , $r$ and $t$ will be referred to as "table" parameters.

From the equations presented in this section, the pseudo-spring stiffness $Q$ can be evaluated for any grillage. Since the table parameters are known, the departure parameter $D$ can be calculated from Eqn. (6.16) for any specified value of $C$ , since $W$ comes directly from $C$ by means of Eqn. (6.15). $Q$ can then be calculated from Eqn. (6.12) using the value of $Q_L$ as calculated from Eqn. (6.11). Combining all of these equations gives the general expression for $Q$

$$Q = L(s, C_g)\left[\frac{B}{m + 1}\right]\left\{1 + rW^t\left[\frac{B}{m + 1}\right]^h\right\} \quad . \qquad (6.17)$$

For grillages with one or two girders the table parameters are obtained from Table 1. For three-girder grillages with uniform girders Table 2 is used, whereas if the girders are non-uniform it is necessary to read values from Tables 2 and 3 and interpolate as follows:

$$h = h_2 + \frac{G_f - 1}{3}\left(h_3 - h_2\right)$$

$$r = r_2 + \frac{G_f - 1}{3}\left(r_3 - r_2\right) \qquad (6.18)$$

$$t = t_2 + \frac{G_f - 1}{3}\left(t_3 - t_2\right)$$

in which the subscripts denote the number of the table.

Having obtained the psuedo-spring stiffness $Q_{ij}$ for each stiffener, these values are then substituted into Eqn. (5.2), (5.3) or (5.4) to obtain the dimensionless stiffener end moments $\{M'\}$ and the interaction forces $[R']$ . With the values of $\{M'\}$ and $[R']$ available, each beam of the grillage becomes statically determinate, and further details (such as bending moments at intersections) can be obtained from elementary beam theory formulae. The next section contains two worked examples which serve to illustrate the procedure for using the method and to give an indication of its accuracy and range of applicability.

## 7. WORKED EXAMPLES

### Example 1

The first example is the strength deck of the naval vessel which was illustrated in Figure 1. The structural dimensions are given in Figure 12.

(i) Calculate $B_{ij}$ values using Eqns. (4.4) and (4.6). Hence obtain

$$\left[\frac{B}{m+1}\right] = \begin{bmatrix} 48.0 & 15.2 & 8.81 & 6.75 & 6.22 \end{bmatrix}$$

Note: only the upper left hand portion of the symmetric 2 x 9 matrix need be calculated.

(ii) Calculate the limit coefficient L :

From Eqn. (6.9) for j = 1 , 2 , 3 , 4 and 5 , s = 0.8 , 0.6 , 0.4 , 0.2 and 0 , respectively. Hence from Eqn. (6.10) values of L are 0.675 , 1.105 , 1.406 , 1.580 and 1.626 for j = 1 , 2 , 3 , 4 and 5 , respectively.

(iii) Evaluate the table parameters. Since there are only two girders, Table 1 is to be used. From Eqn. (6.2) it is seen that $\ell = 2$ and since m = 9 (the number of stiffeners) the following values are obtained:

$$\begin{bmatrix} h \end{bmatrix} = \begin{bmatrix} -0.97 & -0.89 & -1.17 & -0.92 & -0.92 \end{bmatrix}$$

$$\begin{bmatrix} r \end{bmatrix} = \begin{bmatrix} 88.00 & 9.64 & -1.06 & -3.87 & -4.69 \end{bmatrix}$$

$$\begin{bmatrix} t \end{bmatrix} = \begin{bmatrix} 0.25 & 0.24 & 0.50 & 0.30 & 0.28 \end{bmatrix} .$$

(iv) Calculate $Q_{ij}$ by substituting the values of L , [B / m + 1] , [h] , [r] and [t] into Eqn. (6.17)

$$\begin{bmatrix} Q_{ij} \end{bmatrix} = \begin{bmatrix} 173 & 46.3 & 7.79 & -6.8 & -10.3 \end{bmatrix} .$$

(v) Noting from Figure 12 and Eqn. (3.3) that C = 20 and substituting C and [Q] into Eqn. (5.3) calculate the dimensionless stiffener end moments

$$\{M'\} = (-0.310 , \quad -0.506 , \quad -0.643 , \quad -0.720 , \quad -0.741) \quad ;$$

and the dimensionless girder-stiffener interaction forces

$$\begin{bmatrix} R' \end{bmatrix} = \begin{bmatrix} 0.613 & 0.289 & 0.636 & -0.063 & -0.098 \end{bmatrix} .$$

(vi) These results are dimensionalised using Eqn. (4.8) to yield the matrix of girder-stiffener interaction forces [R] and the vector of stiffener end moments. The latter are (in kilo Newton metres)

$$\left\{M_B\right\} = \left\{-483 \ , \quad -792 \ , \quad -1004 \ , \quad -1124 \ , \quad -1156\right\} \quad .$$

This result compares very favourably with the exact result obtained by using a rigorous frame analysis method

$$\left\{M_B\right\}_{exact} = \left\{-468 \ , \quad -781 \ , \quad -1002 \ , \quad -1124 \ , \quad -1158\right\} \quad .$$

(vii) Using [R] and $\left\{M_B\right\}$ , analyse the stiffeners and girders independently using beam theory, assuming that the entire load is borne directly by the stiffeners and transferred to the girders via the interaction forces [R] . Figure 13 is a plot of the central stiffener bending moment $M_{s5}(y)$ and the exact result obtained using the frame analysis method. The two results are in exact agreement. Figure 14 gives the bending moment in the girders. Once again, there is close agreement with the exact result.

Example 2

The second example has been chosen to provide a severe test for the method. The 7 x 3 grillage shown in Figure 15 has low values of $B_{ij}$ and hence the values of $Q_{ij}$ depart quite markedly from the limit value $Q_L$ . Also, both girder and stiffener ends have been made clamped to further increase the departure from $Q_L$ . The procedure is the same as for the first example except that Eqn. (6.8) is used in place of Eqn. (6.10), and Tables 2 and 3 are used (instead of Table 1) together with Eqn. (6.18) since this is a non-uniform three-girder grillage $(G_f = 2.5)$ . Values of the central stiffener bending moment and the girder bending moments are plotted in Figures 16 and 17. It is evident that in spite of the severe choice of grillage properties, the results of the GRIDFORM method agree well with the exact results.

| | | | j = 1 | j = 2 | j = 3 | j = 4 | j = 5 |
|---|---|---|---|---|---|---|---|
| | | 3 | -0.98 | -0.98 | | | |
| | | 4 | -0.96 | -0.93 | | | |
| | | 5 | -0.93 | -0.99 | -0.96 | | |
| ℓ = 1 | m | 6 | -0.90 | -0.52 | -0.94 | | |
| | | 7 | -0.89 | -0.69 | -0.94 | -0.92 | |
| | | 8 | -0.91 | -0.79 | -0.98 | -0.90 | |
| | | 9 | -0.95 | -0.83 | -1.19 | -0.89 | -0.88 |
| | | 3 | -0.98 | -0.97 | | | |
| | | 4 | -0.96 | -0.92 | | | |
| | | 5 | -0.93 | -0.97 | -0.97 | | |
| ℓ = 2 | m | 6 | -0.91 | -0.66 | -0.96 | | |
| | | 7 | -0.93 | -0.85 | -0.97 | -0.97 | |
| | | 8 | -0.94 | -0.88 | -0.99 | -0.94 | |
| | | 9 | -0.97 | -0.89 | -1.17 | -0.92 | -0.92 |
| | | 3 | -0.94 | -1.00 | | | |
| | | 4 | -0.95 | -1.01 | | | |
| | | 5 | -0.93 | -1.21 | -0.98 | | |
| ℓ = 3 | m | 6 | -0.92 | -0.83 | -0.97 | | |
| | | 7 | -0.89 | -0.84 | -0.97 | -0.94 | |
| | | 8 | -0.88 | -0.86 | -1.22 | -0.93 | |
| | | 9 | -0.87 | -0.86 | -0.59 | -0.94 | -0.93 |
| | | 3 | -0.97 | -1.08 | | | |
| | | 4 | -0.95 | -1.12 | | | |
| | | 5 | -0.95 | -1.18 | -0.99 | | |
| ℓ = 4 | m | 6 | -0.94 | -0.93 | -1.00 | | |
| | | 7 | -0.93 | -0.92 | -1.00 | -0.98 | |
| | | 8 | -0.92 | -0.92 | -1.19 | -0.98 | |
| | | 9 | -0.91 | -0.96 | -0.62 | -0.97 | -0.97 |

Table 1a Values of h for $G_f = 0$

| | | | j | | | | |
|---|---|---|---|---|---|---|---|
| | | | 1 | 2 | 3 | 4 | 5 |
| | | 3 | 7.21 | -5.91 | | | |
| | | 4 | 17.68 | -4.46 | | | |
| | | 5 | 28.42 | -2.98 | -6.67 | | |
| ℓ = 1 | m | 6 | 41.11 | 0.30 | -6.04 | | |
| | | 7 | 54.79 | 2.87 | -4.76 | -6.24 | |
| | | 8 | 74.64 | 6.57 | -3.46 | -5.84 | |
| | | 9 | 101.40 | 10.90 | -2.05 | -5.13 | -5.98 |
| | | 3 | 5.60 | -1.30 | | | |
| | | 4 | 11.50 | -2.34 | | | |
| | | 5 | 18.61 | -1.83 | -4.73 | | |
| ℓ = 2 | m | 6 | 25.27 | 0.20 | -4.29 | | |
| | | 7 | 43.16 | 2.75 | -3.60 | -5.00 | |
| | | 8 | 62.14 | 5.96 | -2.42 | -4.56 | |
| | | 9 | 88.00 | 9.64 | -1.06 | -3.87 | -4.69 |
| | | 3 | 13.00 | -1.30 | | | |
| | | 4 | 32.30 | -1.50 | | | |
| | | 5 | 56.60 | -2.42 | -6.64 | | |
| ℓ = 3 | m | 6 | 86.70 | 3.27 | -5.74 | | |
| | | 7 | 113.30 | 8.35 | -3.94 | -5.94 | |
| | | 8 | 151.50 | 15.44 | -2.30 | -5.47 | |
| | | 9 | 195.60 | 23.20 | 0.00 | -4.55 | -5.70 |
| | | 3 | 0.00 | 0.00 | | | |
| | | 4 | 0.00 | 0.00 | | | |
| | | 5 | 41.30 | 0.00 | 0.00 | | |
| ℓ = 4 | m | 6 | 65.60 | 0.00 | 0.00 | | |
| | | 7 | 95.90 | 7.54 | -2.90 | -4.60 | |
| | | 8 | 130.80 | 13.40 | -1.37 | -4.26 | |
| | | 9 | 165.30 | 20.10 | 0.00 | -3.46 | -4.44 |

Table 1b  Values of r for $G_f = 0$

| | | | j | | | | |
|---|---|---|---|---|---|---|---|
| | | | 1 | 2 | 3 | 4 | 5 |
| | | 3 | 0.28 | 0.29 | | | |
| | | 4 | 0.29 | 0.31 | | | |
| | | 5 | 0.27 | 0.30 | 0.27 | | |
| $\ell = 1$ | m | 6 | 0.23 | 0.15 | 0.26 | | |
| | | 7 | 0.19 | 0.12 | 0.24 | 0.21 | |
| | | 8 | 0.20 | 0.10 | 0.28 | 0.21 | |
| | | 9 | 0.20 | 0.12 | 0.41 | 0.22 | 0.20 |
| | | 3 | 0.31 | 0.35 | | | |
| | | 4 | 0.35 | 0.24 | | | |
| | | 5 | 0.21 | 0.38 | 0.33 | | |
| $\ell = 2$ | m | 6 | 0.36 | 0.31 | 0.34 | | |
| | | 7 | 0.31 | 0.26 | 0.32 | 0.31 | |
| | | 8 | 0.29 | 0.26 | 0.34 | 0.31 | |
| | | 9 | 0.25 | 0.24 | 0.50 | 0.30 | 0.28 |
| | | 3 | 0.27 | 0.30 | | | |
| | | 4 | 0.28 | 0.33 | | | |
| | | 5 | 0.25 | 0.36 | 0.27 | | |
| $\ell = 3$ | m | 6 | 0.11 | 0.14 | 0.26 | | |
| | | 7 | 0.24 | 0.14 | 0.25 | 0.23 | |
| | | 8 | 0.18 | 0.14 | 0.39 | 0.23 | |
| | | 9 | 0.16 | 0.16 | 0.32 | 0.23 | 0.22 |
| | | 3 | 0.35 | 0.31 | | | |
| | | 4 | 0.33 | 0.31 | | | |
| | | 5 | 0.33 | 0.30 | 0.34 | | |
| $\ell = 4$ | m | 6 | 0.33 | 0.33 | 0.30 | | |
| | | 7 | 0.29 | 0.27 | 0.33 | 0.32 | |
| | | 8 | 0.28 | 0.27 | 0.42 | 0.31 | |
| | | 9 | 0.25 | 0.24 | 0.40 | 0.30 | 0.30 |

Table 1c Values of $t$ for $G_f = 0$

| | | | j | | | | |
|---|---|---|---|---|---|---|---|
| | | | 1 | 2 | 3 | 4 | 5 |
| | | 3 | -0.59 | -0.59 | | | |
| | | 4 | -0.60 | -0.59 | | | |
| | | 5 | -0.60 | -0.80 | -0.65 | | |
| ℓ = 1 | m | 6 | -0.61 | 0.80 | -0.66 | | |
| | | 7 | -0.66 | -0.60 | -0.77 | -0.71 | |
| | | 8 | -0.70 | -0.65 | -0.84 | -0.71 | |
| | | 9 | -0.74 | -0.66 | -1.00 | -0.72 | -0.70 |
| | | 3 | -1.67 | -1.70 | | | |
| | | 4 | -1.41 | -1.30 | | | |
| | | 5 | -1.28 | -1.30 | -1.31 | | |
| ℓ = 2 | m | 6 | -1.21 | -1.22 | -1.24 | | |
| | | 7 | -1.18 | -1.08 | -1.22 | -1.21 | |
| | | 8 | -1.17 | -1.10 | -1.22 | -1.16 | |
| | | 9 | -1.18 | -1.11 | -1.30 | -1.12 | -1.13 |
| | | 3 | -0.56 | -0.60 | | | |
| | | 4 | -0.54 | -0.54 | | | |
| | | 5 | -0.56 | -0.75 | -0.51 | | |
| ℓ = 3 | m | 6 | -0.57 | -0.40 | -0.53 | | |
| | | 7 | -0.61 | -0.54 | -0.70 | -0.60 | |
| | | 8 | -0.63 | -0.57 | -0.86 | -0.62 | |
| | | 9 | -0.64 | -0.59 | -0.39 | -0.70 | -0.64 |
| | | 3 | -1.41 | -1.58 | | | |
| | | 4 | -1.35 | -1.49 | | | |
| | | 5 | -1.29 | -1.20 | -1.28 | | |
| ℓ = 4 | m | 6 | -1.23 | -1.06 | -1.23 | | |
| | | 7 | -1.19 | -1.09 | -1.28 | -1.16 | |
| | | 8 | -0.87 | -0.76 | -1.93 | -0.84 | |
| | | 9 | -0.85 | -0.76 | -0.89 | -0.89 | -0.80 |

Table 2a    Values of h for $G_f = 1$

| | | j | | | | |
|---|---|---|---|---|---|---|
| | | 1 | 2 | 3 | 4 | 5 |
| | 3 | 0.56 | -0.74 | | | |
| | 4 | 1.70 | -0.78 | | | |
| | 5 | 3.06 | -0.86 | -1.30 | | |
| ℓ = 1 | m 6 | 4.50 | 0.00 | -1.34 | | |
| | 7 | 12.00 | 0.97 | -2.00 | -2.20 | |
| | 8 | 17.90 | 1.20 | -1.70 | -2.50 | |
| | 9 | 28.60 | 4.10 | -1.00 | -2.40 | -2.70 |
| | 3 | 34.10 | -14.20 | | | |
| | 4 | 36.70 | -5.50 | | | |
| | 5 | 40.00 | -2.30 | -6.30 | | |
| ℓ = 2 | m 6 | 51.00 | 0.00 | -4.85 | | |
| | 7 | 64.20 | 4.10 | -3.70 | -5.20 | |
| | 8 | 82.30 | 7.90 | -2.10 | -4.50 | |
| | 9 | 105.00 | 11.90 | -0.25 | -3.80 | -4.70 |
| | 3 | 1.46 | -1.40 | | | |
| | 4 | 2.00 | -0.60 | | | |
| | 5 | 6.30 | -0.32 | -0.71 | | |
| ℓ = 3 | m 6 | 6.10 | 0.13 | -0.45 | | |
| | 7 | 13.40 | 0.91 | -0.69 | -0.97 | |
| | 8 | 22.10 | 2.00 | -1.05 | -1.58 | |
| | 9 | 37.80 | 4.90 | 0.13 | -1.87 | -2.00 |
| | 3 | 43.00 | -5.00 | | | |
| | 4 | 66.50 | -5.20 | | | |
| | 5 | 95.50 | 0.00 | -5.70 | | |
| ℓ = 4 | m 6 | 129.00 | 4.10 | -4.60 | | |
| | 7 | 164.00 | 10.20 | -3.00 | -4.70 | |
| | 8 | 63.50 | 7.10 | -1.60 | -2.30 | |
| | 9 | 75.00 | 9.60 | 0.73 | -2.00 | -2.70 |

Table 2b Values of r for $G_f = 1$

| | | | j | | | | |
|---|---|---|---|---|---|---|---|
| | | | 1 | 2 | 3 | 4 | 5 |
| | | 3 | 0.56 | 0.47 | | | |
| | | 4 | 0.49 | 0.42 | | | |
| | | 5 | 0.54 | 0.50 | 0.43 | | |
| $\ell$ = 1 | m | 6 | 0.53 | 0.38 | 0.42 | | |
| | | 7 | 0.44 | 0.26 | 0.35 | 0.38 | |
| | | 8 | 0.43 | 0.65 | 0.36 | 0.32 | |
| | | 9 | 0.39 | 0.29 | 0.46 | 0.32 | 0.32 |
| | | 3 | 0.30 | 0.31 | | | |
| | | 4 | 0.32 | 0.32 | | | |
| | | 5 | 0.33 | 0.34 | 0.35 | | |
| $\ell$ = 2 | m | 6 | 0.35 | 0.35 | 0.35 | | |
| | | 7 | 0.35 | 0.30 | 0.34 | 0.35 | |
| | | 8 | 0.34 | 0.30 | 0.40 | 0.34 | |
| | | 9 | 0.34 | 0.32 | 0.70 | 0.32 | 0.32 |
| | | 3 | 0.30 | 0.29 | | | |
| | | 4 | 0.47 | 0.37 | | | |
| | | 5 | 0.28 | 0.54 | 0.51 | | |
| $\ell$ = 3 | m | 6 | 0.51 | 0.55 | 0.53 | | |
| | | 7 | 0.35 | 0.44 | 0.49 | 0.43 | |
| | | 8 | 0.35 | 0.39 | 0.37 | 0.39 | |
| | | 9 | 0.25 | 0.22 | 0.57 | 0.26 | 0.30 |
| | | 3 | 0.38 | 0.71 | | | |
| | | 4 | 0.46 | 0.64 | | | |
| | | 5 | 0.48 | 0.45 | 0.46 | | |
| $\ell$ = 4 | m | 6 | 0.46 | 0.31 | 0.41 | | |
| | | 7 | 0.45 | 0.32 | 0.46 | 0.38 | |
| | | 8 | 0.23 | 0.15 | 0.49 | 0.23 | |
| | | 9 | 0.23 | 0.18 | 0.57 | 0.25 | 0.18 |

Table 2c Values of $t$ for $G_f = 1$

| | | j = 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| ℓ = 1 | m 3 | -0.59 | -0.66 | | | |
| | 4 | -0.64 | -0.59 | | | |
| | 5 | -0.66 | -0.54 | -0.61 | | |
| | 6 | -0.64 | 0.60 | -0.58 | | |
| | 7 | -0.55 | -0.50 | -0.49 | -0.43 | |
| | 8 | -0.59 | -0.30 | -0.55 | -0.48 | |
| | 9 | -0.62 | -0.40 | -0.80 | -0.48 | -0.45 |
| ℓ = 2 | m 3 | -1.80 | -1.88 | | | |
| | 4 | -1.61 | -1.55 | | | |
| | 5 | -1.48 | -1.35 | -1.40 | | |
| | 6 | -1.40 | -1.32 | -1.32 | | |
| | 7 | -1.36 | -1.11 | -1.34 | -1.26 | |
| | 8 | -1.33 | -1.14 | -1.30 | -1.20 | |
| | 9 | -1.33 | -1.16 | -1.32 | -1.18 | -1.15 |
| ℓ = 3 | m 3 | -0.76 | -0.83 | | | |
| | 4 | -0.80 | -0.80 | | | |
| | 5 | -0.80 | -0.57 | -0.77 | | |
| | 6 | -0.80 | -0.54 | -0.76 | | |
| | 7 | -0.60 | -0.35 | -0.50 | -0.46 | |
| | 8 | -0.59 | -0.44 | -0.72 | -0.44 | |
| | 9 | -0.58 | -0.46 | -1.90 | -0.42 | -0.40 |
| ℓ = 4 | m 3 | -1.41 | -1.58 | | | |
| | 4 | -1.35 | -1.49 | | | |
| | 5 | -1.29 | -1.20 | -1.28 | | |
| | 6 | -1.23 | -1.06 | -1.23 | | |
| | 7 | -1.19 | -1.09 | -1.28 | -1.16 | |
| | 8 | -0.87 | -0.76 | -1.93 | -0.84 | |
| | 9 | -0.85 | -0.76 | -0.89 | -0.89 | -0.80 |

Table 3a    Values of h for $G_f = 4$

| | | | j | | | | |
|---|---|---|---|---|---|---|---|
| | | | 1 | 2 | 3 | 4 | 5 |
| | | 3 | 3.40 | -4.90 | | | |
| | | 4 | 9.80 | -2.90 | | | |
| | | 5 | 14.40 | -1.00 | -3.80 | | |
| $\ell = 1$ | m | 6 | 19.10 | 0.00 | -3.90 | | |
| | | 7 | 10.30 | 0.00 | -1.10 | -1.10 | |
| | | 8 | 15.20 | 0.40 | -1.10 | -1.20 | |
| | | 9 | 19.90 | 0.90 | -1.40 | -1.00 | -1.10 |
| | | 3 | 99.00 | -15.00 | | | |
| | | 4 | 117.00 | -9.70 | | | |
| | | 5 | 118.00 | -5.00 | -12.50 | | |
| $\ell = 2$ | m | 6 | 138.00 | 0.00 | -9.80 | | |
| | | 7 | 174.00 | 9.40 | -7.30 | -10.10 | |
| | | 8 | 200.00 | 17.00 | -3.70 | -8.70 | |
| | | 9 | 253.00 | 25.10 | -3.20 | -6.60 | -8.40 |
| | | 3 | 18.90 | -24.00 | | | |
| | | 4 | 52.90 | -8.80 | | | |
| | | 5 | 92.20 | -0.50 | -11.00 | | |
| $\ell = 3$ | m | 6 | 148.00 | 1.20 | -8.10 | | |
| | | 7 | 30.20 | 0.82 | -1.00 | -1.40 | |
| | | 8 | 37.70 | 1.80 | -1.40 | -1.00 | |
| | | 9 | 35.80 | 2.60 | -11.80 | -0.70 | -0.71 |
| | | 3 | 11.10 | -1.25 | | | |
| | | 4 | 15.80 | -1.30 | | | |
| | | 5 | 230.20 | 0.00 | -1.40 | | |
| $\ell = 4$ | m | 6 | 34.80 | 1.10 | -1.20 | | |
| | | 7 | 42.70 | 2.50 | -0.75 | -1.20 | |
| | | 8 | 17.10 | 1.75 | -0.40 | -0.53 | |
| | | 9 | 18.30 | 2.40 | 0.20 | -0.50 | -0.56 |

Table 3b Values of r for $G_f = 4$

| | | | j | | | | |
|---|---|---|---|---|---|---|---|
| | | | 1 | 2 | 3 | 4 | 5 |
| | | 3 | 0.21 | 0.23 | | | |
| | | 4 | 0.27 | 0.25 | | | |
| | | 5 | 0.31 | 0.33 | 0.25 | | |
| ℓ = 1 | m | 6 | 0.27 | 0.28 | 0.27 | | |
| | | 7 | 0.33 | 0.34 | 0.34 | 0.24 | |
| | | 8 | 0.34 | 0.24 | 0.35 | 0.30 | |
| | | 9 | 0.35 | 0.36 | 0.39 | 0.33 | 0.30 |
| | | 3 | 0.20 | 0.30 | | | |
| | | 4 | 0.47 | 0.60 | | | |
| | | 5 | 0.49 | 0.35 | 0.45 | | |
| ℓ = 2 | m | 6 | 0.48 | 0.35 | 0.41 | | |
| | | 7 | 0.45 | 0.30 | 0.43 | 0.38 | |
| | | 8 | 0.46 | 0.32 | 0.48 | 0.36 | |
| | | 9 | 0.45 | 0.34 | 0.04 | 0.38 | 0.34 |
| | | 3 | 0.20 | 0.08 | | | |
| | | 4 | 0.26 | 0.32 | | | |
| | | 5 | 0.30 | 0.27 | 0.27 | | |
| ℓ = 3 | m | 6 | 0.27 | 0.37 | 0.28 | | |
| | | 7 | 0.23 | 0.09 | 0.29 | 0.20 | |
| | | 8 | 0.20 | 0.26 | 0.28 | 0.25 | |
| | | 9 | 0.30 | 0.33 | 0.73 | 0.34 | 0.34 |
| | | 3 | 0.20 | 0.30 | | | |
| | | 4 | 0.47 | 0.60 | | | |
| | | 5 | 0.49 | 0.35 | 0.45 | | |
| ℓ = 4 | m | 6 | 0.48 | 0.35 | 0.41 | | |
| | | 7 | 0.45 | 0.30 | 0.43 | 0.38 | |
| | | 8 | 0.46 | 0.32 | 0.48 | 0.36 | |
| | | 9 | 0.45 | 0.34 | 0.04 | 0.38 | 0.34 |

Table 3c    Values of $t$ for $G_f = 4$

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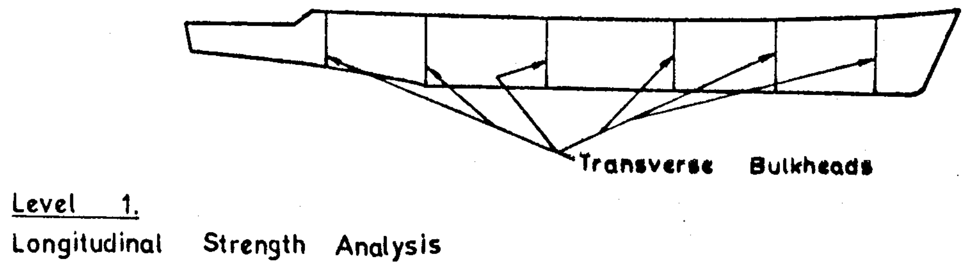
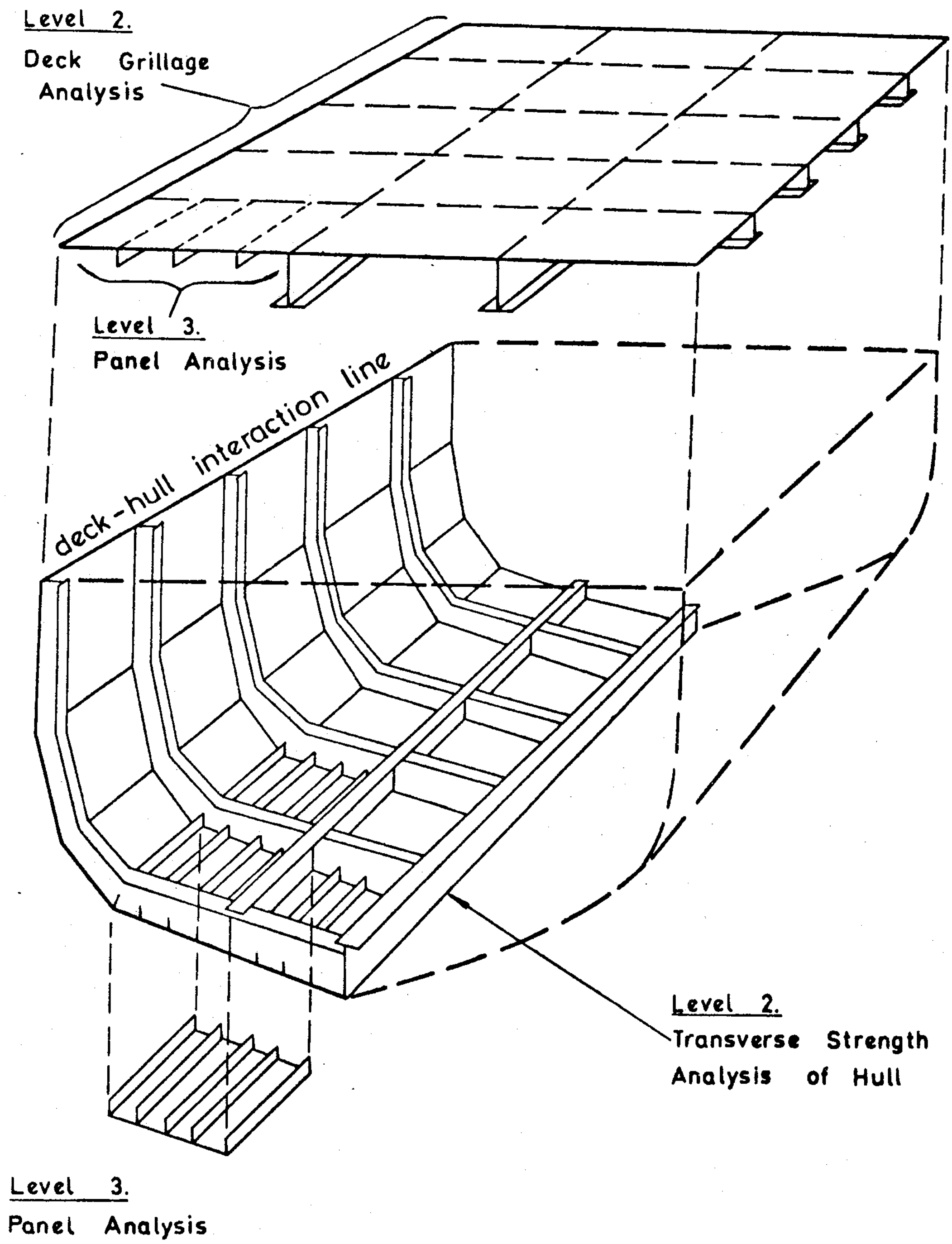
3. Chang, P.Y. "A Simple Method for Elastic Analysis of Grillages". Journal of Ship Research, June 1968.

4. Schade, H.A. "Design Curves for Cross-Stiffened Plating under Uniform Bending Load". Trans. S.N.A.M.E., Vol. 49, 1941, p.154.




(a) The hull as a series of interbulkhead substructures




(b) An interbulkhead substructure (deck raised)

FIG. 1. LONGITUDINALLY FRAMED HULL AND DECK STRUCTURE

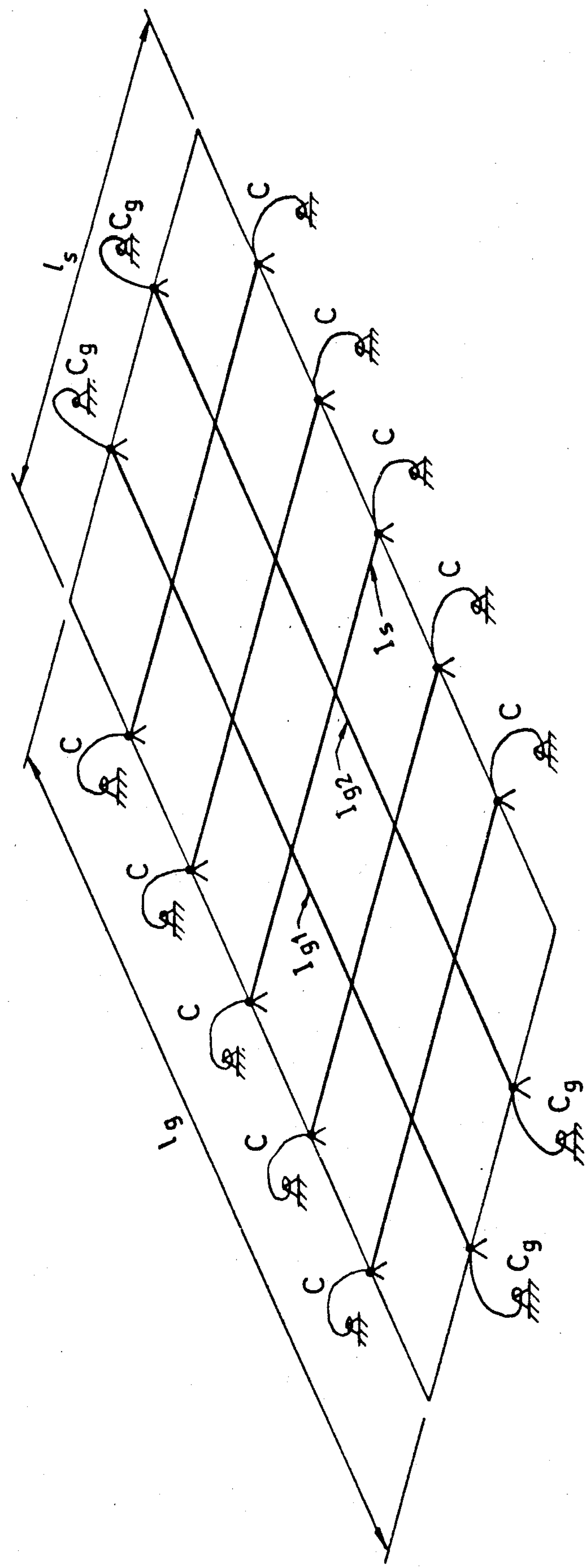



FIG. 2. BOUNDARY CONDITIONS FOR AN ISOLATED GRILLAGE

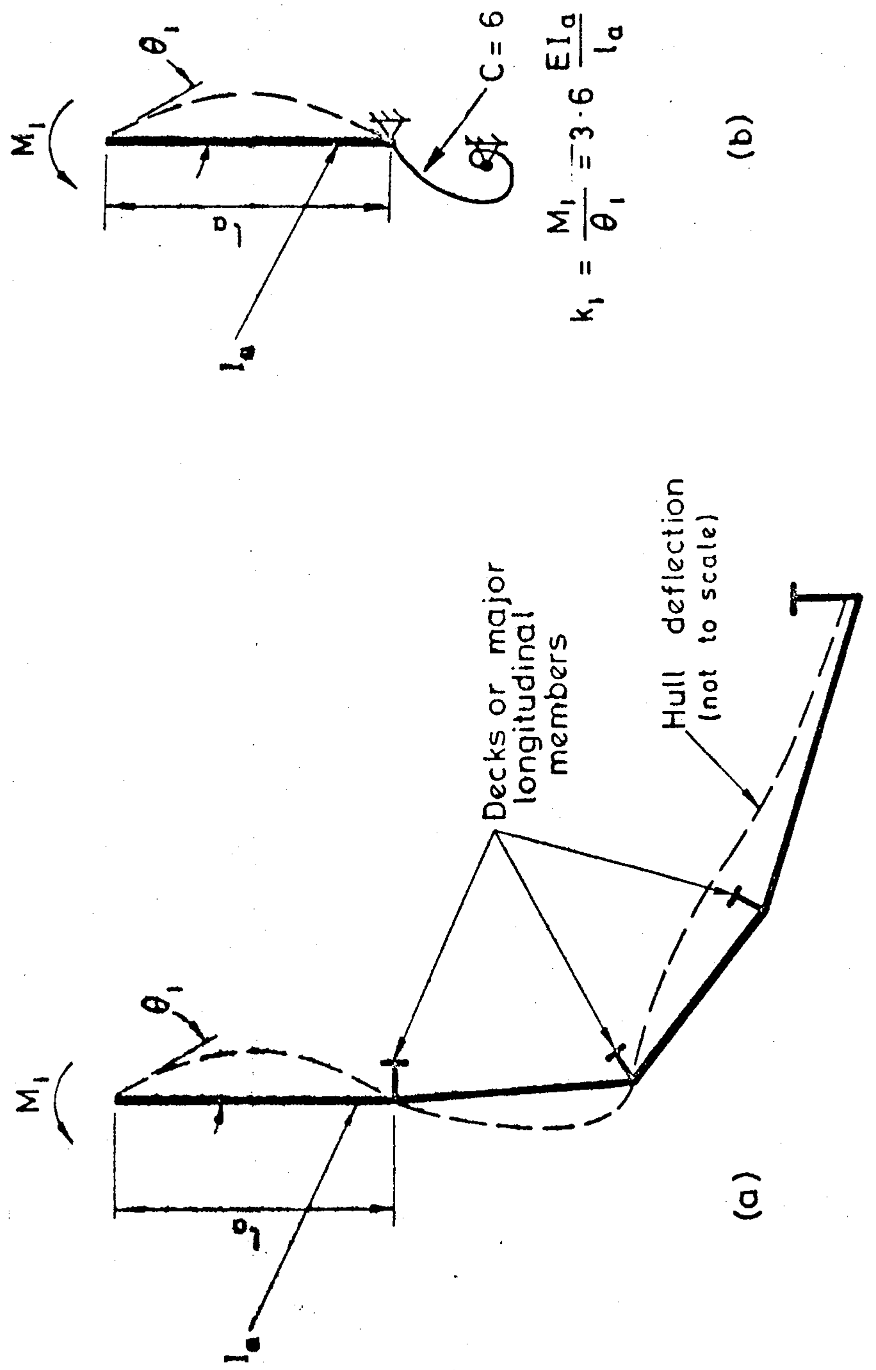
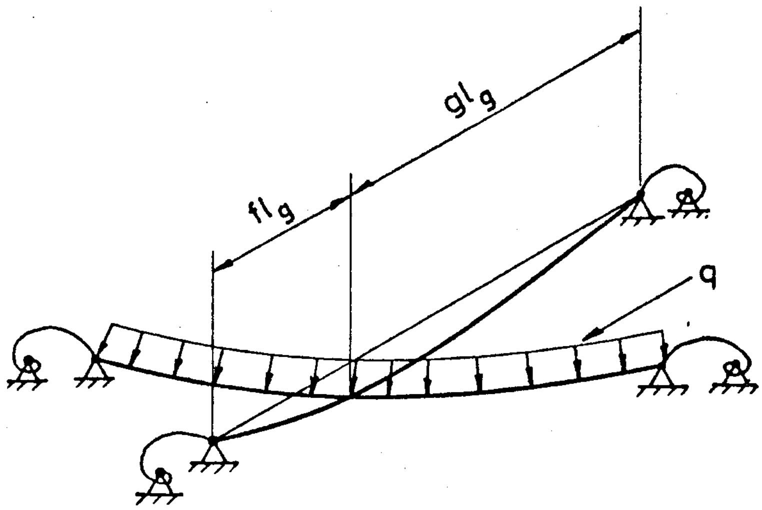
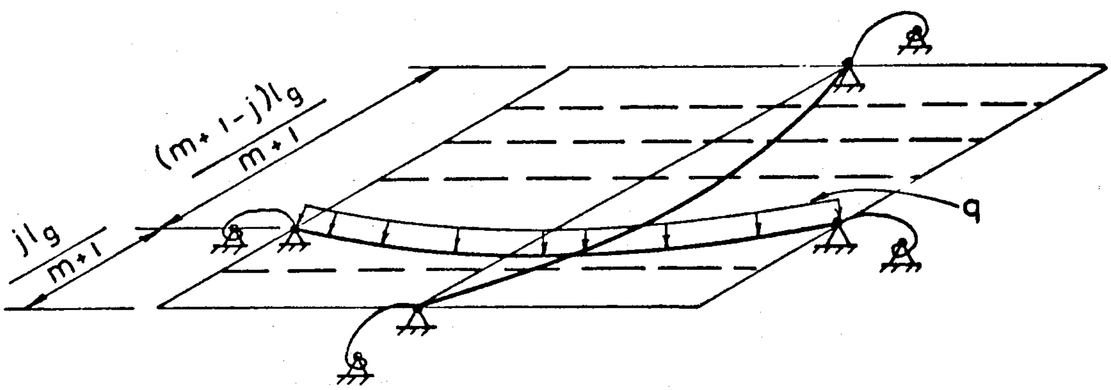


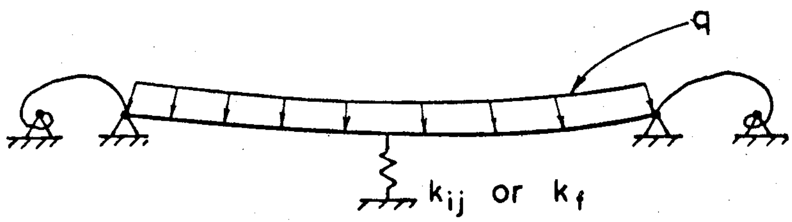

FIG. 3. HULL DEFLECTION DUE TO MOMENT LOAD




(a) Single stiffener supported by one girder

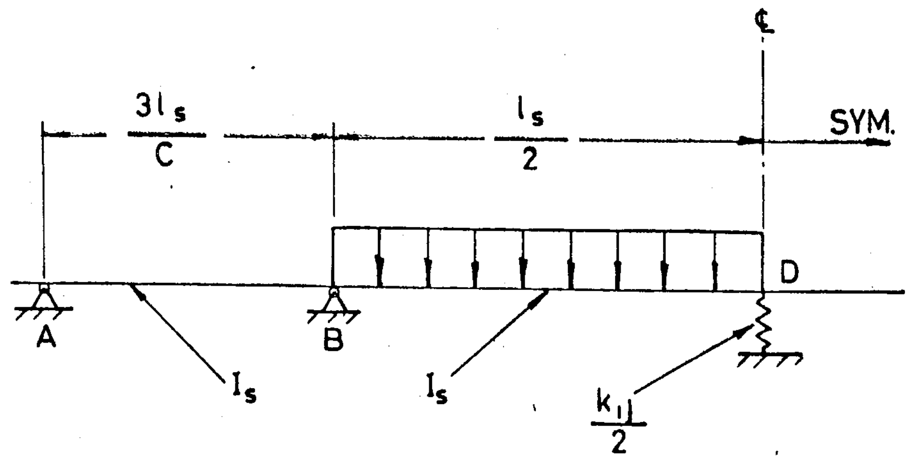



(b) j'th stiffener alone, supported by one girder

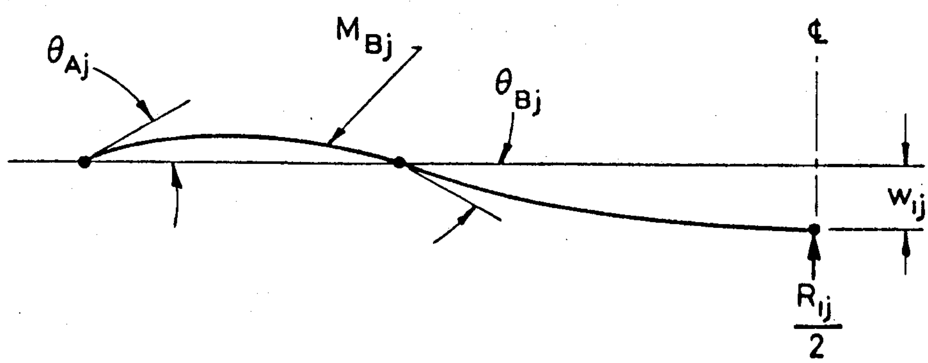



(c) Stiffener supported by equivalent spring

FIG. 4. STIFFENER ON GIRDER SUPPORT




(a) Beam on Spring Model




(b) The unknowns for Eqn. (4.2.5)

FIG. 5. MODEL FOR A BEAM ON A SPRING SUPPORT, CORRESPONDING TO A 1 – GIRDER GRILLAGE

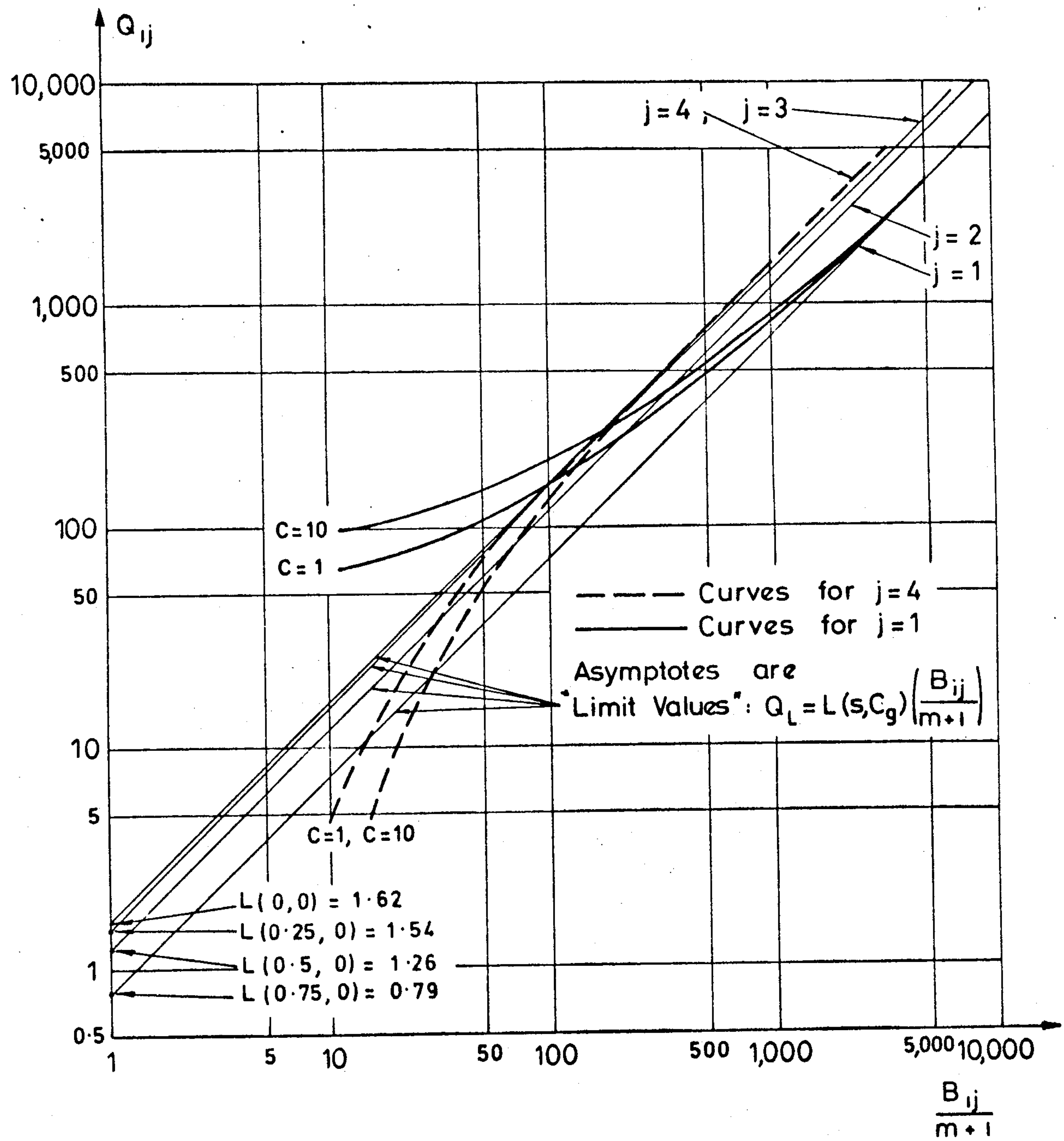



FIG. 6. VARIATION IN PSEUDO-SPRING STIFFNESS

n = 1, m = 7, G = 0

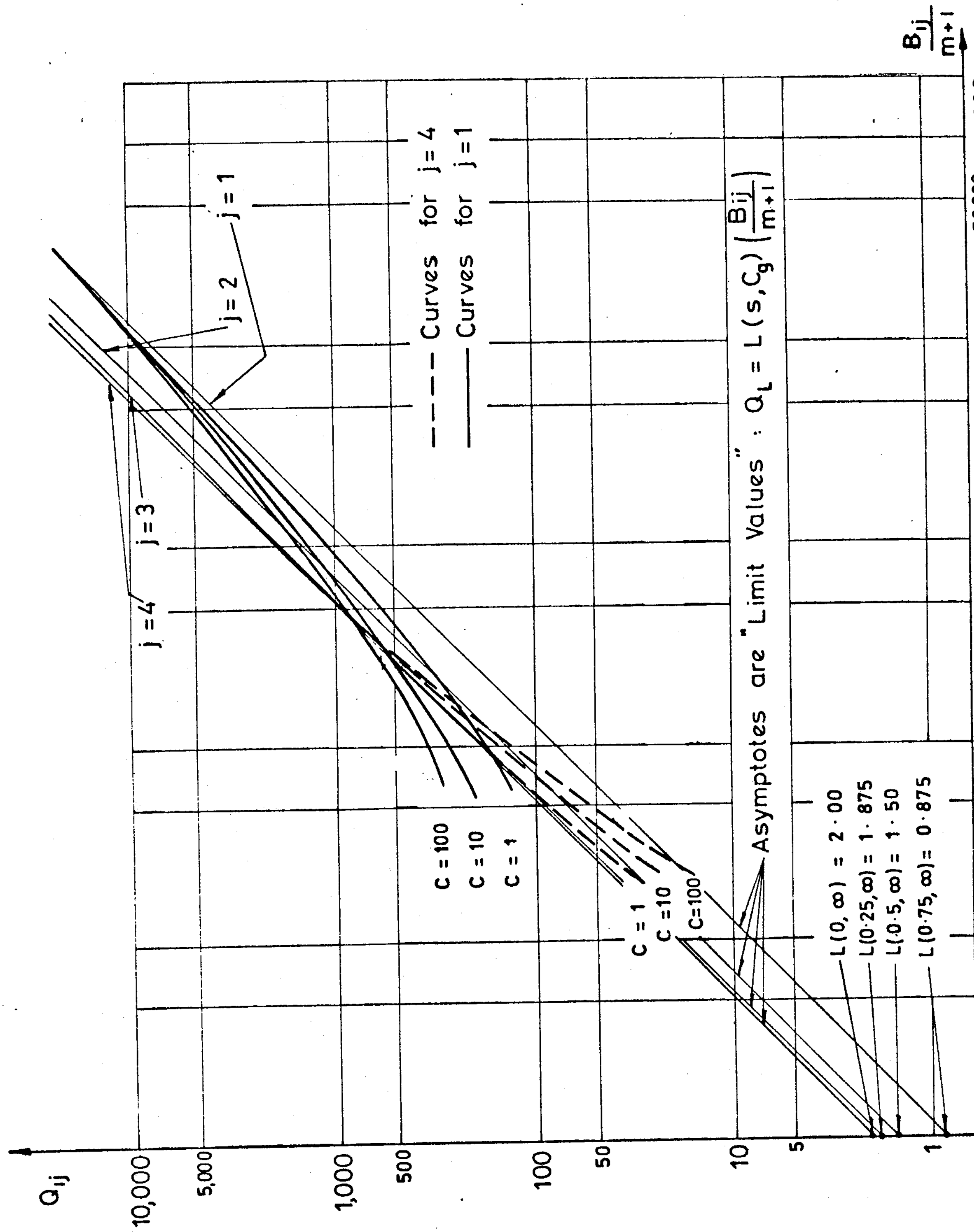



FIG. 7. VARIATION IN PSEUDO-SPRING STIFFNESS

$n = 3$, $i = 1$, $m = 7$, $C_g = \infty$

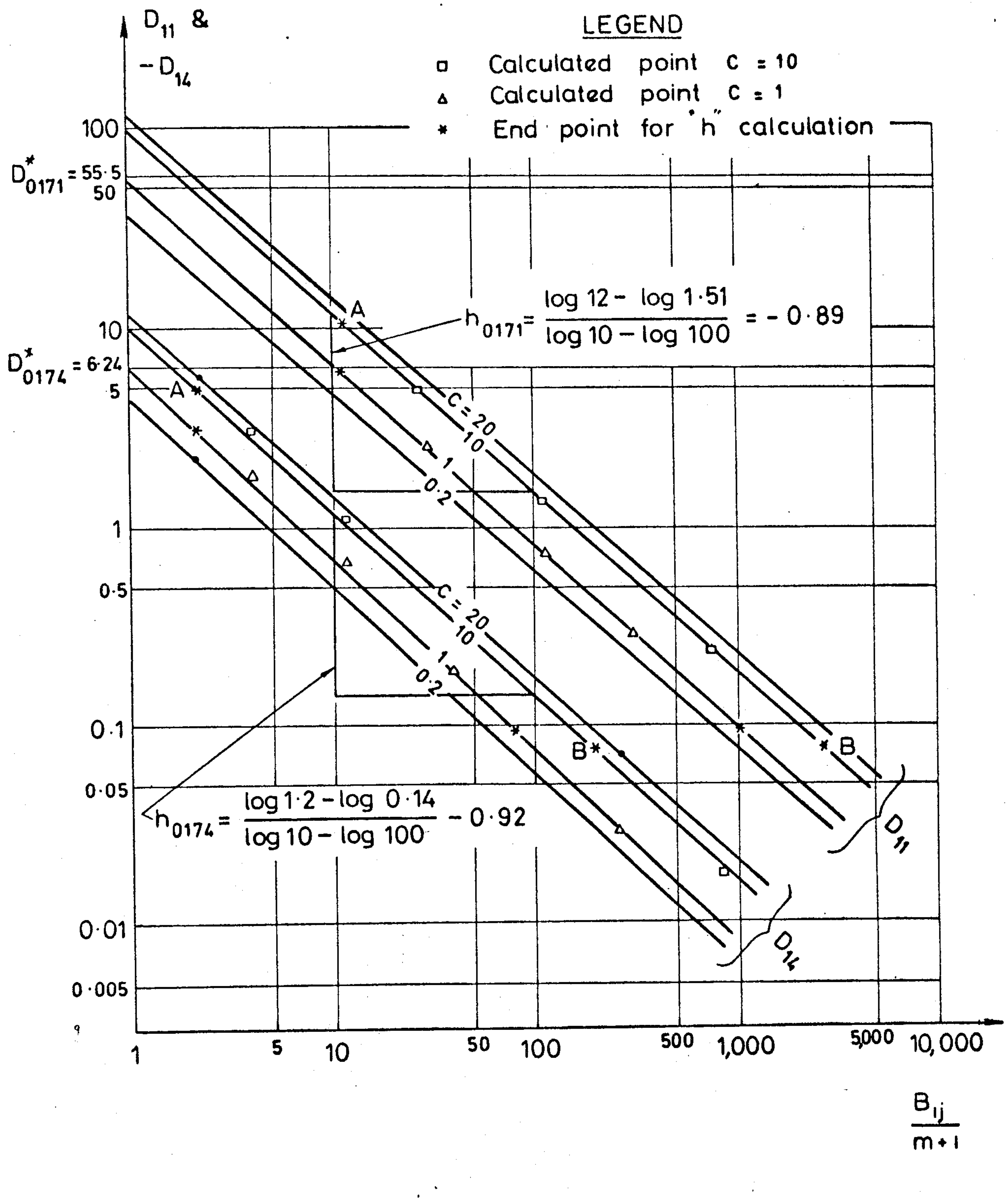



FIG. 8. DEPARTURE PARAMETER vs $\frac{B_{ij}}{m+1}$ FOR $j = 1$ & $4$, $n = 1$, $m = 7$, $C_g = 0$

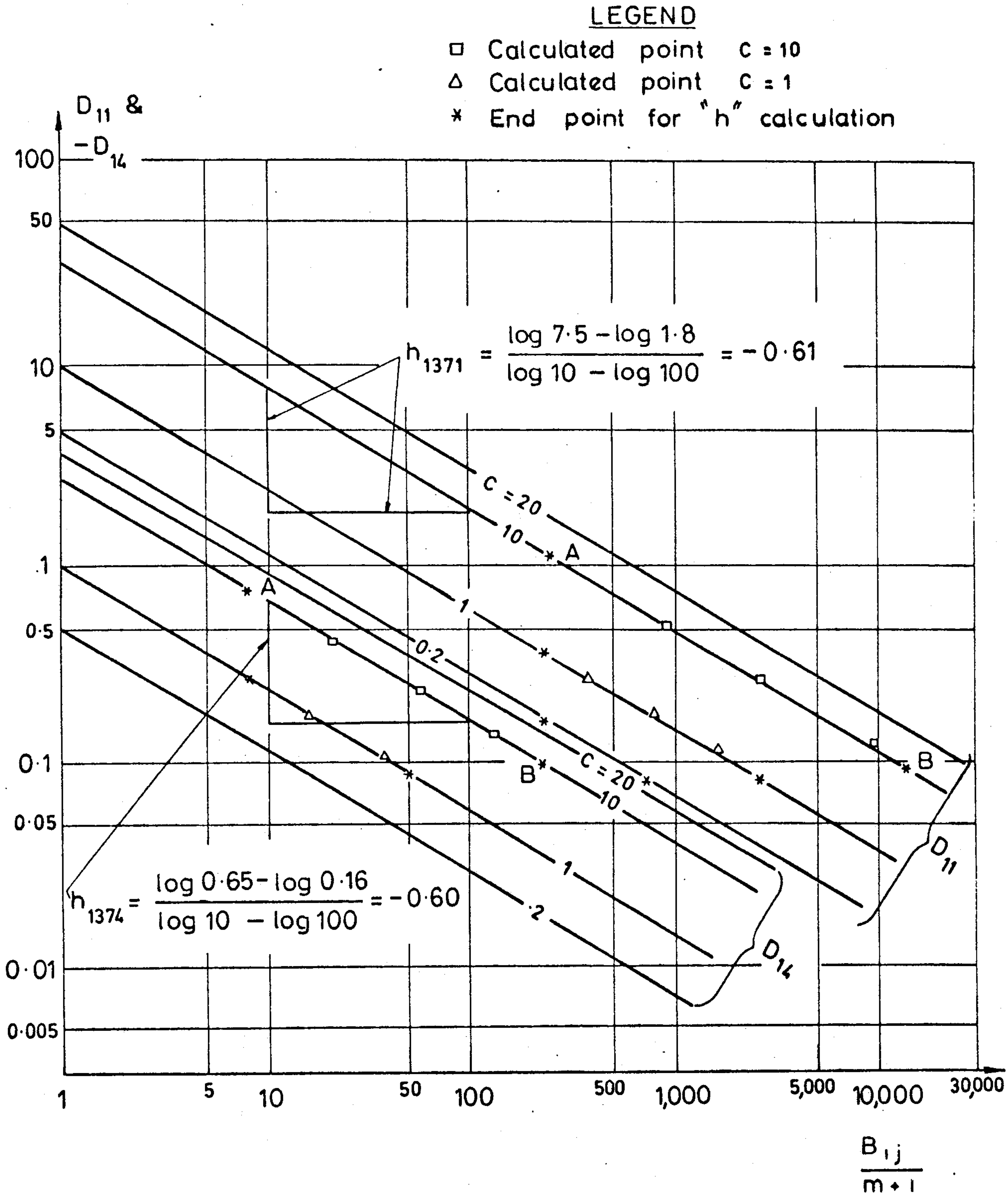



FIG. 9. DEPARTURE PARAMETER vs $\frac{B_{ij}}{m+1}$ FOR $j = 1$ & $4$, $n = 3$, $i = 1$, $m = 7$, $C_g = \infty$

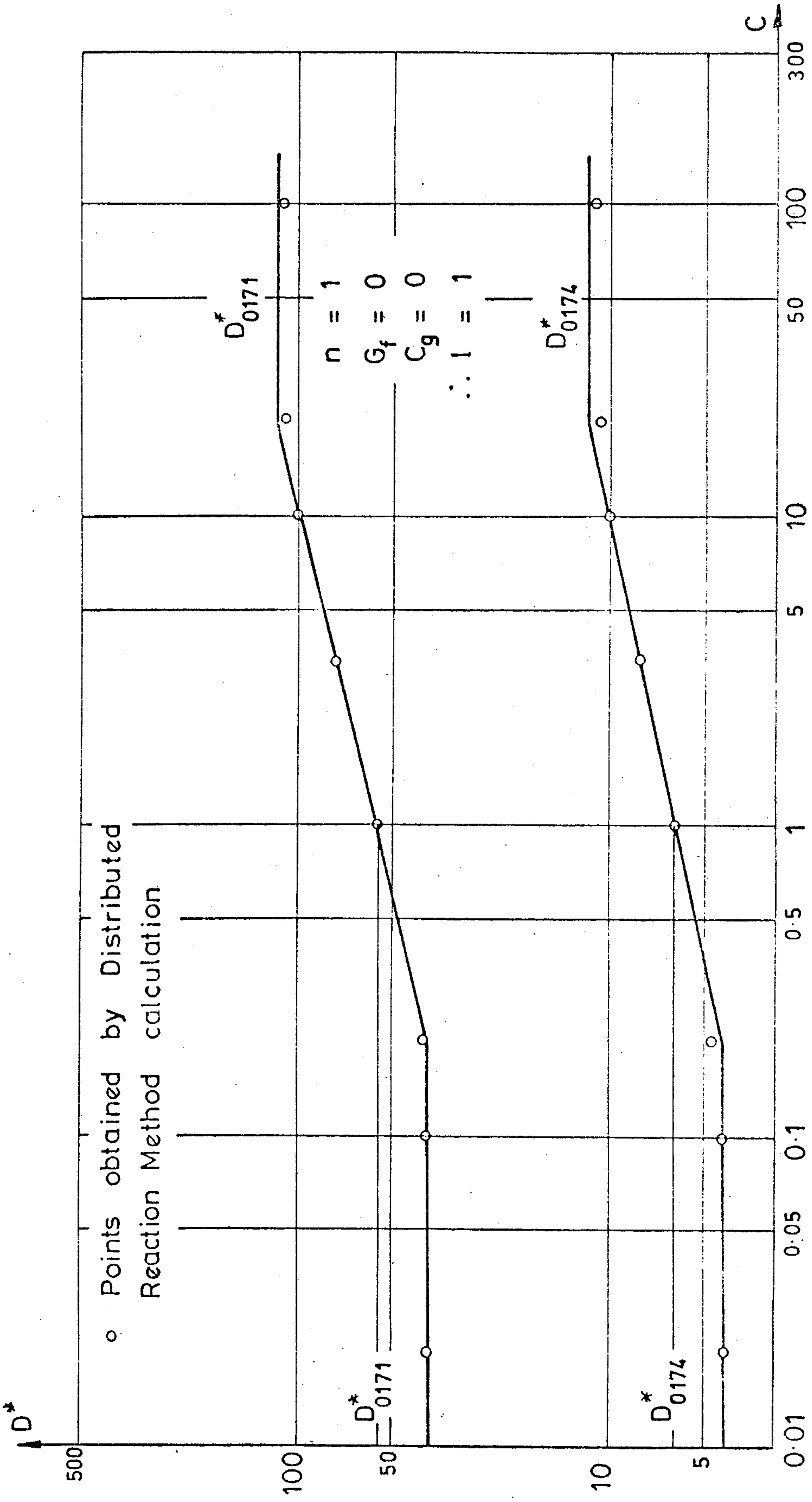



FIG. 10. INTERCEPT PARAMETER, $D^*_{017j}$ vs C FOR j = 1 & 4

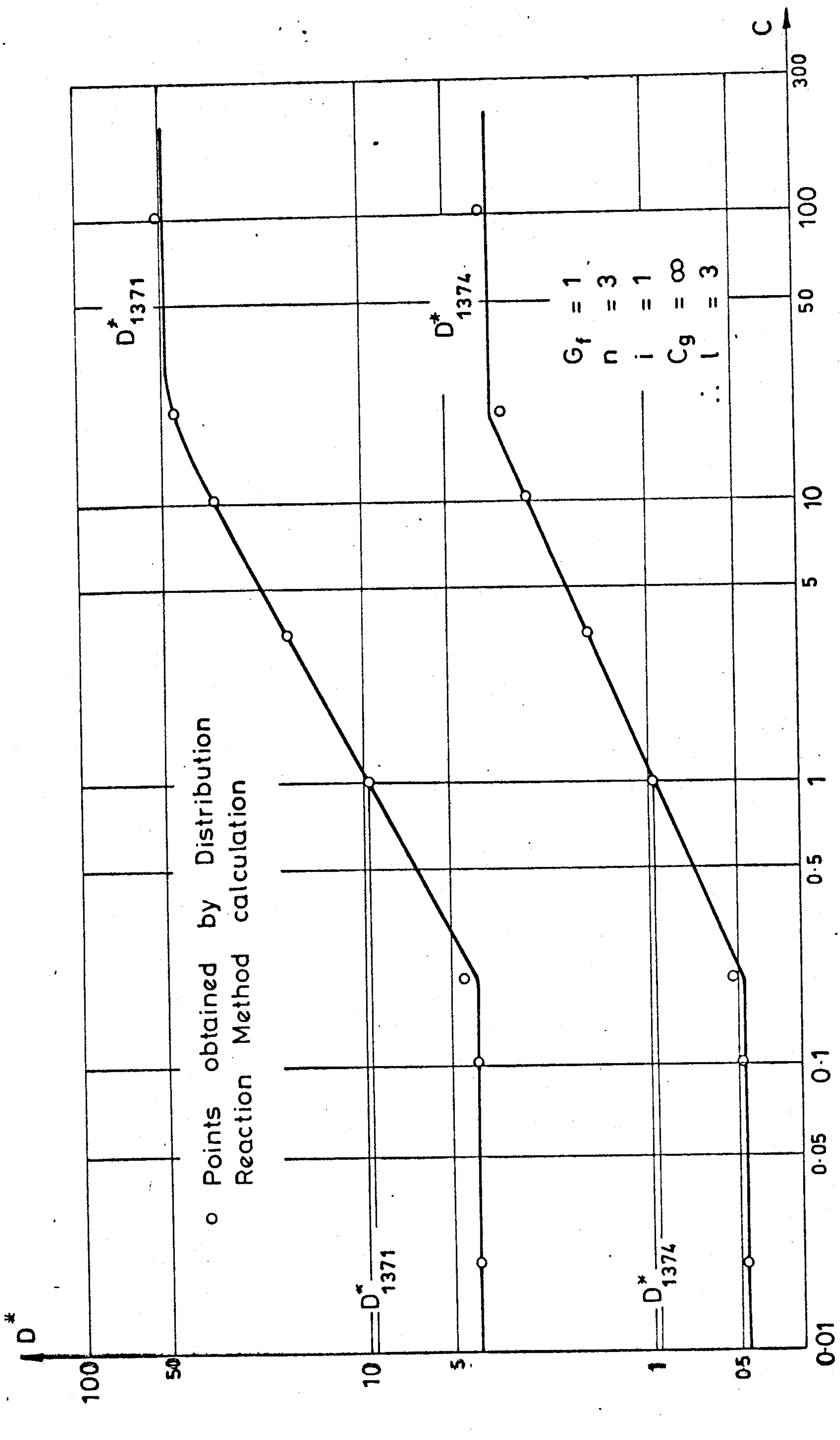
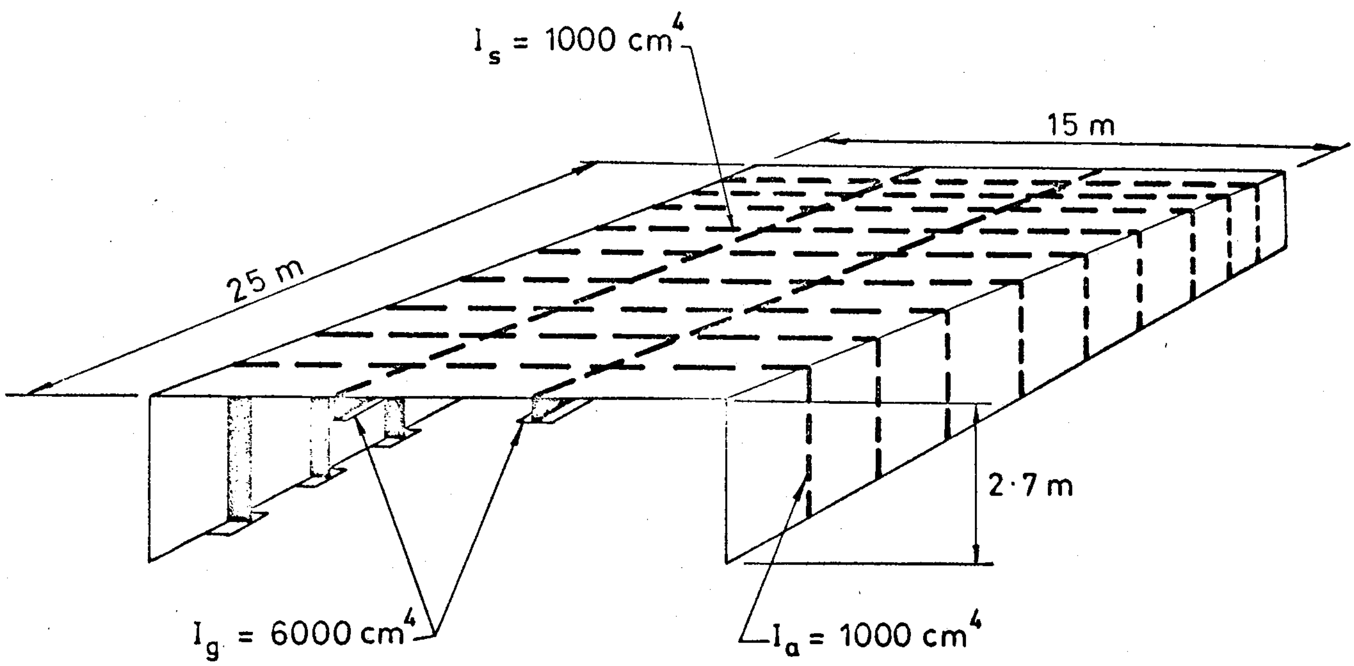



FIG. 11. INTERCEPT PARAMETER, $D^{*}_{137j}$ vs C FOR j = 1 & 4

Deck Load = $2 \cdot 5\ N/cm^2$




$$C = \frac{3 \cdot 6 \times 15}{2 \cdot 7} = 20$$

FIG. 12. STRUCTURAL DIMENSIONS FOR EXAMPLE 1

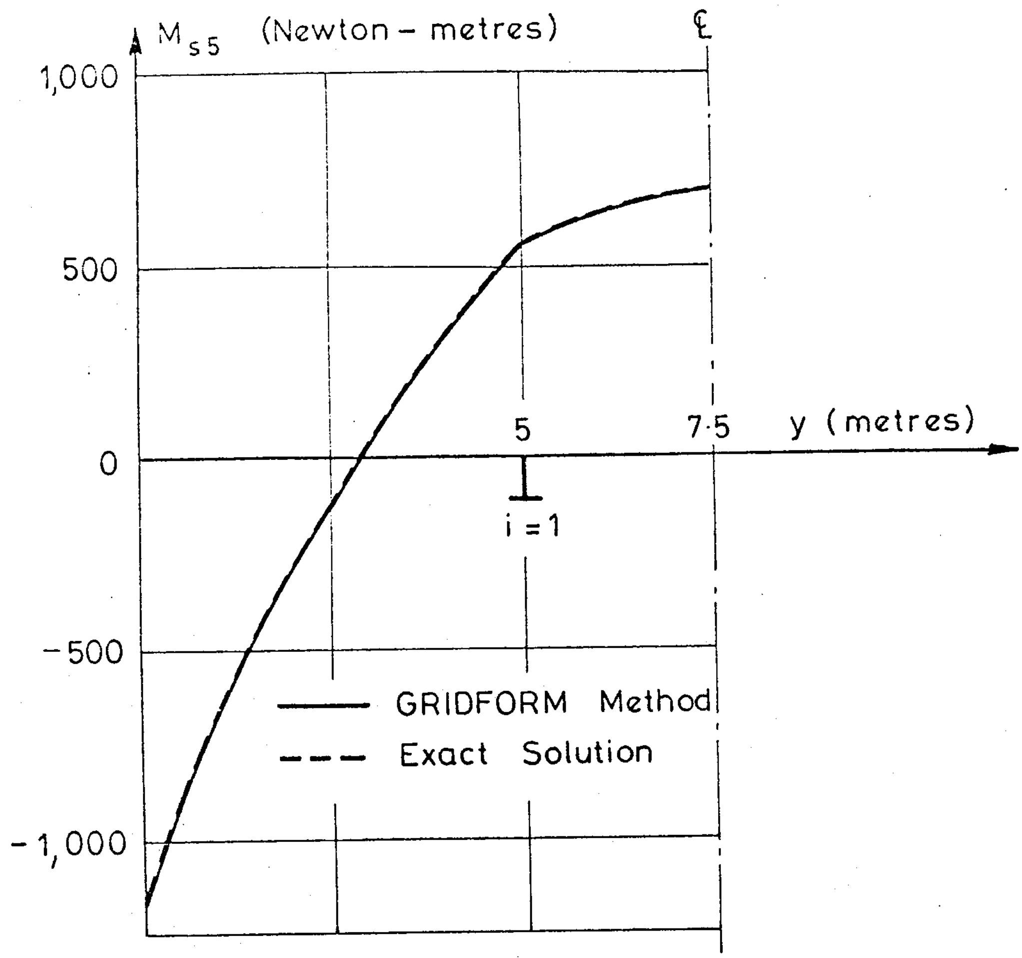



FIG. 13. BENDING MOMENT ALONG CENTRAL STIFFENER

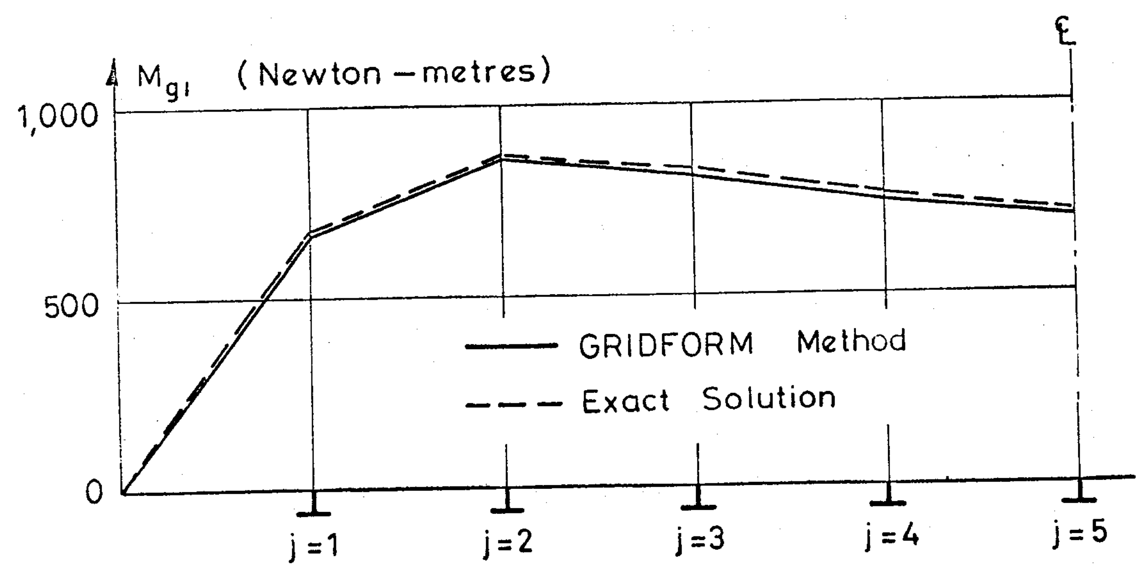
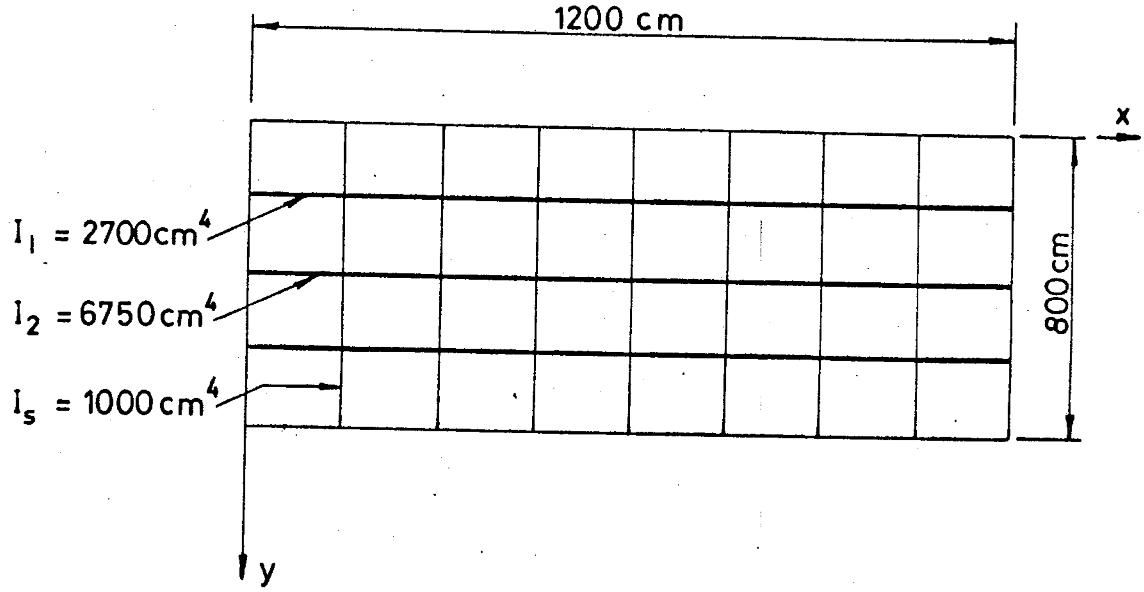



FIG. 14. BENDING MOMENT ALONG GIRDERS




$C = C_g = \infty$

$p = 1$ Newton / cm$^2$

FIG. 15. 7 x 3 GRILLAGE SUBJECT TO 1 N/cm$^2$ UNIFORM LOADING

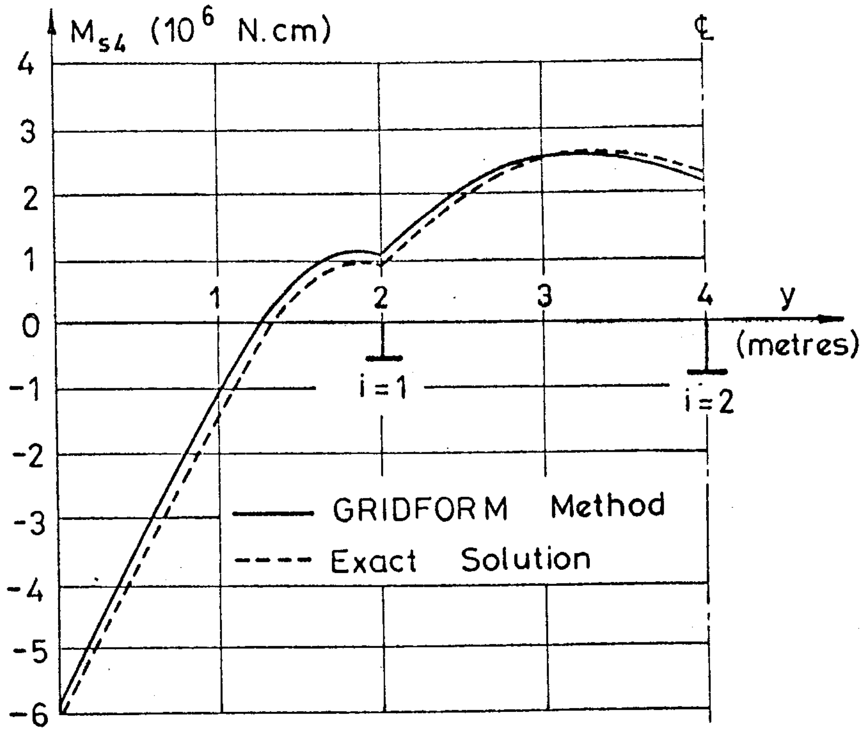



FIG. 16. CENTRAL STIFFENER BENDING MOMENT; EXAMPLE 2

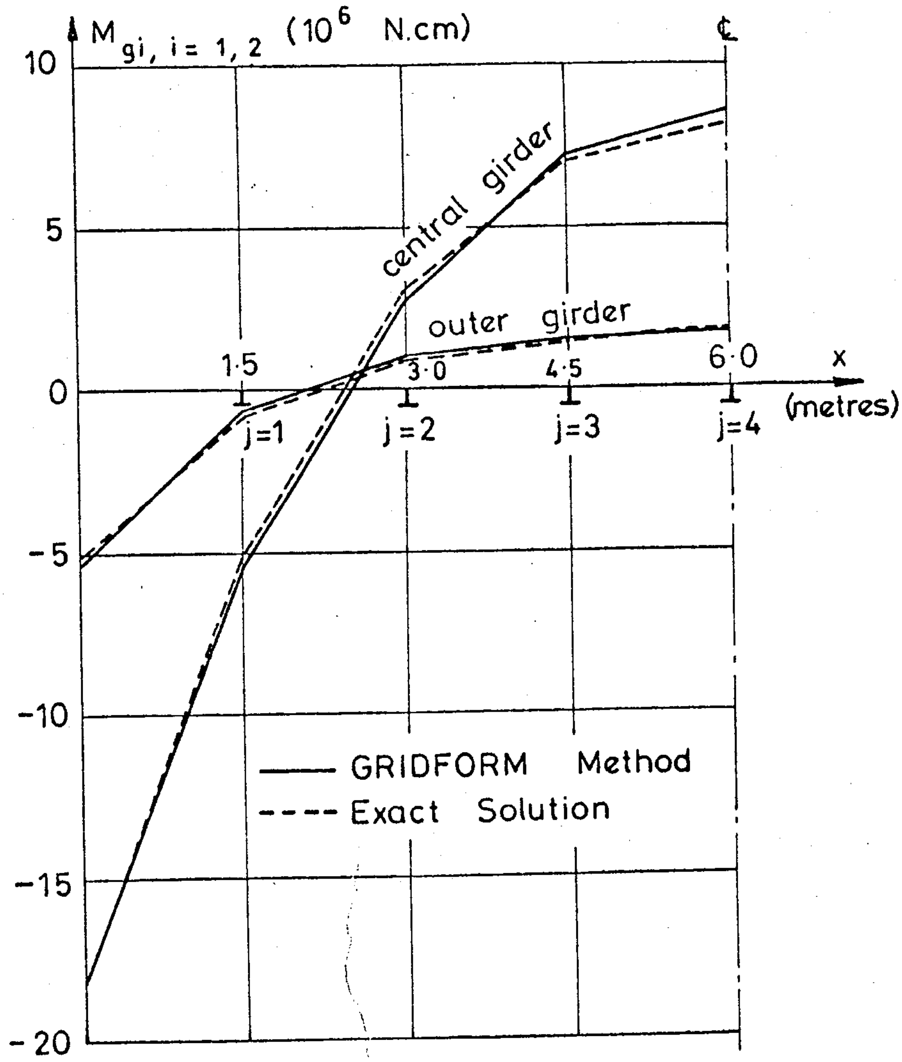



FIG. 17. GIRDER BENDING MOMENTS; EXAMPLE 2