A COMPREHENSIVE METHOD FOR THE AUTOMATED OPTIMIZATION OF SHIP STRUCTURES

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A COMPREHENSIVE METHOD FOR THE AUTOMATED OPTIMIZATION OF SHIP STRUCTURES

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ABSTRACT

A comprehensive rationally-based method is presented for the automated optimum structural design of ships. The method contains three principal features:

- **a rapid, design-oriented finite element program developed especially for structural optimization;**
- **a comprehensive set of subroutines for the accurate estimation of the various modes of collapse and unserviceability;**
- **an optimization method based on a new form of sequential linearization which is capable of solving the resulting large-scale, non-linear, highly constrained optimization problem. The objective may be any continuous non-linear function of the design variables, such as weight or cost.**

The method is illustrated by applying it to the optimum design of a typical general cargo ship, having the same overall dimensions and design loads as for the SD14 ship. Also, since this vessel has proven to be a successful and cost-effective design, it was used as the basis for determining the load factors and cost ratios for the sample problem. The problem contains 74 design variables and 542 constraints, of which 220 are non-linear. The program achieves the optimum design in 4.5 minutes of CPU time on a CYBER 72, which corresponds to a commercial cost of approximately $45.

1. INTRODUCTION

This symposium is concerned with "practical design methods", and in the field of ship structures these words are usually interpreted as the use of simplified and easy-to-use formulae for the scantlings, obtained from the rules of a classification society. This approach saves time in the design office and, since the ship must obtain the approval of the classification society, it also saves time in the approval process. The disadvantage is that, because they are simple, these formulae have large in-built margins, of unknown magnitude. They therefore cannot give a truly efficient design and, in some cases, the extra steel may represent a significant cost penalty in the life of the ship. More importantly, these formulae involve a number of simplifying assumptions and can only be used within certain limits. Outside of this range they may be quite inaccurate, and the history of structural design abounds with examples of structural failures - in ships, bridges and aircraft - which occurred when a standard, time-honoured, "practical" method was used, unknowingly, beyond its limits of validity. For this reason, and also for the potential cost benefits, structural designers are increasingly adopting what is termed a "rational" design procedure, in which the designer seeks to determine quantitatively as many as possible of the factors affecting the safety and performance of the structure throughout its life, and to use this information in order to determine that design which optimizes the performance and which provides adequate safety. This process involves far more calculation, but that can be more than offset by using computers. In that case rational design becomes automated, or computer-aided, optimum structural design. This type of design, once it has been developed, is actually far more "practical" than the earlier type. Therefore, during the past several years ship structural specialists have been working on the development of such a process. This type of design involves two main aspects, analysis and optimization, each of which has presented substantial challenges which are only now being met:

(i) ANALYSIS: the accurate prediction of the structure's response (stress, deflection, etc.) and capability (failure or limit values of response) as a function of the design variables. For large complex structures such as ships, the required accuracy demands some form of three-dimensional finite element analysis. In addition, there is a need for accurate and efficient methods for calculating the various modes of ultimate strength and other limit responses, most of which are highly non-linear.

(ii) OPTIMIZATION (or REDESIGN): the application of a systematic method for determining the design variables which minimize (or maximize a specified objective (e.g. least cost) while satisfying the constraints. For large complex structures such as ships, the constraints are numerous and many of them, particularly the collapse constraints. are highly non-linear. Likewise the objective, be it least weight or least cost, is a non-linear function of the design variables. Rational design seeks to optimize, and optimization of statically indeterminate structures is necessarily iterative, i.e. it requires repeated structural analysis which, with standard finite element programs, is computationally expensive. Moreover, the existing optimization methods for large, non-linear problems are themselves computationally expensive. Therefore the total computational cost quickly becomes prohibitive for large structures.

In the authors' estimation, this challenge could only be met by the development and integration of an optimization-oriented finite element analysis, and a structure-oriented optimization method. Therefore, during the past four years the authors have developed:

(i) a rapid, design-oriented finite element program, which provides only the information required for optimization, but which is sufficiently accurate for design;

(ii) a method for optimal redesign which is capable of efficiently solving the optimization problem posed by a large complex structure having multiple non-linear constraints.

The principal features of these two developments are presented in references [1] [2] and [3]. On the basis of these the authors have recently developed a large-scale, comprehensive computer program known as MAESTRO (Method for Automated Evaluation and STRuctural Optimization). Although intended mainly for ship structures, the methodology of MAESTRO is quite general and can be applied to other semi-monocoque structures, such as the spans of a steel box-girder bridge.

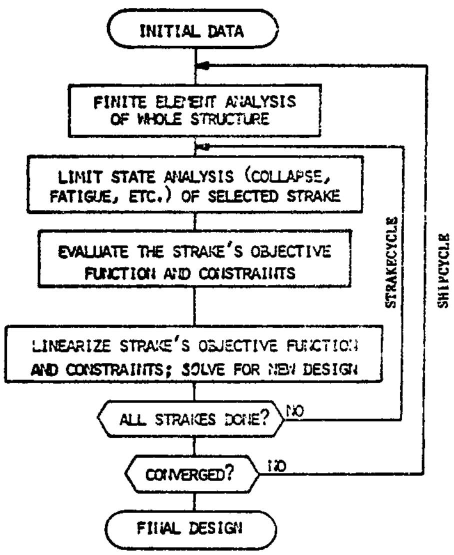


FIGURE 1: MAESTRO PROGRAM

Figure 1 summarizes the MAESTRO program. The input consists of the overall dimensions, the material properties, the design load systems and arbitrary starting values for the design variables. The structural analysis is performed by RANSAP, a design-oriented finite element package. The optimization or redesign is performed by SLIP2 (Sequential Linear Programming -- 2nd order) on a strake-by-strake basis (see Figure 2 for the meaning of the term "strake"). For each strake, the current stresses and safety margins are calculated. The constraints and the objective function are then linearized using first and second-order derivative information, and the resulting linear programming problem is solved for the new size variables. When all of the strakes have been redesigned, a check is made whether the overall design process has converged satisfactorily. If not, a modified longitudinal bending stress distribution is determined from the new size variables and another RANSAP finite element analysis begins the next design cycle.

Because the redesign is done on a strake-by strake basis, the total problem is divided into a series of sub-problems, each of which involves no more than 14 design variables and approximately 75 constraints, thus achieving a more rapid and efficient solution, and significantly decreasing computational costs.

Unfortunately, space limitations do not permit a description of the optimization method, SLIP2. A full description is given in reference [3] which is available from the authors.

2. DESIGN-ORIENTED STRUCTURAL ANALYSIS

2.1 Structural Modelling

As noted in Section 1, structural optimization is an iterative process, requiring repeated structural analyses. This requires that the structural analysis method be extremely rapid and yet be sufficiently comprehensive to give all significant stresses in all major members of the structure. None of the standard finite element programs presently available possesses this combination. They are large, complicated general purpose programs, applicable to a wide range of structures, and they are intended to be used for analysis only, and not as part of an optimization method.

Hence, one of the most important aspects of the MAESTRO project has been the development of a special design-oriented method of finite element analysis, which would perform a rapid and efficient analysis of ship structures. The key step in achieving this, and the main link between the analysis and the optimization phases, is the structural modelling scheme.

In a ship structure the transverse bulkheads divide the structure into compartments which may conveniently be treated as substructures since the bulkheads eliminate the transverse degrees of freedom at these boundaries. Because of the complex three-dimensional nature of both loading and structure, the minimum extent of structure which must be modelled is an entire compartment. On the other hand, a single compartment is sufficient because the transverse bulkheads greatly restrict the degree of interaction between adjacent compartments, and the effect of adjacent compartments may be estimated and introduced in the form of supports and loads at the boundaries.

Moreover, there are usually a few key compartments that dominate the design because of either large loads or a diminished amount of structure, or both. Most of these key compartments occur in the midship half length where the hull girder bending moment is largest. Once these compartments have been optimized, the structural dimensions of intermediate and adjacent compartments can be obtained by interpolation because of the need for longitudinal continuity of structure.

Another feature of the MAESTRO design philosophy is that each compartment is assumed to be longitudinally prismatic, i.e. within any compartment there is no variation, in the longitudinal direction, of either overall transverse dimensions or individual member sizes. The assumption of

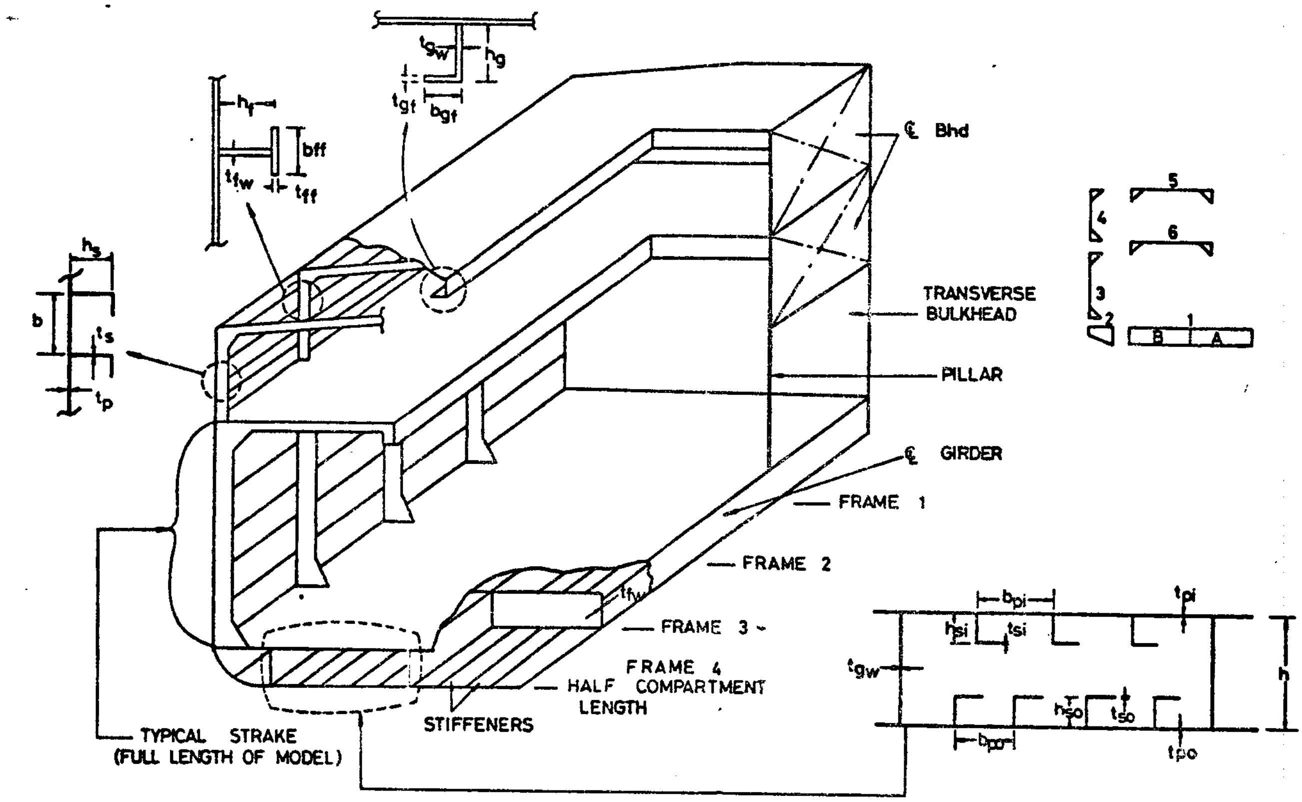


FIGURE 2: STANDARD SHIP - GEOMETRY AND DESIGN VARIABLES

uniform overall transverse dimensions is a reasonable approximation because in ship structures the taper in one compartment length is relatively small, especially for key compartments.

The program also contains a feature by which specific elements can be deleted, and this permits the modelling of rectangular openings and discontinuous beams or panels, such as the partial centreline bulkhead in Figure 2.

The assumption of uniform member sizes within each strake means that all transverse frames are identical, each longitudinal girder is of constant cross section, and the plating thickness and stiffener sizes are constant within each strake. This is not excessively restrictive because such uniformity is usually required for efficient, low cost fabrication. Obviously, it is of assistance in optimization since it greatly reduces the number of design variables. As shown in Figure 2, this modelling produced, at most, 14 size variables per strake:

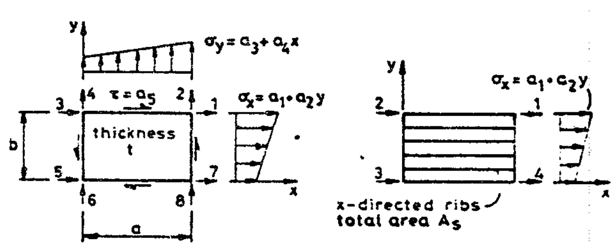
- four for the stiffened plating
- four for the longitudinal girder associated with the strake (if any)
- six for the transverse frame segments within the strake (or four if there are no brackets).

2.2 Elements Used in RANSAP

On the basis of the above modelling scheme, the special finite element program RANSAP was developed to be both simple and rapid, while yet providing the required accuracy. Its simplicity is illustrated by the fact that it requires only three types of elements:

2.2.1 Stiffened Panel Element

A special rectangular, multi-ribbed plane stress element was developed for modelling the panels of stiffened plating. Details of this element are given in reference [4] in which it is shown to give better results than other types of stiffened panel elements, which either lump the ribs at the panel edges or use an orthotropic element. The element is non-conforming, and has constant shear stress and linearly varying direct stresses, as shown in Figure 3. The main advantage of the element is that it possesses sufficient accuracy to allow relatively large stiffened panels to be modelled by a single element, thus decreasing both the storage requirements and the solution time. Another significant advantage is that the element stiffness matrix is expressed analytically, which allows a more rapid evaluation - and re-evaluation - than the usual numerically integrated elements.



(a) The Turner Stress Rectangle (b) The Multi-rib Element

FIGURE 3: STIFFENED PANEL ELEMENT

Bending of stiffened plating is essentially a local phenomenon; i.e., it has little effect on the overall behaviour of the structure. Therefore the bending stiffness of the panels is not included in the RANSAP finite element model, and the stiffener and plate bending stresses are calculated to a satisfactory degree of accuracy by means of standard formulae.

2.2.2 Beam Element

The second type of element is a special beam element which has been adapted for modelling beams which are attached to plating and which may have bracketed ends. The plating acts as one of the flanges of the beam and the program takes this into account by assuming an effective breadth for this "plate flange". The neutral axis location and the moment of inertia of the composite beam are automatically calculated by the program. For longitudinal beams the plate thickness used in calculating the effective flange breadth is an "effective" plate thickness, obtained by smearing the longitudinal stiffeners into the plate.

2.2.3 Strut element

The strut element is simply a pin-jointed bar which can connect any two nodes in the compartment. Its most common use is for representing pillars but it can also be used to represent screen bulkheads, passageways, and other portions of structure which are not primary structural members but which, because of their in-plane rigidity, do contribute to the stiffness of the overall structure.

2.2.4 Double bottom element

In addition to the three basic elements MAESTRO contains a special element for modelling a double bottom strake. In this case there are eleven design variables per strake, as shown in Figure 2.

2.3 Accuracy and Speed of RANSAP

The accuracy of RANSAP is demonstrated in reference [5], in which the hull structure of a Whitby class frigate was analysed using both RANSAP and the general purpose finite element program STARDYNE. In order to give a rigorous check, the STARDYNE analysis used a fine mesh modelling, with each stiffener being modelled as a separate bar element. In general, the RANSAP results (deflection, bending moments, stresses) were within 3% of the fine mesh results.

To gauge the speed of RANSAP, the RANSAP structural model was set up and solved twice, once using STARDYNE and once using RANSAP, on a CDC 6600 computer. STARDYNE required 40 CP seconds, while RANSAP required only 9 CP seconds. Since then, various improvements have been made to RANSAP which further reduce the time to approximately 5 CP seconds.

3. REQUIREMENTS FOR RATIONAL DESIGN

3.1 Safety Requirements and Probability

At its most basic level, the measure of structural safety is the probability of failure, i.e. the probability that one or more of the loads will exceed the capability of the structure. A probabilistic basis is particularly appropriate for ship structures since the principal loads - those arising from waves - are themselves essentially probabilistic. But in reality most of the other loads are also stochastic, and most of the principal factors which determine capability (e.g. material properties, accuracy of analysis, quality of workmanship) also display statistical variations. Hence a fully rational design process must be based on some minimum acceptable risk of failure. Once this is established, such a process may be defined as the determination of all of the design variables so as to optimize the objective while at the same time not exceeding this minimum acceptable risk.

The specification of the minimum acceptable risks is primarily the responsibility of regulatory authorities, reflecting the wishes of society. However, even when these risk levels are established and available, it is difficult for a designer to work in purely probabilistic terms, dealing with the statistical distributions of the various loads and other design factors. It is easier to deal with the characteristic values for each of these distributions, and hence the preferred approach is for the regulatory authorities to specify the characteristic values of the various loads and the margins which must be provided between these characteristic loads and the corresponding capabilities, such that the required degree of safety is achieved. In order to do this the statistical distributions must first be investigated and established, with particular attention being paid to the spread or variance of each distribution.

The field of ship structures has not yet progressed as far as other structural areas such as aircraft structures and, more recently, large box girder bridges. The latter have much in common with ships, and many features of the recently developed rational design procedure for box girder bridges are directly applicable to ships. One such feature is the use of separate load factors as a way of providing the required safety margins and of combining these to give the required overall degree of safety. This and other aspects of rational design are discussed in the next section.

3.2 Load Factors and Materials Factors

Rational design requires that a clear distinction be made between the various degrees of failure, of which there are at least two: collapse and unserviceability. It also requires for each of these degrees the explicit consideration of all of the modes in which a structure may fail. In modern design terminology these failure modes are referred to as limit states, because each of them is defined as a state in which one or more of the responses reach some limiting value(s). For the case of stress response this limiting value will be denoted as σ_{limit}. It may be the yield stress σ_Y or it may be a derived quantity such as buckling stress, in which case it will be written as $\sigma_{limit}(\underline{X})$ to indicate that its value depends on the design variables $\underline{X}$.

In modern box girder bridge design the required margin between a response and its limit value is specified by means of load factors, which multiply the loads, and limit response factors, which divide (and hence reduce) the limit response value. There are two sets of load factors. One set, denoted herein as $\underline{\alpha}$, corresponds to the various limit states and the other set, denoted as $\underline{\beta}$, corresponds to the various load types. The values of $\underline{\alpha}$ are chosen according to the degree of seriousness of the corresponding limit state, and the values of $\underline{\beta}$ for the various load types are chosen according to the degree of variability or uncertainty of that load type. Each load which participates in a particular limit state is multiplied by the two appropriate load factors.

There are several sets of limit response factors, each corresponding to some source of uncertainty or variation in the limit response value such as material properties, accuracy of calculation, quality of workmanship, etc. The most significant is the materials factor, which accounts for variations in material properties. Most of the other limit response factors can be included within the materials factor and hence for simplicity only this factor is considered in this paper. The set of materials factors, denoted $\underline{\gamma}$, consists of a separate factor for each portion of the structure which is of a different material or of different material quality. This factor is applied as a divisor to each limit response thereby reducing it below its characteristic value.

In terms of these factors the requirement of a safety margin becomes the requirement (or "constraint", in optimization terminology) that the maximum response to the factored loads must not exceed the factored limit value of that response. For example, for some collapse mode which is stress dependent (say, panel buckling) there would be a constraint on that design that the design variables $\underline{X}$ must be such that

$$\max_{\ell=1,L}\left[\sigma_\ell(\alpha_c\beta_1 Q_1\ ,\ \alpha_c\beta_2 Q_2\ ,\ldots)\right] < \frac{1}{\gamma_j}\sigma_{limit}(\underline{X})$$

where L = total number of loadcases

Q_i = i^{th} load, within the ℓ^{th} loadcase, which participates in this mode of collapse

α_c = load factor for collapse (α_s for serviceability)

β_i = load factor for Q_i, depending on its degree of uncertainty or variability

γ_j = materials factor for the j^{th} panel, reflecting the quality of its material and of its fabrication, and the accuracy of calculation of $\sigma_{limit}(\underline{X})$

σ_ℓ = in-plane compressive stress resulting from the loads in the ℓ^{th} loadcase.

In some cases, for simplicity, the above three factors are combined into a "global" load factor, N, which depends only on the type of failure, the loadcase, and the material. In this case a typical constraint (say, for collapse) becomes simply

$$\max_{\ell=1,L}\left[N_{c\ell j}\sigma(Q_1, Q_2,\ldots)\right] < \sigma_{limit}(\underline{X})$$

where $N_{c\ell j} = \alpha_c\beta_\ell\gamma_j$.

However, this simplification requires that for each loadcase β is single-valued (i.e. that all of the relevant loads have the same variability) the value chosen being whatever is the largest value of β for all of the loads of that loadcase. It also implies that there is a linear load-response relationship, so that both α and β can be applied to the response instead of to the load.

4. CONSTRAINTS

4.1 Classification of Constraints

From a mathematical point of view there are three types of constraints: non-linear, linear, and minimum-maximum value. Linear constraints restrict the proportion between one design variable and another, in order to avoid unbalanced combinations (e.g. large stiffeners attached to thin plating). Minimum/maximum constraints arise from considerations of availability or fabrication (e.g. an upper limit on plate thickness) or practicality (e.g. a lower limit of thickness, to avoid flimsy, easily dented plates).

From the point of view of the extent of structure which is involved there are two types: overall constraints, which relate to the entire structure and its action as a box girder, and strake constraints, which apply to stiffened panels, cross beams and girders. Purely local effects (e.g. local stress concentrations) are not dealt with in MAESTRO since these are more properly a part of detail design rather than overall structural design.

Finally, constraints may also be classified according to the limit state to which they refer, the two main types being collapse and unserviceability. A structure may become unserviceable due to: (i) yield, (ii) local buckling, (iii) excessive deflection/flexibility, (iv) fatigue, cracking, or other progressive type of damage. Fatigue is often considered as a separate limit state. In MAESTRO it is catered for by making an exhaustive search for the largest values of those stresses which are fluctuating stresses (defined by the user) and imposing the constraint(s) that these peak values be less than the permissable values, which are also user-specified, according to the expected number of cycles.

For a large stiffened steel structure the modes of collapse and unserviceability are numerous, complex, and highly non-linear. In particular, there are several collapse modes which require a lengthy and in some cases iterative calculation - a complete subroutine rather than a few formulae. The authors are unaware of any other structural optimization method which is able to deal with a full set of comprehensive and rigorous ultimate strength constraints. Moreover, the existence of these routines in MAESTRO makes the program very useful for checking existing structures or evaluating proposed designs or design alterations. Also, by making appropriate thickness reductions to allow for corrosion, the program can be used to predict structural life. In all cases the program identifies the mode of collapse, the location, the load combination responsible, and the margin by which the constraint is violated. The following section gives a brief summery of the collapse and unserviceability constraints in MAESTRO.

4.2 Collapse and Unserviceability Constraints

Panel; Collapse

1. PCSB: Panel Collapse, Stiffener Buckling. Interaction of plate buckling (elastic or inelastic) due to in-plane stresses $\underline{\sigma}$ (longitudinal, transverse and shear) and stiffener collapse as a beam column due to the combination of axial compression and lateral pressure p.

2. PCST: Panel Collapse, Stiffener Tripping. Interaction of plate buckling (as above) and stiffener collapse due to torsional instability under the action of $\underline{\sigma}$ and p.

3. PCCB: Panel Collapse, Combined Buckling. Collapse of orthotropic panel (stiffeners and plating acting together) by interactive buckling (elastic or inelastic) due to $\underline{\sigma}$.

4. PCOB: Panel Collapse, Overall Buckling. Collapse of gross orthotropic panel (involving buckling of one or more frames) by interactive buckling (elastic or inelastic) due to $\underline{\sigma}$.

5. PCMY: Panel Collapse, Membrane Yield. Collapse due to extensive through-the-thickness yielding (Von Mises criterion) under the action of the in-plane stresses $\underline{\sigma}$.

Panel; Serviceability

6. PYTF, 7. PYTP, 8. PYCF, 9. PYCP: Panel Yield, Tension or Compression, in the Flange (of the stiffener) or in the Plating, due to the continued action of longitudinal stress and lateral pressure p.

10. PYPB: Panel Yield, Plate Bending. Surface yield of plating due to plate bending between stiffeners.

11. PSLB: Panel Serviceability, Local Buckling. Plate buckling between stiffeners (elastic or inelastic).
Note: local buckling of stiffeners (flange or web) is dealt with by means of proportionality constraints.

Girder; Collapse

12. GCPH: Girder Collapse; Plastic Hinge. Collapse due to formation of one hinge, if girder is simply supported at the bulkheads, or three hinges if clamped.

13. GCCF: Girder Collapse, Compression, Flange/Plating/Tripping.

14. GCCP, 15. GCCT: Girder collapses under combined axial load and bending either: (a) as a beam-column, when the outer fiber stress on the compression side reaches yield, or (b) due to torsional buckling (rotation of the beam web along its line of attachment to the plating).

Girder; Serviceability

16. GYTF, 17. GYTP: Girder Yield, Tension, Flange/Plating. Combined axial load and bending cause tensile yield in the outer fiber.
Note: Local buckling of girder web or flange is dealt with by means of proportionality constraints.

Frame; Collapse

Note: all frame failure modes are checked at both ends of the frame segment (i.e. at the strake edges) and at the centre, in order to allow for brackets. Therefore the number of constraints is tripled.

18-20. FCPH: Frame Collapse, Plastic Hinge. Collapse due to the formation of <u>one</u> plastic hinge within this frame segment, either at one of the ends or at the centre.
Note: Requiring only one hinge is conservative, and also covers all of the various multistrake mechanisms of collapse which might occur.

Frame; Serviceability

21-23. FYTF, 24-26. FYTP, 27-29. FYCF, 30-32. FYCP: Frame Yield, Tension or Compression, in the Flange or in the Plating, due to the combination of axial stress and bending stress.
Note: Local buckling of frame web or frame flange is dealt with by means of proportionality constraints.

4.3 Treatment of Multi-Strake Constraints

As defined in MAESTRO, a strake is an important part of the hull girder, such that collapse of a strake could, for all practical purposes, be considered as collapse of the structure. The error in this assumption is small and it lies on the conservative side. It has the great advantage that it allows all of the constraints to be treated as strake constraints, thus permitting the strake-by-strake approach of MAESTRO. However, the separate treatment of the strakes also requires that the stresses in a particular strake are affected mainly by design changes within that strake. This is true for all "secondary" stresses (i.e. stresses which are due to transverse and/or local loads) and hence any constraints which involve these stresses can be satisfied by making changes within that strake. But, this is not the case with the hull girder bending stress, σ_1. Since this stress depends on the overall section modulus, and hence on all strakes in differing degrees, there are two problems which must be overcome for a strake-by-strake approach:

(i) A change of design variables in a particular strake will influence σ_1 in that strake. If this effect is not accounted for during the strake redesign, but is delayed until the next finite element analysis, then convergence will be slow.

(ii) If a constraint is violated because σ_1 is too large, the redesign should not be restricted to only that strake because this would cause disproportionate member sizes and would probably prevent convergence. The optimal solution requires changes to several strakes, particularly the strakes in the flanges (i.e. strength deck and bottom).

These problems are overcome by two special features of MAESTRO.

(a) <u>Inclusion of Stress Derivative Term for σ_1.</u>

The effect which a change of scantlings in a particular strake has on the primary stress in that strake is accounted for by including the derivative of σ_1 with respect to the design variables in the formulation of the linearized constraint functions. As shown in reference [3], the SLIP2 optimization method obtains the linearized form $g_L(\underline{X})$ of a general non-linear constraint function $g(\underline{X})$ in the following form

$$g_L(\underline{X}) = g(\underline{X}^\circ) + \sum_i^N (X_i - X_i^\circ)\Delta_i$$

in which $\underline{X}$ = vector of N design variables ($N \leq 14$)

$\underline{X}^\circ$ = current values of $\underline{X}$

and Δ_i is a special type of derivative of g with respect to X_i (a secant rate of change instead of a tangent) which is computed using second order finite differences. This computation uses the current values of $\underline{X}$ and of σ_1, thus treating σ_1 as if it were constant, and the interaction between σ_1 and $\underline{X}$ is then accounted for as follows:

$$\Delta_i = (\Delta_i)_{\sigma_1 = \text{const.}} + \frac{\partial g}{\partial \sigma_1}\frac{\partial \sigma_1}{\partial X_i}$$

A rapid and simple algorithm for calculating the stress derivative $\frac{\partial \sigma_1}{\partial X_i}$ is presented in reference [7]. An expression for $\frac{\partial g}{\partial \sigma_1}$ for each constraint may be obtained by simply differentiating the expression which defines $g(\underline{X})$ for that constraint, treating all terms other than σ_1 as constant.

(b) Control of σ_1 by Preliminary Scaling of Flanges.

Even though the stress derivative term for σ_1 is included for each strake, it is also necessary to make some allowance for multi-strake resizing in order to reduce excessive values of σ_1 in a realistic and effective manner. In MAESTRO this is done in the following manner. Before the strake-by-strake optimization is commenced, each strake is tested as to whether the factored value of σ_1 for that strake exceeds the yield stress. If so, then the plate thickness in the bottom and in the strength deck is "scaled up" (by two separate scaling factors) such that the worst stress excess is exactly eliminated, while keeping the total added area to a minimum. As shown in reference [7], the stress derivatives mentioned above allow this problem to be formulated as a two-variable linear programming problem and hence the solution requires negligible time.
It should be noted that this scaling process does not constitute optimization of the flange; it is merely a preliminary to the strake-by-strake optimization, in order to obtain an overall multi-strake redistribution of material which is of the correct general nature, so that the individual strake optimizations will not produce artificially large changes and consequent oscillation and divergence. Details are given in reference [7].

Deflection of the hull girder, δ_1, depends on the overall moment of inertia and hence a constraint on overall structure flexibility is another instance of global or multi-strake constraint. It can be dealt with in a similar manner to that for σ_1 using deflection derivatives $\frac{\partial g}{\partial \delta_1}$ and $\frac{\partial \delta_1}{\partial X_i}$.

In the future development of MAESTRO these global constraints will be dealt with explicitly, using the Dantzig-Wolfe decomposition algorithm [8].

5. DETERMINATION OF GLOBAL LOAD FACTORS AND COST COEFFICIENTS

5.1 Determination of Load and Materials Factors

Apart from wave bending, there is as yet little information available regarding values of load factors and materials factors which would give an acceptable minimum risk, and it is hoped that the various regulatory authorities and research groups will make a strong effort to determine these, as has already been done by their counterparts in the field of box girder bridges. One way of obtaining approximate values is to analyse a series of ship designs which have proven to be acceptable and to extract the overall safety factors which are implicit in those designs. The steps involved in this process are:

(i) determine the characteristic or design loads.
(ii) calculate all of the responses to these loads.
(iii) calculate the limit values of all of these responses, both for collapse and for unserviceability.
(iv) divide (iii) by (ii) to obtain N, the global load factor.
(v) repeat for all parts of the structure.

This type of analysis can determine the global load factors for each strake, for collapse and serviceability, but it cannot determine the separate values of α, β and γ. Even then it is a massive task and it could only be done by means of a program like MAESTRO which includes both finite element analysis, for calculating all of the responses, and a complete set of subroutines for calculating all of the various collapse and unserviceability limit response values.

5.2 Objective Function for Least Cost Optimization

The optimization aspect of rational design requires the mathematical definition of the objective which is to be optimized. For commercial vessels least cost is an immediate and generally satisfactory objective. For the particular case of optimization of ship scantlings it is sufficient to include only those costs which are directly dependent on scantlings. Other aspects of the structure, such as the topology and geometry (e.g. compartment length, location of major members, type of framing system) are usually determined, or at least strongly influenced by, non-structural requirements. Hence during scantling optimization these quantities should be held fixed. If desired, their effect can be found by making a parametric series of optimum designs.

The cost is defined on a per-strake basis using two alternative cost functions, one for ordinary single-plated strakes and another for double bottom strakes. The cost function for a singly plated strake is

$$\text{Cost} = \sum^{NS} (C_{Base} + C_{sp} + C_g + C_f) \qquad (5.1)$$

where NS = number of strakes, and all of the four cost terms are on a per strake basis.

C_{Base} = base cost for each strake, which is constant and represents fixed costs, i.e. all costs which may be taken as being independent of scantlings. In the sample problem, for simplicity, the base cost for all strakes was set at zero.

C_{sp} = cost of the stiffened panel for each strake. In the current version of MAESTRO this is defined as

$$C_{sp} = \frac{A}{d}\left[\rho_1\left(Bt_p + s(1+F_1)\,h_s t_s\right) + \rho_2 s\right] \quad (5.2)$$

in which s is the number of stiffeners:

$$s = \frac{B}{b} - 1$$

t_p, h_s and t_s are the panel scantlings (see Fig. 2)

A is the compartment length

d is the frame spacing

B is the width of the strake

ρ_1 is the material cost coefficient in terms of cost per unit volume

ρ_2 is the labour cost coefficient in terms of cost per unit length of stiffener

F_1 is the ratio of stiffener flange width to stiffener web height.

C_g = cost of the girder for each strake. In this case the material cost only is considered to be variable. The labour cost is assumed to be independent of scantlings and hence forms part of the base cost. Therefore

$$C_g = \rho_1 N_g A (h_g t_{gw} + b_{gf} t_{gf}) \quad (5.3)$$

N_g is the number of girders per strake.

C_f = cost of frame. In this case too, only the material cost is considered to be variable. Hence

$$C_f = \rho_1 N_f B (h_f t_{gf} + b_{ff} t_{ff}) \quad (5.4)$$

N_f is the number of frames per strake:

$$N_f = \frac{A}{d} - 1$$

For optimization the absolute values of ρ_1 and ρ_2 are not required; it is only necessary to have their correct relative magnitude. This information can be determined by a careful study of shipyard costs, as has been done for tankers in [6]. Alternatively it can be obtained approximately by "working backwards" from designs which are known to be cost-efficient. This is, however, another large task, and one which can only be done with facility by a comprehensive structural optimization program such as MAESTRO.

5.3 Calculation of Current Load Factors and Cost Coefficients

The first task in the present project was to determine realistic "current practice" values for both the global load factors and for the relative magnitude of the two cost coefficients by the approach suggested above, i.e. by applying MAESTRO to existing ship designs which have proven to be both safe and cost-efficient. Ideally these basis design(s) should be:

(i) modern, in order to reflect current design practice materials, fabrication methods and quality control;

(ii) for general purpose ships, rather than highly specialized types;

(iii) design(s) in which some extra effort has been made to employ a rational design process and to achieve an optimum (least cost) design.

In the authors' estimation the best way of satisfying the above requirements was to choose one of the recently developed and highly successful standard ship designs, such as the Freedom ship or the SD14. The latter was chosen because the required information was ready to hand. However, at the time when this study was made MAESTRO did not yet have the capability of modelling mixed framing systems, and so the transversely framed side shell of the SD14 was converted to longitudinal framing, as shown in Figure 2. This has no material effect on the results of the study; it simply means that the basis design is very much like, but not exactly the same as, the SD14. It will be referred to herein as the "standard ship".

A total of five loadcases were used as shown in Tables 1 and 2. The vessel was modelled using six strakes, as shown in Figure 2. An extensive series of optimizations was performed using various values for the global load factors and for the cost coefficients for each strake, in order to determine the values for which the resultant optimum scantlings matched those of the standard ship.

The results for the various strakes were quite consistent, giving a value of about 1.5 for the global load factor for collapse, and about 1.25 for the global load factor for serviceability. The ratio of cost coefficients which generated the desired standard ship was

ρ_1/ρ_2 = cost per unit volume ÷ cost per unit length of stiffening

= 11.43

This ratio is, of course, dependent on the length units which are used. The above figure is for lengths measured in feet, and hence the results indicate that if welding costs were, for example, $1 per foot, then on this scale the cost of steel would be about $11.43 per cubic foot. For a typical stiffened panel the two portions of the total cost are usually about equal, or at least are of the same order of magnitude.

6. COST EFFECTIVENESS OF MAESTRO

6.1 Sample Problem

In order to validate the MAESTRO philosophy and to demonstrate its speed and cost effectiveness, the program was used for the automated structural design of a general purpose cargo vessel. In order to make the exercise as realistic as possible, the overall dimensions, design loads and other design specifications were based on the SD14, obtained from [9].

The specific compartment modelled was the number four hold, just aft of midships. The com-

Component	Type of Load
I	A symmetric hogging moment of 94500 ft.tons
II	A symmetric sagging moment of 94500 ft.tons
III a	Deep draft (hogging) of 40 ft.
b	Deep draft (sagging) of 18 ft.
c	Light draft (hogging) of 18 ft.
d	Light draft (sagging) of 6.5 ft.
IV a	2.33 psi on strength deck and 2.85 psi on lower deck.
b	8.50 psi on tank top.

TABLE 1: COMPONENTS OF LOAD CASES

partment was modelled using six strakes (Figure 2), which resulted in a total of 74 design variables and 542 constraints of which 220 are non-linear. The design load system is made up of five separate load cases. The components of the load cases are given in Table 1 and the design load system is summarized in Table 2.

The min - max and proportionality constraints are summarized in Tables 3 and 4 respectively. The non-linear constraints are described in Section 4.2. MAESTRO automatically deletes any design variables and constraints which are not relevant for a particular strake. For example, the design variables and constraints related to a girder are automatically suppressed for strakes 2 and 3 (side shell).

Initially, the height of the double bottom and the web height of the deck beams were treated as variables. It was found that from a structural standpoint the optimum height of the double bottom and of the deck beams was approximately 30 inches and 27 inches respectively. It was concluded that these dimensions were determined for the SD14 from non-structural considerations. Accordingly, in the MAESTRO design these dimensions were fixed at the corresponding SD14 values, using a standard MAESTRO feature for this purpose.

The program was run from an arbitrary initial feasible (A.I.F.) design and an arbitrary initial infeasible (A.I.I.) design. For the A.I.F. design, the initial variables are taken as one half of their respective maximum values (which are themselves arbitrary). For the A.I.I. design, the initial variables for each strake are set at their respective minimum values,

6.2 Stability and Convergence

The changes in the total cost from one shipcycle to the next are shown in Figure 4. The initial cost of the A.I.F. design is 102.97 units. There is a dramatic decrease to 43.37 units in the first shipcycle. At the end of the fourth and fifth shipcycles the cost is 39.24 and 39.21 units respectively. The five cycles required 273 seconds of CPU time on a CYBER 72 which corresponds to a commercial cost ($600 per hour) of approximately $45.

In order to check the stability of the solution MAESTRO was permitted to run for 10 shipcycles. The variation in cost after the fourth shipcycle does not exceed 1.4 percent.

Load Case	Description	Components
1	Hogging deep draft & b.mom,hold empty	I,IIIa,IVa
2	Sagging deep draft & b.mom,hold empty	II,IIIb,IVa
3	Hogging deep draft & b.mom,hold full	I,IIIa,IVb
4	Hogging light draft & b.mom,hold full	I,IIIc,IVb
5	Sagging light draft & b.mom,hold full	I,IIId,IVb

TABLE 2: DESIGN LOAD SYSTEM: LOAD CASES

	Min.	Max.
S	1	10
t	0.4	3.0
h_s	1.0	24.0
t_s	0.2	2.0

	Min.	Max.
h_g	12.0	65.0
t_{gw}	0.3	2.0
h_{gf}	5.0	50.0
t_{gf}	0.4	2.0

	Min.	Max.
h_f	5.0	65.0
t_{fw}	0.2	2.0
h_{ff}	5.0	50.0
t_{ff}	0.2	2.0

TABLE 3: MIN - MAX CONSTRAINTS (Inches)

All Strakes	Shell & Decks
$h_s \geq 36t_s$	$b_{ff} \geq 0.2h_f$
$t_s \leq 2t_{ps}$	$b_{ff} \leq 0.8h_f$
$t_{fw} \leq 2t_{ps}$	$t_{fw} \geq 0.5t_{ff}$
$h_f \geq h_s$	$t_{fw} \leq 2.0t_{ff}$
Bottom & Decks	**Decks**
$t_{gw} \leq 2t_{ps}$	$t_{gw} \geq 0.5t_{gf}$
$t_{gw} \geq 0.25t_{fw}$	$t_{gw} \leq 2t_{gf}$
$t_{gw} \leq 4t_{fw}$	$b_{gf} \geq 0.2h_g$
$h_g \leq 120t_{gw}$	$b_{gf} \leq 0.8h_g$
$h_f \leq 120t_{gw}$	$h_g \geq h_f$

TABLE 4: PROPORTIONALITY CONSTRAINTS

The strake costs are summarized in Figure 5. The percentage differences in cost between the fourth and fifth shipcycles are shown in brackets. For each strake the cost has stabilized by the fourth shipcycle. This is an indication that the design is at least a local optimum.

In order to verify that the solution is a global optimum the problem was re-solved using an arbitrary initial infeasible design. From this starting point the first feasible design was generated in the third shipcycle (Figure 4). At the end of the fourth and fifth shipcycles the cost is 37.36 units and 37.47 units respectively. For the fifth shipcycle the cost is 4.4 percent different from that of the A.I.F. design. In order to verify that these are in fact the same design, a comparison was made between the active constraints for all strakes for both the A.I.F. and the A.I.I. designs. For the bottom, strength deck and second deck the correspondence is exact, and in the side shell only two constraints are different. The active constraints for the bottom and the strength deck are shown in Tables 5 and 6. The locations and load cases which governed the design are also shown in the tables.

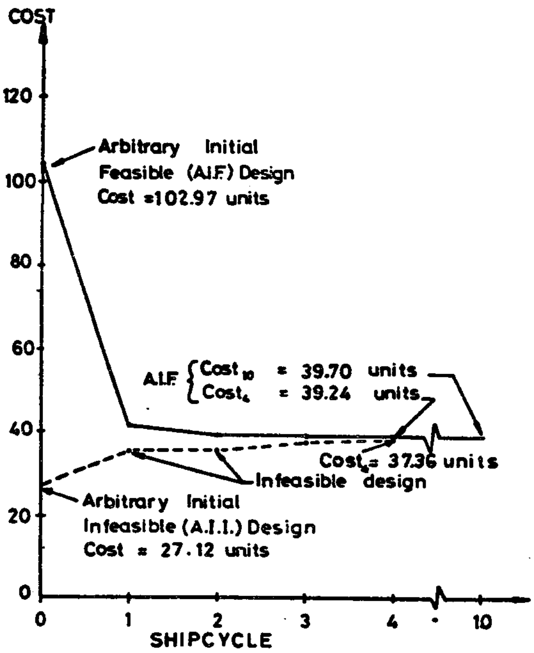


FIGURE 4 : RELATIVE STRUCTURAL COSTS FOR SHIP

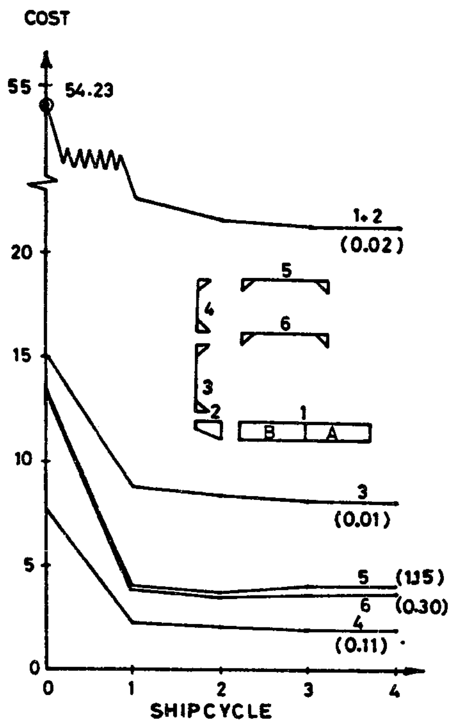


FIGURE 5 : STRAKE COSTS - ARBITRARY INITIAL FEASIBLE DESIGN

On the basis of the above evidence the authors conclude that for all practical purposes the MAESTRO design represents a global optimum because:

(a) both the overall cost and the individual strake costs achieve a stable minimum value.

(b) from two arbitrary starting points (one which is feasible and the other which is not) MAESTRO converges to the same design as verified by comparing the active constraints.

6.3 Further Details of the MAESTRO Design

The details of the MAESTRO design for the double bottom and the strength deck are shown in Tables 8 and 9 respectively. The rationalised MAESTRO designs are also shown.

As indicated earlier, the optimization method used in MAESTRO requires the sequential linearization of the non-linear constraints. In order to avoid oscillation of the design, a few constraints are accumulated from one strake to the next. Such constraints are identified in the tables with the prefix ACC. To improve convergence, a limit has

	Active Constr. 1A	Active Constr. 1B	Frame Number	Critical Loadcase 1A	Critical Loadcase 1B
Outer Bottom, Girder, Frame	$h_{so} = 36t_{so}$				
	$h_g = 120t_{gw}$				
	$h_f = 120t_{fw}$				
	PYCF	PCSB	4	1	
	PYPB		4	1	
	ACC.PYPB		4	1	
Tank Top	$t_{si} = 36h_{si}$				
	PYTF	PYCF	4	5	3
	PYPB		4	5	5
	ACC.PYPB		4	4	5

TABLE 5: ACTIVE CONSTRAINTS - DOUBLE BOTTOM

Active Constr. (A.I.F. & A.I.I.)	Frame Number	Critical Loadcase
$t_{gf} = 0.4$		
$h_s = 36t_s$		
$h_g = 120t_{gw}$		
$b_{gf} = 0.2h_g$		
$t_{gw} = 0.5t_{ff}$		
PCSB	2	2
PCOB	4	5
ACC.PCOB	4	5
GYTF	4	1
FYFC	4	1

TABLE 6: ACTIVE CONSTRAINTS - STRENGTH DECK

	Initial (A.I.F.)	Shipcycle 5		Rationalised	
		1A	1B	1A	1B
s_o	7	4.66	5.64	5	6
t_{po}	1.20	0.58	0.62	0.60	
h_{so}	16.00	10.13	11.21	11.00	
t_{so}	1.20	0.28	0.31	0.30	
s_i	7	3.90	5.66	4	6
t_{pi}	1.20	0.48	0.46	0.5	
h_{si}	16.00	8.11	7.54	8.00	
t_{si}	1.20	0.23	0.21	0.25	
t_{gw}	1.20	0.39	0.39	0.40	
t_{fw}	1.20	0.39	0.39	0.40	
h	FIXED AT 46.5			46.5	

TABLE 7: DOUBLE BOTTOM - DESIGN VARIABLES (inches)

	Initial (A.I.F.)	Shipcycle 5	Rationalised
s	7	5.02	5
t_p	1.20	0.44	0.50
h_s	16.00	8.65	8.50
t_s	1.20	0.24	0.25
h_g	40.00	48.46	48.50
t_{gw}	1.20	0.40	0.40
b_{gf}	40.00	16.16	16.00
t_{gf}	1.20	0.46	0.50
h_f	FIXED AT	14.00	14.00
t_{fw}	1.20	0.46	0.50
b_{ff}	20.00	11.20	11.50
t_{ff}	1.20	0.92	1.00

TABLE 8: STRENGTH DECK - DESIGN VARIABLES (inches)

been imposed on the change in any variable from one strake to the next. For this reason, the values shown in Tables 7 and 8 do not exactly satisfy the active constraints shown in Tables 5 and 6.

6.4 Principal Attributes of MAESTRO

The above example illustrates the two principal attributes of MAESTRO:

(i) the MAESTRO design process is rapid, low-cost, accurate and versatile. The calculation of scantlings requires many man-hours by normal techniques, and if the design is to be particularly efficient, or if it needs to be rationally based due to some non-standard features, then it requires many man-weeks of iterative design. In contrast to this, MAESTRO can produce a rationally-based optimal design for less than $50. Using the SD14 design specifications and SD14-derived values for load factors and costs, and starting from quite arbitrary initial scantlings, MAESTRO produced a design which is quite similar to the SD14. Since the SD14 is a prototype ship, it is likely that special efforts were made to make it an efficient and cost effective design. Hence the rapid convergence of MAESTRO to the SD14 design does much to demonstrate the inherent accuracy and sound philosophy (and hence trustworthiness) of MAESTRO.

(ii) since the resulting design is optimal (based on the actual costs, or whatever other measure of merit is specified) it will yield significant savings, due to lower initial cost and/or increased profitability, compared to a non-optimal "rules-designed" vessel.

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