

OPTIMUM DESIGN OF PROPELLERS

USING SERIES DATA

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ABSTRACT

The design of a marine screw propeller, regardless of the method, is a laborious task, and in most cases requires many iterations to obtain a propeller that satisfies the essential requirements at high efficiency.

A FORTRAN computer program has been developed, utilizing a mathematical optimisation procedure in finding the characteristics of a marine propeller for maximum efficiency in a particular application. The program uses an optimisation technique called the "Better Point Algorithm", and employs mathematically faired data of B-series screws. The design is subject to assurances of non-cavitation and adequate blade strength, and considerations of delivered power and vibration. The program has applications in the preliminary design of almost any non-cavitating marine propeller, and the detailed design of any light to moderately loaded propeller of the B-series.

SYMBOLS AND ABBREVIATIONS

A_p	: Projected blade area	(m ²)
B.A.R.	: Blade area ratio = $\frac{\text{Expanded blade area}}{\text{Disc area}}$	
B_p	: Loading coefficient (based on power)	
B_u	: Loading coefficient (based on thrust)	
c	: Chord length at any radius	(m)
$c_{0.75R}$: Chord length at 0.75R	(m)
C_1	: Coefficient for blade thickness determination	
C_B	: Block coefficient	
C_D	: Drag coefficient of blade	
D	: Propeller diameter	(m)
dhp	: Engine delivered horsepower	(metric hp)
dhpc	: Power absorbed by screw	(metric hp)
D_{LG}	: Distance from leading edge to generating line of a blade section	(mm)
D_{LM}	: Distance from leading edge to maximum thickness of a blade section	(mm)
f_v	: Hull vibration frequency	(cycles/min.)
h	: Height from 0.7R on vertical upright blade to water surface	(m)
i	: Tangent of one half of the angle of rake	
J	: Advance coefficient	
K_Q	: Torque coefficient	
n	: Propeller revolutions per second	
N	: Propeller revolutions per minute	
NSMB	: Netherlands Ship Model Basin	
P	: Propeller pitch (at 0.7R)	
P_D	: Torque power	(metric hp)
P_T	: Thrust power	(metric hp)

P/D	: Pitch/Diameter ratio
q	: Relative fluid velocity over the blade at $0.7R$ (m/s)
Q	: Torque (Nm)
r	: Radius (variable) (m)
R	: Propeller radius (m)
R_n	: Reynolds Number
$R_{n0.75R}$: Reynolds Number at $0.75R$
rpm	: Revolutions per minute
S_c	: Maximum allowable compressive stress in blade (kPa)
shp	: Engine shaft horsepower (metric hp)
S_R	: Real slip
T	: Thrust (N)
T_{AP}	: Draft at aft perpendicular (m)
t/c	: Thickness to chord ratio
$t/c_{0.75R}$: Thickness to chord ratio at $0.75R$
t_o	: Thickness at zero radius (extrapolated to shaft centreline) (m)
t_r	: Thickness at a characteristic radius (mm)
t_{te}	: Thickness of trailing edge (mm)
V_A	: Speed of advance (m/s)
V_S	: Ship speed (m/s)
W_T	: Taylor wake fraction
Y_{b_r}	: Back ordinate of blade section at a character- istic radius (mm)
Y_{f_r}	: Face ordinate of blade section at a character- istic radius (mm)
Z	: Number of blades
Γ	: Strength of circulation flow (m^2/s)
δ	: Taylor advance coefficient

η_0	: Open water efficiency
θ	: Angle for elliptical search
μ	: Torque constant
ν	: Kinematic viscosity (m ² /s)
ρ	: Fluid density (kg/m ³)
σ	: Cavitation number
σ'	: Thrust constant
ϕ'	: Speed constant
ϕ_4	: Coefficient for blade thickness determination
K_T	: Thrust Coefficient

1. PROPELLER OPTIMISATION PROBLEM

1.1 INTRODUCTION

With ship operating costs rising sharply in recent years, there is a need for ensuring the maximum efficiency of the transfer of engine horsepower to propeller thrust. In the process of this transformation of power, it is the propeller that is the final element, and it is the optimisation of the efficiency of the propeller that is the subject of this study.

In the design of marine screw propellers, the objective must be to determine the characteristics of the screw that will absorb the available power at maximum efficiency, yet not cavitate or induce resonant hull vibration, be of sufficient strength, and yet still propel the ship at the required speed. In the past, the search for maximum efficiency has been an essentially tedious and iterative process, and regardless of the method chosen, the use of charts and graphs was unavoidable. With the recent development at the Netherlands Ship Model Basin (NSMB) of sufficient data to express the efficiency of B-screw series propellers as a mathematical function of their various dimensions and characteristics {12},* the design of series propellers may now be computer aided.

This study describes the use of the NSMB data, and data from several other sources, in the development of a FORTRAN

* Numbers in brackets indicate references in Section 7

computer package, called PROPOPT, for the mathematical optimisation of the efficiency of B-screw series propellers. The optimisation, subject to considerations of cavitation, delivered power, blade strength and vibration, is performed using a method called the "Better Point Algorithm".

For most applications of the marine screw propeller, the use of PROPOPT in the selection of propeller dimensions and characteristics will suffice not merely for preliminary design, but also for detail design. In the case of detailed design, the output is suitable for use directly by the draftsman and the manufacturer in the production of the optimum B-series screw.

There are many cases, such as vessels operating under highly loaded conditions (tugs, trawlers, naval vessels, etc.), where screws designed using B-series data may be significantly improved upon using a more theoretical approach. To meet this requirement, the design may be performed by utilising PROPOPT, and the chosen screw may be used as a basis for the ensuing theory.

1.2 THE B-SCREW SERIES

The Wageningen B-screw series, upon which this study is based, is comprised of 21 individual screw series, having blade numbers ranging from two to seven and blade area ratios between 0.3 and 1.05. The characteristics of the B-series are well described in references {3,13,21,22}. A summary of the series is given in Table 1 {21}.

	Number of Blades					
	2	3	4	5	6	7
Blade	0.30	0.35	0.40	0.45	0.50	0.55
Area		0.50	0.55	0.60	0.65	0.70
Ratio		0.65	0.70	0.75	0.80	0.85
		0.80	0.85	0.90		
			1.00	1.05		

Table 1 Summary of B-Screw Series Geometry

Due to the equivalence of blade profile and blade sections of B-series screws, these characteristics may be expressed in the form of formulae {13}, such as equations 21 - 27.

The results of model tests of each individual series are, over a range of P/D from 0.5 to 1.4, expressed in the form of charts such as the K_T - K_Q - J charts {22}. In the recent past the mathematical fairing of these charts has been completed, at NSMB, by means of a regression analysis {12}. As the thrust and torque coefficients are defined for a standard Reynolds No. and blade thickness at a characteristic radius, the effect of different values of these parameters on the coefficients has been determined. As such, the following polynomials have been developed:

$$K_T = f_1 (J, P/D, B.A.R., Z, R_n, t/c)$$

$$K_Q = f_2 (J, P/D, B.A.R., Z, R_n, t/c)$$

The details of these polynomials are shown in Appendix B.

1.3 PROBLEM FORMULATION

The problem is to develop a method by which the characteristics of a marine propeller can be selected for optimum efficiency. As the problem is one of optimisation, an objective function is formulated, the decision variables are selected, and the constraints are framed.

1.3.1 Objective Function

With the criterion for optimisation being efficiency, the objective function is stated as;

$$\text{Maximise: } \eta_o = \frac{J}{2\pi} \frac{K_T}{K_Q}$$

subject to constraints of delivered power, non-cavitation, and free variables lying within allowable ranges. The mathematical model for the solution of the optimisation problem is shown in Appendix A.

1.3.2 Decision Variables

The thrust and torque coefficients, as seen in Appendix B (equations 6 and 8)*, are expressed as polynomials {12} which are functions of advance coefficient, pitch ratio, blade area ratio, number of blades, Reynolds Number and thickness to chord ratio, as shown:

$$K_T = f_1 (J, P/D, B.A.R., Z, R_n, t/c)$$

$$K_Q = f_2 (J, P/D, B.A.R., Z, R_n, t/c)$$

The Reynolds Number is a function of the velocity of fluid flow over the blade, and the chord length of the blade

at a characteristic radius . As the chord length of a B-series screw is a function of diameter, blade area ratio and number of blades (equation 9), the Reynolds Number is expressed as:

$$R_n = f(n, D, B.A.R., Z)$$

The thickness to chord ratio is a function of the blade dimensions, the propeller loading and the maximum allowable blade stress (equations 10, 12) and may be represented as:

$$t/c = f(n, D, P/D, B.A.R., Z, S_c)$$

Furthermore, based on a method developed by Lerbs{13}, it has been shown that a variation in the thickness to chord ratio may be represented by a change in Reynolds Number. This effect has the consequence of affecting values of the torque and thrust coefficients only through a modified Reynolds Number.

The Reynolds Number and the thickness to chord ratio may therefore be eliminated as free variables in the K_T and K_Q polynomials. This will be discussed in details shortly.

With the advance coefficient, J , being a function of the propeller revolution and the diameter, for a constant speed of advance, the open water efficiency - the objective function - may now be expressed as a function of the following free variables:

$$\eta_o = g(n, D, P/D, B.A.R., Z)$$

* Equation numbers refer to equations listed in Appendix B

The number of blades, Z , is not taken as a decision variable because the range of rpm is chosen from considerations of non-resonant hull vibration for a particular number of blades. However, the effect of a variation of blade number may be studied by changing the range of rpm, this range being determined from the hull frequency diagram.

Therefore, the number of blades is taken as a design parameter and not a free variable. Thus, the decision variables are:

$$n, D, P/D, \text{ and } B.A.R.$$

and the objective function is represented by:

$$\eta_0 = f(n, D, P/D, B.A.R.)$$

The details of the objective function are shown in Appendix B, equations 3 - 16.

1.3.3 Constraints

The optimisation is subject to one equality constraint and nine inequality constraints. These are:

Equality: the power available from the engine must equal the power absorbed by the screw.

$$dhp = dhp_c \quad (\text{equation 17})$$

Inequality: the free variables must lie within their allowable ranges.

$$n \geq n_{\min}$$

$$n \leq n_{\max}$$

$$D \geq D_{\min}$$

$$D \leq D_{\max}$$

$$P/D \geq 0.5$$

$$P/D \leq 1.4$$

$$\text{B.A.R.} \geq 0.3$$

$$\text{B.A.R.} \leq 1.05$$

the cavitation number must lie below a critical value

$$\left(\frac{T/A_p}{q} \right) \leq \left(\frac{T/A_p}{q} \right)_{\text{crit}} \quad (\text{equation 20})$$

2. SOLUTION

2.1 METHODOLOGY

In this study, the problem is solved by a mathematical optimisation procedure known as the "Better Point Algorithm", a method which not only provides solutions to non-linear objective functions with inequality constraints, but also satisfactorily deals with the additional problems associated with the inclusion of equality constraints. Further description of the algorithm is not given here, and the reader is referred to reference {8}, where the methodology is well documented. The "Better Point Algorithm", as adapted to the solution of ship design problems, is shown in references {5,8,14,15}.

The optimisation method as used utilizes a minimisation procedure, and it is therefore necessary to restate the objective function:

$$\text{Maximise } \eta_0 = \left| \text{Minimise } (-\eta_0) \right|$$

2.2 PROCEDURE

(a) Thrust and Torque Coefficients

The polynomials for K_T and K_Q are evaluated in two stages.

Firstly: $K_T = f_3 (J, P/D, B.A.R., Z)$ (equation 6)

$K_Q = f_4 (J, P/D, B.A.R., Z)$ (equation 8)

with the Reynolds Number at $0.75R$ assumed to be constant at 2×10^6 , and the thickness to chord ratio as determined by equation 10.

A Reynolds Number greater than 2×10^6 , and a non-standard blade thickness, are represented by changes in the thrust and torque coefficients, viz:

$$\Delta K_T = f_5 (J, P/D, B.A.R., Z, R_n) \quad (\text{equation 15})$$

$$\Delta K_Q = f_6 (J, P/D, B.A.R., Z, R_n) \quad (\text{equation 16})$$

The total value of thrust and torque coefficients are given

$$\text{by } [K_T]_{TOT} = K_T + \Delta K_T$$

$$[K_Q]_{TOT} = K_Q + \Delta K_Q$$

(b) Evaluation of Blade Thickness

The Technique used in this study for the determination of blade thickness for adequate blade strength is not unique to the B-screw series. It is a method developed by Saunders {20}, and is shown below. The critical mode of failure of a propeller blade is, for the purposes of this study, regarded as failure in compression.

$$\frac{C_1 \times \text{shp}}{4.123ND^3Z(t_o/D)^3} + \frac{DN^2i\phi_4}{12788(t_o/D)} = S_c + \frac{D^2N^2}{12788}$$

The first term of this equation represents the compressive stress, the second term shows the increase in compressive stress due to the centrifugal force effect of blade rake, and the right hand side represents the allowable stress. The equation is in imperial units.

The coefficients C_1 and ϕ_4 are dependent on the P/D ratio and are shown in figure 1. They are translated to a usable numerical form by expressing ϕ_4 as a linear function of P/D , and C_1 as a logarithmic function of P/D (figure 2).

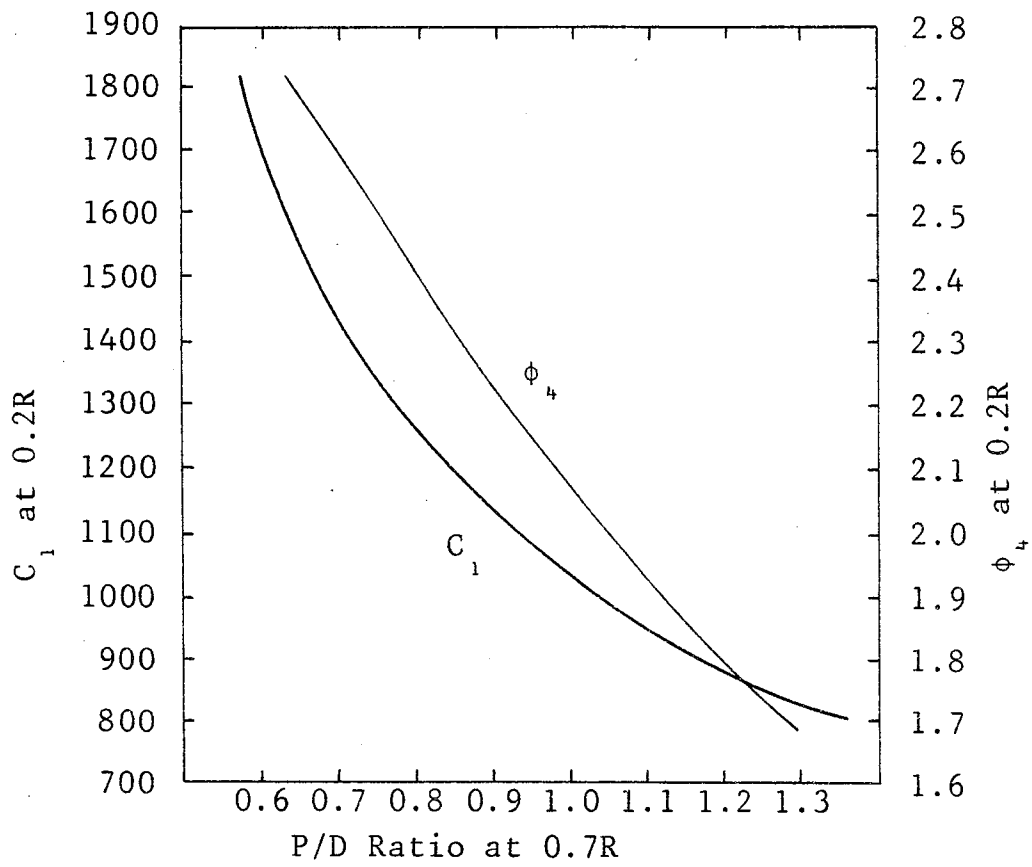


Figure 1 Values of Coefficients, C_1 and ϕ_4

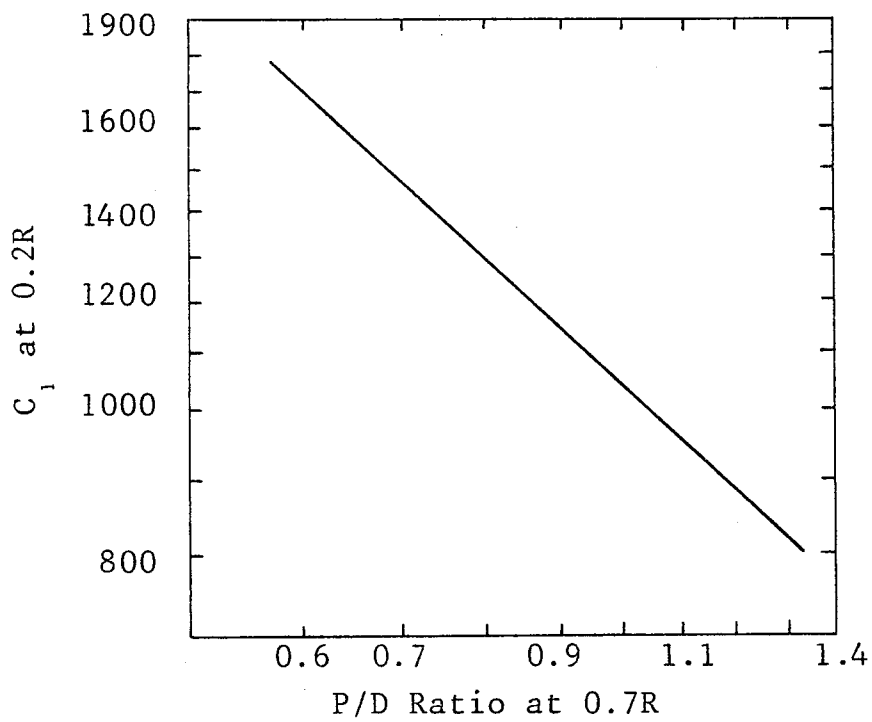


Figure 2 Value of C_1 (Logarithmic Scales)

The coefficient i is the tangent of one-half the angle of rake and, as B-series screws are raked at a standard 15° , this is a constant term.

The equation for t_0 is a cubic, and is solved implicitly, as equation 12 (in metric units).

(c) Reynolds Number

For use in the NSMB thrust and torque polynomials, the Reynolds Number is expressed at a radius ratio of 0.75. It is therefore determined as a function of the chord length at 0.75R, and the resultant of the two velocity vectors - speed of advance, and blade rotational velocity.

The Reynolds Number is given as equation 11 in Appendix B.

(d) Blade Thickness Effect

The effect of a change in the thickness to chord ratio is represented by a specific change in the Reynolds Number, by a relationship developed at NSMB {12}.

This can be explained by considering a plot of the drag coefficient, C_D , of a particular B-series screw against the advance coefficient, J . The effect of an increase in Reynolds Number has been shown to correspond to a vertical shift of the C_D vs J curve. Similarly, it has been shown that an increase or decrease in the thickness to chord ratio also corresponds to a shifting upwards or downwards of the curve.

Based on these effects, a relationship has been developed to relate a change of thickness to chord ratio to a corresponding change of Reynolds Number. This is shown

as equation 14, and it is this modified value that is used in the ΔK_T and ΔK_Q polynomials.

(e) Non-cavitation

As was the case in determining the blade thickness, the criteria used for ensuring non-cavitation is not applicable only to screws of the B-series. Figure 3 shows the experimentally determined boundary curves of Burrill, a mean curve frequently used by NSMB in connection with the B-series, and a theoretical boundary curve by Lerbs {22}.

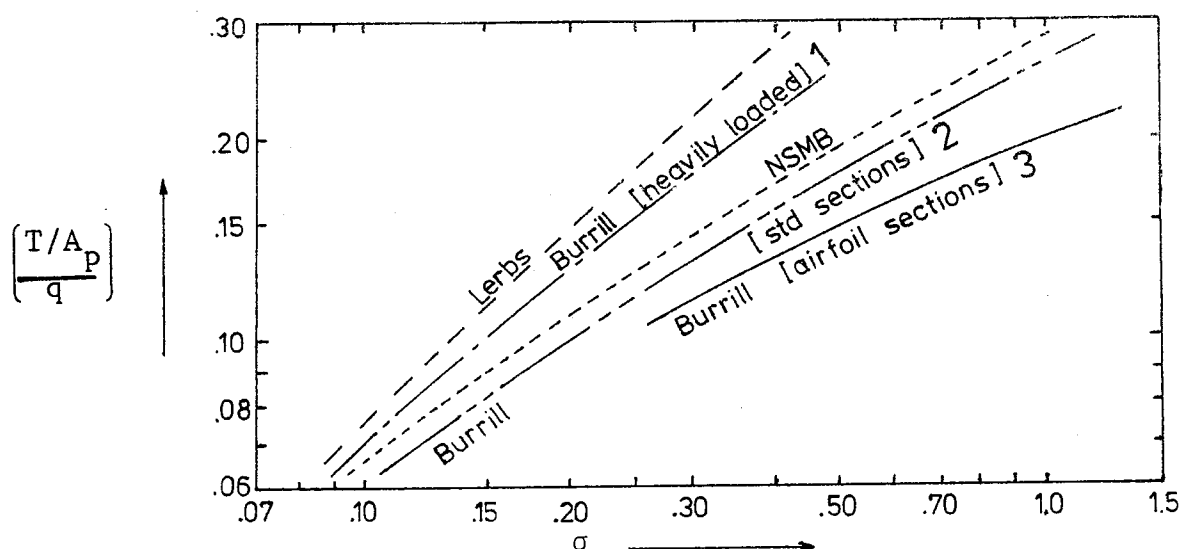


Figure 3 Cavitation Chart

The curves used in this study are those of Burrill. The type of loading of the propeller is specified, and the corresponding curve selected. These relationships are expressed numerically by linearising the logarithmic function in discrete segments, as shown in figure 4 for the example of the standard merchant screw.

The linearisation of these curves actually creates an additional "factor of safety" against cavitation, as the numerical curve lies below the experimental curve. The ratio

of thrust per unit blade area, T/A_p , to the stagnation pressure, $\frac{1}{2}\rho V^2$, is then a logarithmic function of the cavitation number, the nature of the function depending on the range of cavitation number within which the designed screw lies. The functions are given as equation 20 in Appendix B.

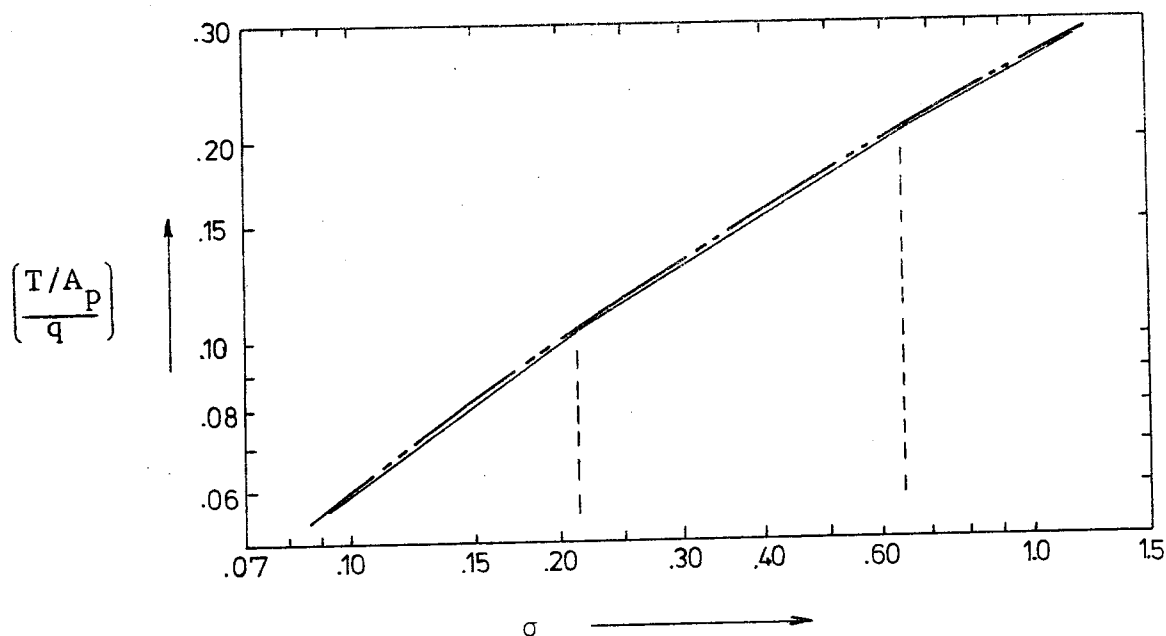


Figure 4 Linearisation of Cavitation Chart

2.3 THE PROGRAM

Based on the B-screw series data and utilizing the "Better Point Algorithm", a FORTRAN code titled PROGRAM PROPOPT has been developed to obtain the characteristics of a marine propeller for optimum efficiency.

The program is divided into three major sections:

(a) Data Entry and Manipulation

- Ship data
- Optimisation data
- Data manipulated into a usable form where necessary
- Check for data set within range of B-screw series data

(b) Optimisation

- Step sizes
- Accuracies and tolerances of constraints
- Characteristics for optimum efficiency

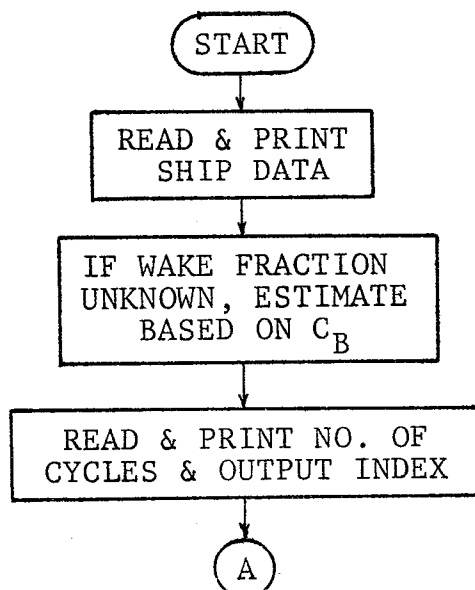
(c) Presentation of Results

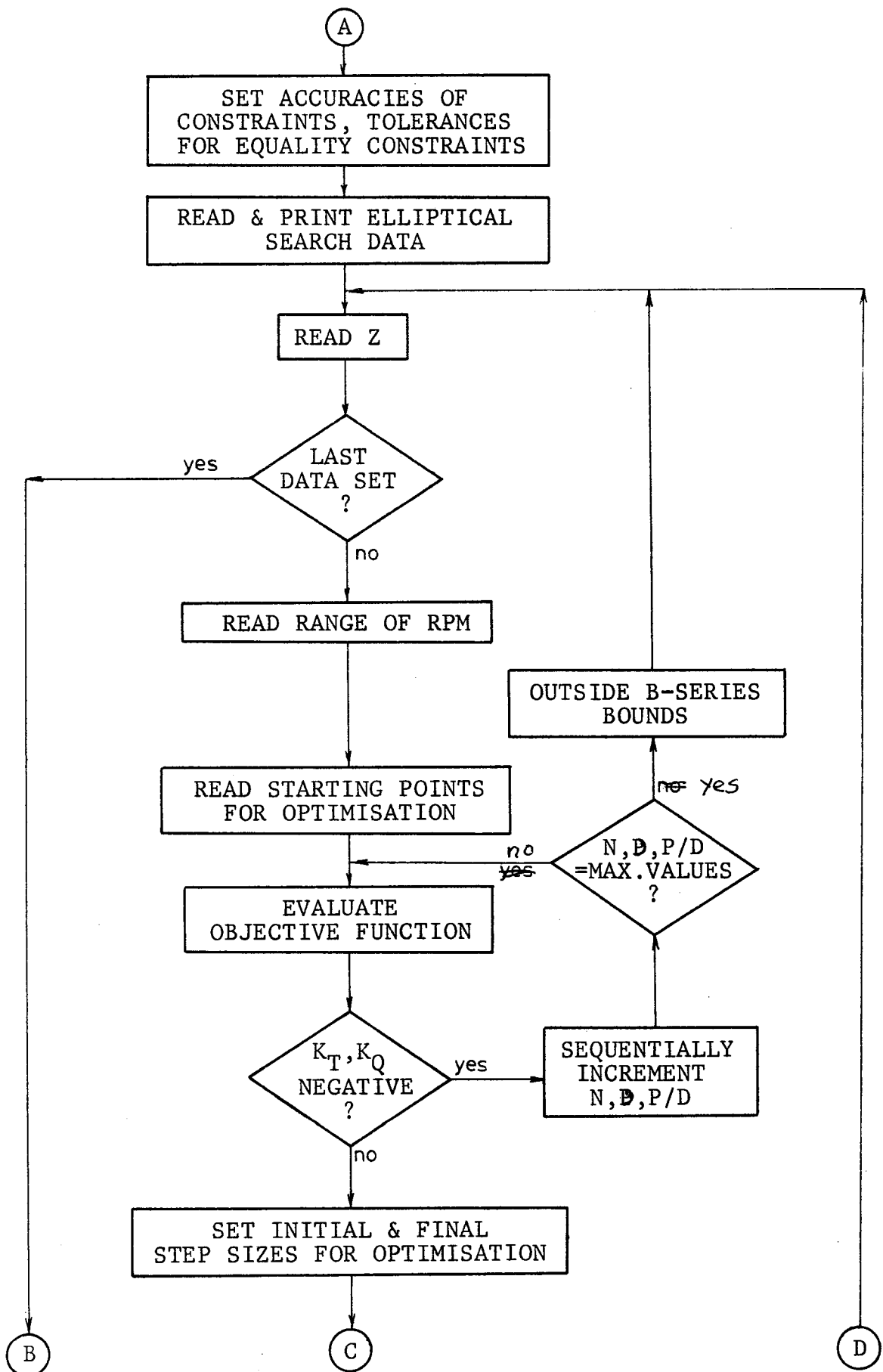
- Optimum efficiency
- Propeller characteristics at optimum efficiency
- Best alternative (if several data sets considered)
- Blade sections (if detail design performed)

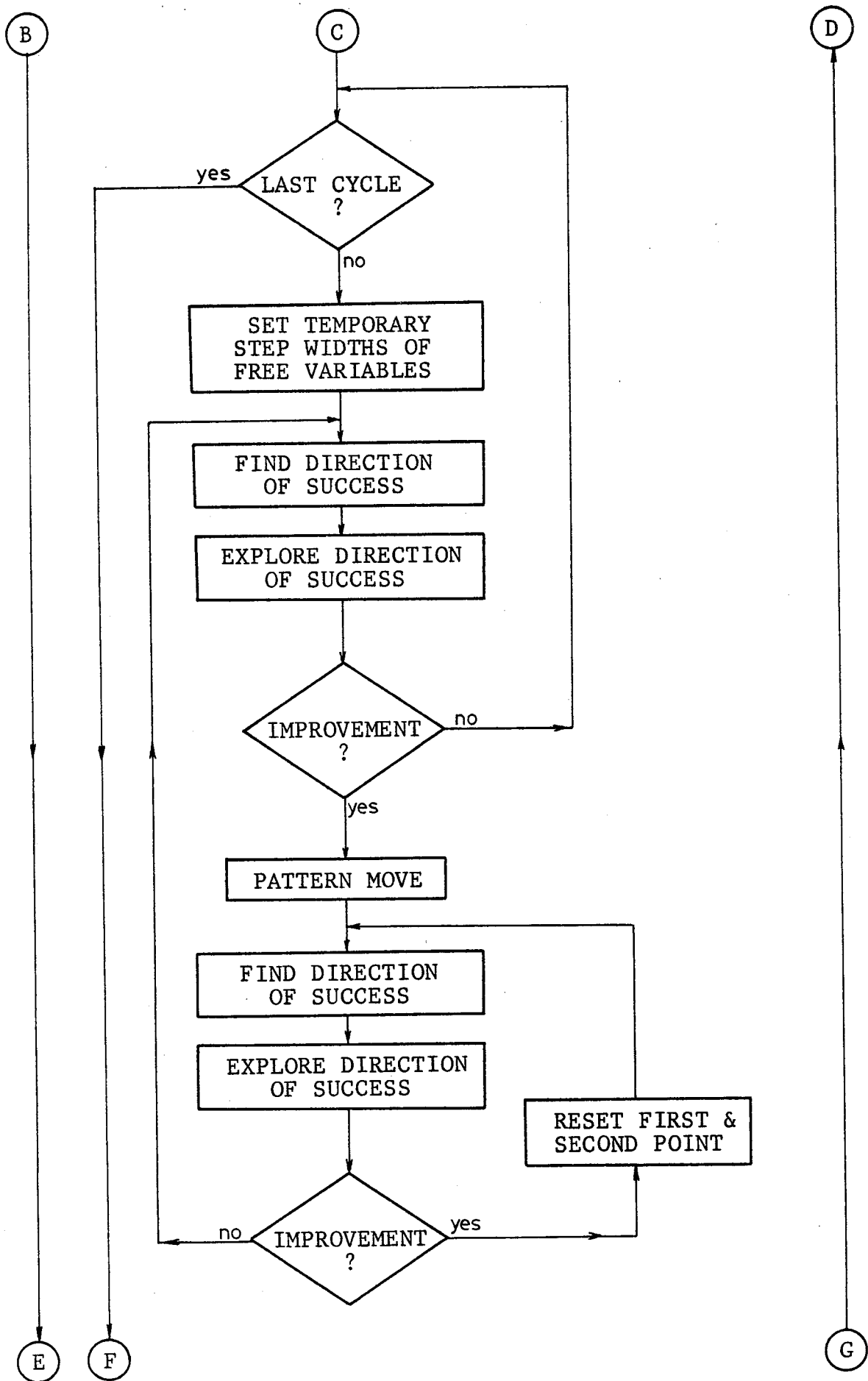
The manner by which PROGRAM PROPOPT performs the functions stated above is best explained by considering firstly the program logic diagram, and secondly the output of a sample design exercise (Appendix D) which will be described shortly.

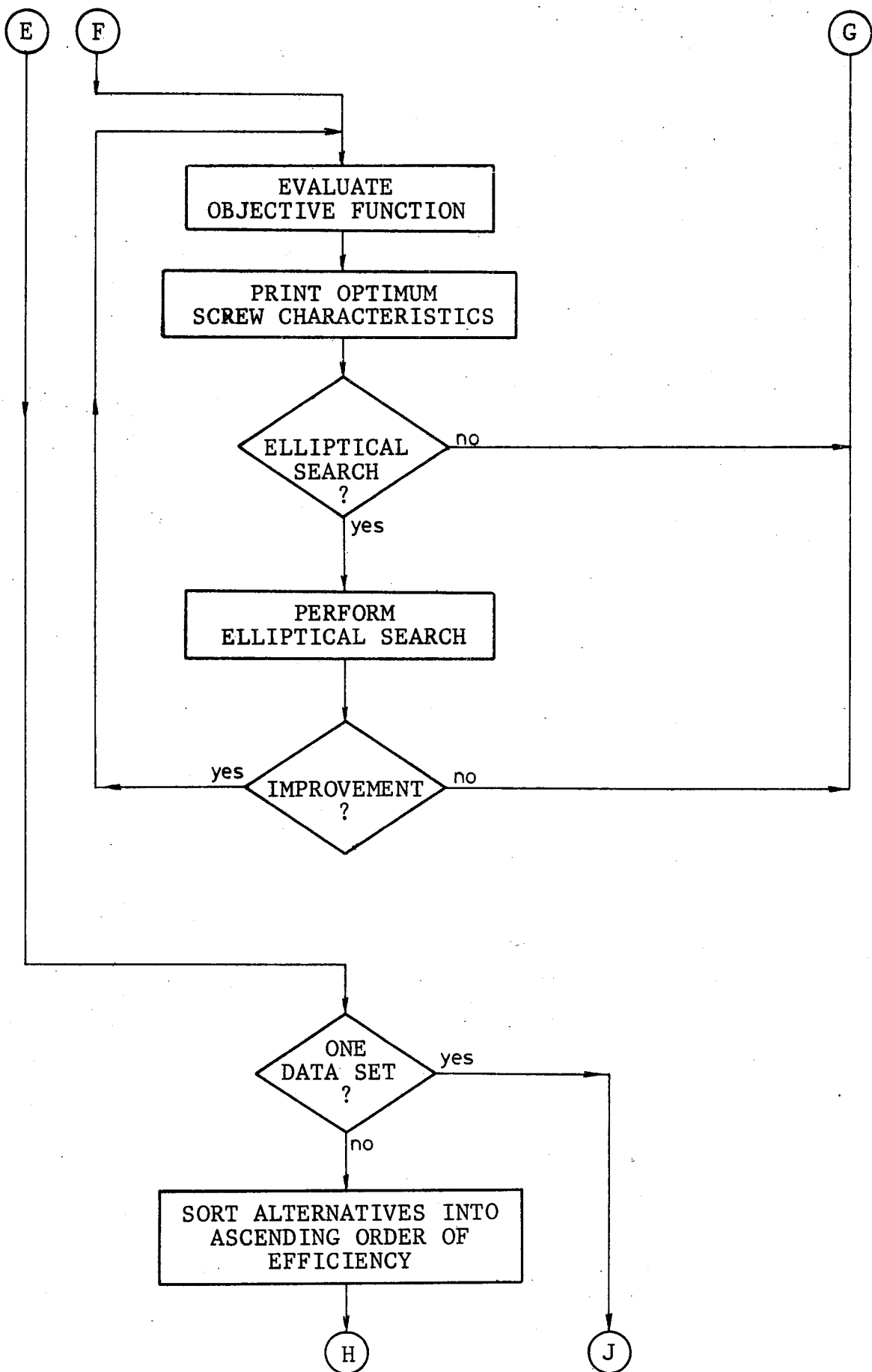
2.3.1 Program Propopt - Logic Diagram

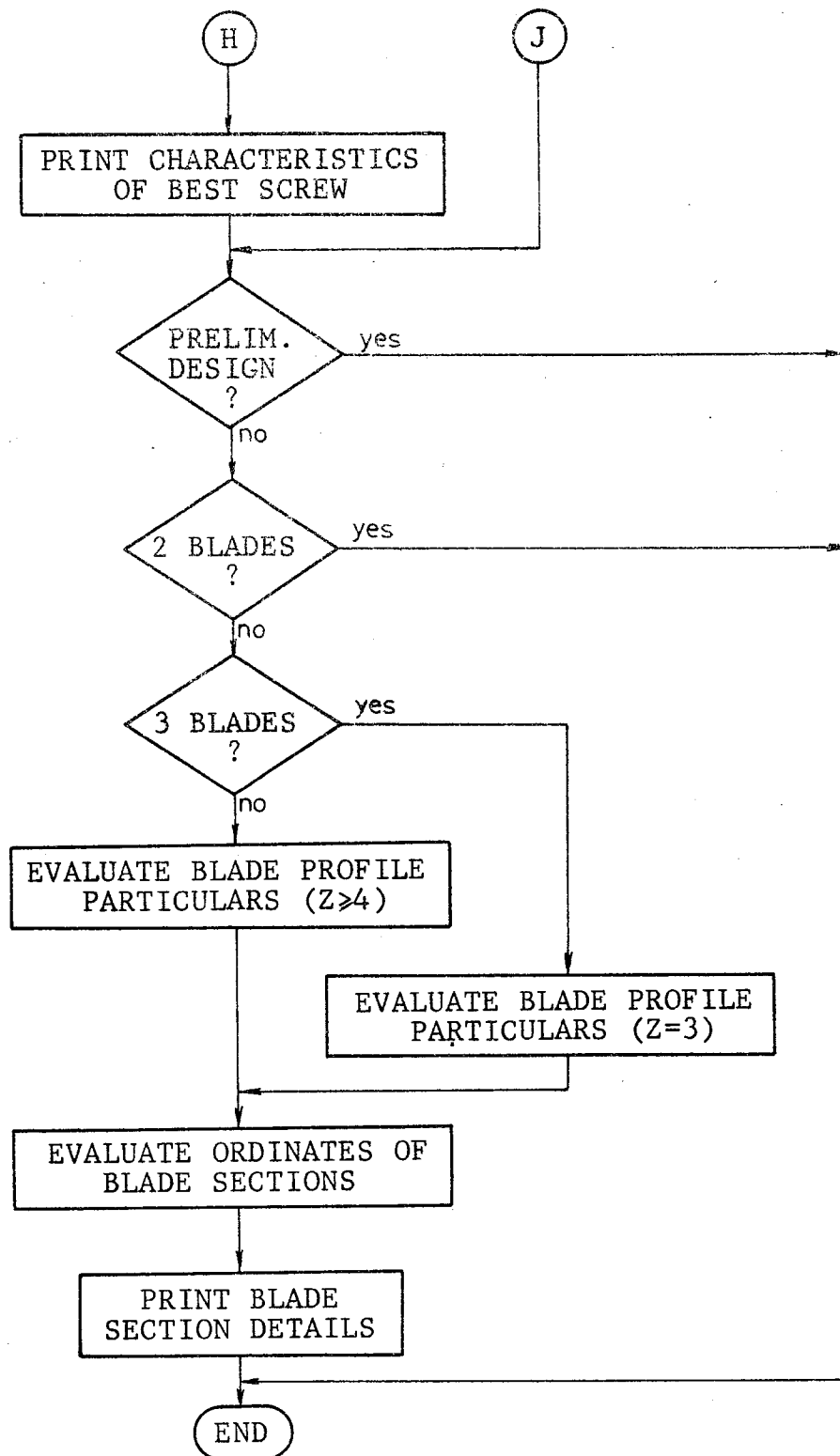
A logic diagram of the main program is presented here. The logic diagrams of each of the 22 subroutines are not given, nor is the program listing, and the reader is referred to reference 5 for these details. The listing may also be obtained from the authors.











2.4 SAMPLE DESIGN EXERCISE

To demonstrate the application of program PROPOPT, the design of a B-series screw is performed here, for a 21,000 tonne displacement single screw containership of typical proportions. Several alternatives are considered in the preliminary design, with the "best" screw being the basis of detailed design.

(a) Preliminary Design

For the purposes of this example, it is assumed that the preliminary hull design has been performed, the power estimates completed, and a preliminary hull frequency diagram prepared.

The vessel characteristics required by PROPOPT are:

21,000 tonne displacement containership

Shaft Horsepower : 11,000 metric horsepower down
one shaft

Ship Speed : 22 knots

C_B : 0.66

T_{AP} : 8.00 m

W_T : 0.34

Max. Blade Stress : 48.0 Mpa
(compression)

Allowable Range of
Diameter : 4.8 m to 5.4 m

Standard Merchant Ship Propeller

From the hull frequency diagram, acceptable ranges of shaft revolutions and the corresponding blade numbers are

determined. Within the allowable ranges of rpm, diameter, P/D ratio and blade area ratio there may exist several local optima, and as such several starting points are considered for each blade number, to find the global optimum. Table 2 displays the effect of varying the starting value of blade area ratio only, on the characteristics at optimum efficiency.

No. of blades : 4
 Range of rpm : 190 - 210
 Starting Points, N : 200
 D : 5.0
 P/D : 0.8
 B.A.R. : Variable

B.A.R.	Variables at optimum				
	N	D	P/D	B.A.R.	η_0
0.45	203.2	4.826	0.65	0.627	0.627
0.5	203.6	4.820	0.65	0.631	0.627
0.6	203.2	4.826	0.65	0.627	0.627
0.7	203.6	4.820	0.65	0.631	0.627
0.8	196.8	4.766	0.70	0.655	0.633

Table 2 Effect of Varying B.A.R. Starting Value Only

The effect of varying the starting values of the other variables is examined, and a summary of the results of preliminary design, and the corresponding starting points are shown in Tables 4 and 3 respectively, for the "best" screw of each blade number. A sample output is included in Appendix C.

Z	Nmin	Nmax	N	D	P/D	B.A.R.
4	190	210	200	5.0	0.8	0.8
5	170	200	185	5.0	0.8	0.8
6	140	170	160	5.0	0.8	0.8

Table 3 Starting Points for Optimisation

Z	Variables at Optimum				
	N	D	P/D	B.A.R.	η_o
4	196.8	4.766	0.70	0.655	0.633
5	187.5	4.886	0.70	0.767	0.629
6	162.0	5.006	0.80	0.859	0.635

Table 4 Summary of Results

The six-bladed propeller is chosen as the "best" screw, yielding an efficiency of 0.635, and is used as the basis for detailed design.

(b) Detailed Design

The output of the detailed design of the six-bladed screw is shown in Appendix C. It is seen that elliptical search is performed at a search angle of 45° , and some improvement is made.

A summary of the results of the optimisation of efficiency of the six-bladed propeller is shown below.

Blades	:	6
rpm	:	160.9
Diameter	:	5.027 m
P/D	:	0.80
B.A.R.	:	0.859
Req'd shp	:	10,813.9 metric hp
Root Thickness	:	238.7 mm
η_0	:	0.636

3. USER MANUAL - PROPOPT

Cards 1,2,3	TITLE	(8A10)
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3 cards to denote project or design title

Card 4 SHP, SPEED,CBLOCK,DRAFT,SIGMAX,ICAVN,
NDES, NSCREW (5G10.0,3I2)

SHP = Available shaft horsepower per shaft

SPEED = Required ship speed (knots)

CBLOCK = Ship block coefficient

DRAFT = Draft at aft perpendicular (m)

SIGMAX = Maximum allowable blade stress (kPa)

ICAVN = Index for choice of cavitation criterion

$$= 1 \text{ for heavily loaded propeller}$$

= 2 for merchant ship propeller (standard section)

= 3 for merchant ship propeller (airfoil section)

NDES = Index for indication of design stage

= 1 for preliminary design

= 2 for detailed design (design for manufacture)

NSCREW = Number of screws

Card 5 DIAMIN, DIAMAX (2G10.0)

DIAMIN = Minimum diameter allowable

DIAMAX = Maximum diameter allowable

Card 6 TITLEA, WAKFAC (A10,G10.0)

(a) If wake fraction known:

TITLEA = input the word "WAKFAC"

WAKFAC = Taylor wake fraction

(b) If wake fraction unknown:

Leave card 6 blank

Card 7 NTCY,L (2I2)

NTCY = Number of cycles to be performed
for the optimisation process
(suggest NTCY = 3)

L = Indicator for interim output
= 0 for no intermediate output
= 1 for full intermediate output
= 2 for partial intermediate output

Card 8 TITLEC,ANGLE (A10,G10.0)

(a) If elliptical search is to be performed:
TITLEC = input the word "ANGLE"
ANGLE = elliptical search angle (suggest 30°)

(b) If elliptical search is not to be performed:
Leave card 8 blank

Card 9 TITLED,ZZ (A10,G10.0)

TITLED = input the word "BLADES"
ZZ = Number of blades (input as REAL variable)

Card 10 RPMMIN,RPMMAX (2G10.0)

RPMMIN = minimum rpm allowable
RPMMAX = maximum rpm allowable

Card 11 V(I),I=1,4 (4G10.0)

V(1) = Starting point for rpm
V(2) = Starting point for diameter
V(3) = Starting point for pitch/diameter ratio
V(4) = Starting point for blade area ratio

If there are other cases to consider, return
to Card 9, if not, go to Card 12

Card 12 Blank Card

4. CONCLUSION

Program PROPOPT is suitable for the detailed design of non-cavitating propellers for any vessel requiring a B-series screw, and for the preliminary design of propellers for almost any vessel.

In searching for the characteristics for optimum efficiency there may exist several local optima. This is more likely if the allowable ranges of rpm and diameter are large. For this reason, several starting points for optimisation should be considered, to ensure that the global optimum is found.

As this work was performed as an undergraduate thesis during 1982, there was insufficient time to perform a large number of case studies to test both the capabilities of the program and its sensitivity to varying starting points. Further experience with running PROPOPT will provide an indication of modifications that may be made to remove the possibility of the dependence of results on starting points.

The work done in this study provides a reliable method of propeller design, and a basis for further studies in the general field of optimisation of ship design.

5. SCOPE FOR PROGRAM EXTENSION

5.1 PROGRAM PROPOPT

Whilst PROPOPT is entirely adequate for the preliminary design of screw propellers for almost any application, its use in detailed design is limited to the design of light and moderately loaded propellers. Whilst this is a large category, it would be of value to increase the range of applications.

The cavitation criteria employed by PROPOPT in detailed design is a curve representing the average cavitation curve for B-series screws, but the likelihood of cavitation is a function of, with other factors, the P/D ratio. Cavitation tests have been performed on the B-series at NSMB, and the results expressed in the form of charts for each series, with the effect of differing P/D ratios being included.

The complexity of these charts render them virtually unusable by the computer, but numerical methods applied to the data, such as a regression analysis, would result in the development of polynomials defining the cavitation curves in terms of propeller characteristics and loading. The results of such an analysis, applied to PROPOPT would widen the scope of the program in detailed design, with more heavily loaded propellers being designed with a greater assurance of non-cavitation.

Yet another possibility is the implementation of circulation theory in the determination of the cavitation characteristics of B-series screws. Information is presently

available to modify PROPOPT, including several additional subroutines enabling greater confidence when designing for non-cavitation.

With finite element methods becoming increasingly common in the field of structural analysis, the thickness distribution of a propeller blade for adequate strength may now be optimised. The incorporation of such a method in PROPOPT would see the program with increased powers, optimising not only efficiency, but also weight, and ultimately cost.

The program is not limited to optimising propeller characteristics only, as it may be employed as an aid in preliminary hull design. The designer may consider a wide range of propeller possibilities in a very short period of time, and therefore has greater freedom in designing the after lines. This suggests that the program may be extended to consider variations in the after lines and therefore the flow into the propeller, by the addition of the wake fraction as a variable, if the program is to be used not merely for preliminary propeller design, but also for preliminary hull design.

5.2 NOZZLE PROPELLERS

In a similar fashion to the fairing of the K_T - K_Q - J curves for the B-series, the fairing of data for nozzle propellers has been performed at NSMB{20}. Unlike the work on the B-screw series, however, the polynomials developed for the thrust and torque coefficients of nozzle propellers apply only to four-bladed screws of a blade area ratio of 0.7,

being functions of advance coefficient and P/D ratio only.

Should the analysis be extended in the future to determine polynomials in the additional variables of number of blades and blade area ratio, then a program may be developed utilizing the same optimisation procedure as PROPOPT, but adapting the method of B-series screw design to nozzle propeller design.

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APPENDIX AMATHEMATICAL MODEL

MATHEMATICAL MODEL

The mathematical model for the solution of the optimisation problem is stated as:

$$\text{Maximise } (\eta_0) = \left| \text{Minimise } (-\eta_0) \right|$$

$$\eta_0 = \frac{J}{2\pi} \frac{K_T}{K_Q}$$

subject to:

$$H_1 = dhp_c - dhp = 0$$

$$G_1 = +n_{\min} - n \leq 0$$

$$G_2 = -n_{\max} + n \leq 0$$

$$G_3 = +D_{\min} - D \leq 0$$

$$G_4 = -D_{\max} + D \leq 0$$

$$G_5 = +0.5 - P/D \leq 0$$

$$G_6 = -1.4 + P/D \leq 0$$

$$G_7 = +0.3 - \text{B.A.R.} \leq 0$$

$$G_8 = -1.05 + \text{B.A.R.} \leq 0$$

$$G_9 = - \left[\frac{T/A_p}{q} \right]_{\text{crit}} + \left[\frac{T/A_p}{q} \right] \leq 0$$

where $\left[\frac{T/A_p}{q} \right]_{\text{crit}}$ is the critical ordinate of the Burrill cavitation function

$\left[\frac{T/A_p}{q} \right]$ is based on the actual thrust, projected area, and velocity of flow over the blade.

APPENDIX BLIST OF EQUATIONS AND TABLES OF COEFFICIENTS

PROGRAM PROPOPTLIST OF EQUATIONS AND TABLES OF COEFFICIENTS

Speed of Advance

$$1. \quad V_A = V_S (1.05 - 0.5 C_B) \quad (\text{m/s})$$

(w_T unknown, SS vessel)

or

$$2. \quad V_A = V_S (1 - w_T) \quad (\text{m/s})$$

Open Water Efficiency

$$3. \quad \eta_o = \frac{J}{2\pi} \cdot \frac{K_T}{K_Q}$$

Advance Coefficient

$$4. \quad J = \frac{V_A}{nD}$$

Thrust Coefficient

$$5. \quad K_T = \frac{T}{\rho n^2 D^4}$$

$$6. \quad K_T = \sum_{i=1}^{39} (C_i (J)^S (P/D)^t (B.A.R.)^u (Z)^v)$$

(see table B.1 for coefficients)

Torque Coefficient

$$7. \quad K_Q = \frac{Q}{\rho n^2 D^5}$$

$$8. \quad K_Q = \sum_{i=1}^{47} (C_i (J)^S (P/D)^t (B.A.R.)^u (Z)^v)$$

(see table B.1 for coefficients)

Chord Length at 0.75R

$$9. \quad c_{0.75R} = \frac{2.073 \text{ D} \times \text{B.A.R.}}{Z}$$

Standard t/c Ratio at 0.75R

$$10. \quad t/c_{0.75R} = \frac{(0.0185 - 0.00125 Z) Z}{2.073 \text{ B.A.R.}}$$

Reynolds No. at 0.75R

$$11. \quad R_{n0.75R} = \frac{c_{0.75R} \sqrt{V_A^2 + (0.75\pi nD)^2}}{v}$$

Minimum Root Thickness for Blade Strength

$$12. \quad t_o = \sqrt[3]{\frac{\frac{4.244 (P/D)^{-0.942} \text{ shp} \times 0.946}{nZ} + 0.588 (2.44 - (P/D)) n^2 D^2 t_o^2}{\frac{S_c}{6.895} + 3.03 n^2 D^2}}$$

$(t/c)'_{0.75R}$ Corresponding to 12

$$13. \quad (t/c)'_{0.75R} = \frac{0.2065 t_o + 0.00238 D}{c_{0.75R}}$$

Effective Reynolds No.

$$14. \quad R'_{n0.75R} = \exp \left[4.6052 + \frac{1+2(t/c)_{0.75R}}{1+2(t/c)'_{0.75R}} \cdot (\ln R_{n0.75R} - 4.6052) \right]$$

Increment in Thrust Coefficient

$$15. \quad \Delta K_T = \sum_{i=1}^9 (C_i (J)^a (P/D)^b (\text{B.A.R.})^c (Z)^d (R_L)^e)$$

$$\text{where } R_L = \log R_{n0.75R} - 0.301$$

(See table B.2 for coefficients)

Increment in Torque Coefficient

$$16. \Delta K_Q = \sum_{i=1}^{13} (C_i (J)^a (P/D)^b (\text{B.A.R.})^c (Z)^d (R_L)^e)$$

(see table B.3 for coefficients)

Power Developed by Screw (fresh water)

$$17. \text{dhp}_c = \frac{204\pi K_Q n^3 D^5}{75}$$

Cavitation Number

$$18. \sigma = \frac{99081 + 11772 h}{q}$$

$$h = 0.95 T_{AP} - 0.85 D$$

$$q = 500 (V_A^2 + (0.7\pi n D)^2)$$

Projected Blade Area

$$19. A_P = \frac{\pi D^2 (1.067 - 0.229 P/D)}{4 \text{ B.A.R.}}$$

Critical Ordinate of Burrill Cavitation Curve

$$20. \left[\frac{T/A_P}{q} \right]_{\text{crit}} = c \sigma^d$$

(see table B.4 for values of c and d)

Screw Geometry

$$21. c = \frac{c_0.75R K_1}{2.15}$$

$$22. D_{LG} = K_2 c \quad (K_1, K_2, K_3 \text{ in table B.5})$$

$$23. D_{LM} = K_3 c$$

$$24. t_r = \left(1 - \frac{r}{R}\right) t_o + 0.007D \quad (D < 4m)$$

$$25. t_r = \left(1 - \frac{r}{R}\right) t_o + 0.002D + 0.02 \quad (D \geq 4m)$$

Blade Section Ordinates

$$26. \quad Y_{f_r} = (t_r - t_{te})W_A$$

$$27. \quad Y_{b_r} = (t_r - t_{te})(W_A + W_B) + t_{te}$$

(see tables B.6 and B.7 for values of W_A and W_B)

Speed of Advance

$$28. \quad V_A = V_S (1.2 - 0.55 C_B) \text{ (m/s)}$$

(w_T unknown, TS vessel)

Thrust Coefficient, K_T					Torque Coefficient, K_Q				
C_i	s	t	u	v	C_i	s	t	u	v
+0.00880496	0	0	0	0	+0.00379368	0	0	0	0
-0.204554	1	0	0	0	+0.00886523	2	0	0	0
+0.166351	0	1	0	0	-0.032241	1	1	0	0
+0.158114	0	2	0	0	+0.00344778	0	2	0	0
-0.147581	2	0	1	0	-0.0408811	0	1	1	0
-0.481497	1	1	1	0	-0.108009	1	1	1	0
+0.415437	0	2	1	0	-0.0885381	2	1	1	0
+0.0144043	0	0	0	1	+0.188561	0	2	1	0
-0.0530054	2	0	0	1	-0.00370871	1	0	0	1
+0.0143481	0	1	0	1	+0.00513696	0	1	0	1
+0.0606826	1	1	0	1	+0.0209449	1	1	0	1
-0.0125894	0	0	1	1	+0.00474319	2	1	0	1
+0.0109689	1	0	1	1	-0.00723408	2	0	1	1
-0.133698	0	3	0	0	+0.00438388	1	1	1	1
+0.00638407	0	6	0	0	-0.0269403	0	2	1	1
-0.00132718	2	6	0	0	+0.0558082	3	0	1	0
+0.168496	3	0	1	0	+0.0161886	0	3	1	0
-0.0507214	0	0	2	0	+0.00318086	1	3	1	0
+0.0854559	2	0	2	0	+0.015896	0	0	2	0
-0.0504475	3	0	2	0	+0.0471729	1	0	2	0
+0.010465	1	6	2	0	+0.0196283	3	0	2	0
-0.00648272	2	6	2	0	-0.00502782	0	1	2	0
-0.00841728	0	3	0	1	-0.030055	3	1	2	0
+0.0168424	1	3	0	1	+0.0417122	2	2	2	0
-0.00102296	3	3	0	1	-0.0397722	0	3	2	0
-0.0317791	0	3	1	1	-0.00350024	0	6	2	0
+0.018604	1	0	2	1	-0.0106854	3	0	0	1
-0.00410798	0	2	2	1	+0.00110903	3	3	0	1
-0.000606848	0	0	0	2	-0.000313912	0	6	0	1
-0.0049819	1	0	0	2	+0.0035985	3	0	1	1
+0.0025983	2	0	0	2	-0.00142121	0	6	1	1
-0.000560528	3	0	0	2	-0.00383636	1	0	2	1
-0.00163652	1	2	0	2	+0.0126803	0	2	2	1
-0.000328787	1	6	0	2	-0.00318278	2	3	2	1
+0.000116502	2	6	0	2	+0.00334268	0	6	2	1
+0.000690904	0	0	1	2	-0.00183491	1	1	0	2
+0.00421749	0	3	1	2	+0.000112451	3	2	0	2
+0.0000565229	3	6	1	2	-0.0000297228	3	6	0	2
-0.00146564	0	3	2	2	+0.000269551	1	0	1	2
					+0.00083265	2	0	1	2
					+0.00155334	0	2	1	2
					+0.000302683	0	6	1	2
					-0.0001843	0	0	2	2
					-0.000425399	0	3	2	2
					+0.0000869243	3	3	2	2
					-0.0004659	0	6	2	2
					+0.0000554194	1	6	2	2
$R_n = 2 \times 10^6$									

Table B.1 Coefficients of the K_T and K_Q Polynomials

C_i	a	b	c	d	e
+0.000353485	0	0	0	0	0
-0.00333758	2	0	1	0	0
-0.00478125	1	1	1	0	0
+0.000257792	2	0	1	0	2
+0.0000643192	2	6	0	0	1
-0.0000110636	2	6	0	0	2
-0.0000276315	2	0	1	1	2
+0.0000954	1	1	1	1	1
+0.0000032049	1	3	1	2	1

Table B.2 Coefficients of ΔK_T Polynomial for Reynolds
No. Effect ($R_n > 2 \times 10^6$)

C_i	a	b	c	d	e
-0.000591412	0	0	0	0	0
+0.00696898	0	1	0	0	0
-0.0000666654	0	6	0	0	0
+0.0160818	0	0	2	0	0
-0.000938091	0	1	0	0	1
-0.00059593	0	2	0	0	1
+0.0000782099	0	2	0	0	2
-0.0000052199	2	0	1	1	1
-0.00000088528	1	1	1	1	2
+0.0000230171	0	6	0	1	1
-0.00000184341	0	6	0	1	2
-0.00400252	0	0	2	0	1
+0.000220915	0	0	2	0	2

Table B.3 Coefficients of ΔK_Q Polynomial for Reynolds
No. Effect ($R_n > 2 \times 10^6$)

Cavitation Criteria	σ	m	n
1	≥ 0.175	0.46	0.789
	< 0.175	0.53	0.881
2	≤ 0.217	0.32	0.728
	$0.217 < \sigma \leq 0.65$	0.27	0.559
	> 0.65	0.28	0.624
3	≥ 0.63	0.20	0.398
	< 0.63	0.21	0.514

Cavitation Criteria

1 = heavily loaded propeller

2 = merchant ship propeller (standard section)

3 = merchant ship propeller (airfoil section)

Table B.4 Coefficients for Equation 20.

r/R	3 Bladed Screws			4, 5, 6, 7 Bladed Screws		
	K_1	K_2	K_3	K_1	K_2	K_3
0.2	1.662	0.617	0.35	1.633	0.616	0.35
0.3	1.882	0.613	0.35	1.832	0.611	0.35
0.4	2.050	0.601	0.351	2.000	0.599	0.35
0.5	2.152	0.586	0.355	2.120	0.583	0.355
0.6	2.187	0.561	0.389	2.186	0.558	0.389
0.7	2.144	0.524	0.443	2.268	0.526	0.442
0.8	1.970	0.463	0.479	2.127	0.481	0.478
0.9	1.582	0.351	0.500	1.657	0.400	0.500
1.0	0.0	0.0	0.0	0.0	0.0	0.0

Table B.5 Coefficients for Equations 21, 22, 23

r/R	-1.	-.8	-.6	-.4	-.2	0	+.2	+.4	+.6	+.8	+.9	+.95	+1.0
0.2	.2826	.1967	.1207	.0592	.0172	0	.0049	.0304	.0804	.1685	.2353	.2821	.3560
0.3	.2306	.1333	.0623	.0202	.0033	0	.0027	.0148	.0503	.1191	.1760	.2186	.2923
0.4	.1467	.0630	.0214	.0044	0	0	0	.0033	.0189	.0637	.1088	.1467	.2818
0.5	.0522	.0190	.0040	0	0	0	0	0	.0034	.0311	.0500	.0778	.1278
0.6	0	0	0	0	0	0	0	0	0	.0006	.0067	.0169	.0382
0.7-1.0	0	0	0	0	0	0	0	0	0	0	0	0	0

Table B.6 Coefficient W_A of Equations 26 and 27

0.2	0	.3060	.5842	.7984	.9446	1	.9750	.8875	.7277	.4777	.2840	.1560	0
0.3	0	.3360	.6195	.8265	.9583	1	.9750	.8920	.7520	.5130	.3197	.1890	0
0.4	0	.3500	.6353	.8415	.9645	1	.9725	.8933	.7593	.5220	.3235	.1935	0
0.5	0	.3569	.6439	.8456	.9639	1	.9710	.8880	.7478	.5039	.3056	.1750	0
0.6	0	.3583	.6415	.8426	.9613	1	.9690	.8790	.7200	.4620	.2720	.1485	0
0.7	0	.3600	.6400	.8400	.9600	1	.9675	.8660	.6840	.4140	.2337	.1240	0
0.8	0	.3600	.6400	.8400	.9600	1	.9635	.8520	.6545	.3765	.2028	.1050	0
0.9-1.0	0	.3600	.6400	.8400	.9600	1	.9600	.8400	.6400	.3600	.1900	.0975	0

Table B.7 Coefficient W_B of Equation 27

APPENDIX CSAMPLE DESIGN EXERCISE - OUTPUT

PROGRAM PRO:OPT
 A PROGRAM BY ROBERT CURRAN (UNSW P.E. THESIS 1982) FOR THE SELECTION OF
 RPN, DIAMETER, P/D RATIO AND BLADE AREA RATIO OF WAGENINGEN R-SERIES SCREWS
 FOR OPTIMUM EFFICIENCY.

SELECTION OF PROPELLER CHARACTERISTICS FOR OPTIMUM EFFICIENCY
 FOR THE PRELIMINARY DESIGN OF A

21,000 TONNE SINGLE SCREW CONTAINERSHIP.

AVAILABLE SHAFT HORSEPOWER : 11000. HP

SHIP SPEED : 22.0 KNOTS

BLOCK COEFFICIENT : .6600

DRAFT (AT AFT PUMP.) : 8.000 M.

MAX. ALLOWABLE BLADE STRESS : 48004.00 KPA

MERCHANT SHIP PROPELLER (STANDARD SECTION)

DATA LIES BETWEEN 4.200 M. AND 5.4 M.

WAKE FRACTION : .3400

PRELIMINARY DESIGN ONLY

SINGLE SCREW VESSEL

NUMBER OF CYCLES : 3
 INDICATOR FOR INTERIM OUTPUT : 0

ELLIPTICAL SEARCH WILL NOT BE PERFORMED

CASE 1 - NUMBER OF BLADES : 4.0

RPM LIPS BETWEEN 190.00 AND 210.00

COORDINATES OF STARTING POINT :

200.00000 5.00000 -80000 -80000

OBJECTIVE FUNCTION VALUE AT STARTING POINT : -552558722

1 RESULT OF DIRECT SEARCH OF OPTIMUM OF THE OBJECTIVE FUNCTION

VALUE OF VARIABLES AT OPTIMUM EFFICIENCY

SHAFT REVS PER MINUTE = 196.8

DIAMETER = 4.766 M.

P/D RATIO = .2700

F.A.R. = .2639

NUMBER OF BLADES = 4.

DELIVERED HORSEPOWER = 10659.7

ROOT THICKNESS = .2649 M.

OPTIMUM EFFICIENCY = .633208

VALUE OF CONSTRAINTS AT THE POINT :

-4.35266

-0.01433

-0.31500

.03400

VALUE OF CONSTRAINTS AT THE POINT :

-1.63400

-0.20000

-0.70000

-0.35500

VALUE OF CONSTRAINTS AT THE POINT :

-0.79500

-0.07034

EVALUATION OF THE PROCESS OF DIRECT SEARCH OF OPTIMISATION :

NUMBER OF TEST STEPS : 26

NUMBER OF STEP IN THE DIRECTION :

NUMBER OF FUNCTION EVALUATION : 87

DELIVERED HORSEPOWER = 10659.7

REQUIRED SHAFT HORSEPOWER = 10989.4

BLADE THICKNESS (AT ZERO RADIUS) = .264.9MM

END OF PROGRAM PROPOPT

PROGRAM PRO-OPT
 A PROGRAM BY ROBERT J. HUNTER (UNSW S.E.) FOR THE SELECTION OF
 PROPELLER DIAMETER, PITCH AND PLACEMENT FOR A RATIO OF WARPINGEN R-SERIES SCREWS
 FOR OPTIMUM EFFICIENCY.

SELECTION OF PROPELLER CHARACTERISTICS FOR OPTIMUM EFFICIENCY
 FOR THE DETAIL DESIGN OF A

21,000 TONNE SINGLE SCREW CONTAINERSHIP.

AVAILABLE SHAFT HORSEPOWER : 11000. HP

SHIP SPEED : 22.0 KNOTS

BLOCK COEFFICIENT : .6600

DRAFT (AT AFT PERP.) : 8.000 M.

MAX. ALLOWABLE BLADE STRESS : 48000.00 KPA

MERCHANT SHIP PROPELLER (STANDARD SECTION)

SHA. LIES BETWEEN 4.000 M. AND 5.400 M.

WAKE FRACTION : .3400

DETAIL DESIGN - BLADE SECTIONS REQUIRED

SINGLE SCREW VESSEL

NUMBER OF CYCLES : 3

INDICATOR FOR INTERIM OUTPUT : 0

ELLIPTICAL SEARCH: ANGLE = 45.00

CASE 1 - NUMBER OF BLADES : 6.0

RPM LIES BETWEEN 140.00 AND 170.00

COORDINATES OF STARTING POINT :

160.00000 5.00000 20000 .80000

OBJECTIVE FUNCTION VALUE AT STARTING POINT : -.629011167

1 RESULT OF DIRECT SEARCH OF OPTIMUM OF THE OBJECTIVE FUNCTION

VALUE OF VARIABLES AT OPTIMUM EFFICIENCY

SHAFT REVS PER MINUTE = 162.0

DIAMETER = 5.006 M.

P/D RATIO = .800

F.A.R. = .859

NUMBER OF BLADES = 6.

DELIVERED HORSEPOWER = 10680.6

ROOT THICKNESS = .2381 M.

OPTIMUM EFFICIENCY = .634831

VALUE OF CONSTRAINTS AT THE POINT :

-11.46959 -.20833 -.21167 -.20600

VALUE OF CONSTRAINTS AT THE POINT :

-.39400 -.30000 -.60000 -.55875

VALUE OF CONSTRAINTS AT THE POINT :

-.16125 -.00200

EVALUATION OF THE PROCESS OF DIRECT SEARCH OPTIMISATION

NUMBER OF TEST STEPS : 14
NUMBER OF STEPS IN THE DIRECTION OF FUNCTION EVALUATION : 41

IELIP = 1

1 RESULT OF DIRECT SEARCH OF OPTIMUM OF THE OBJECTIVE FUNCTION

VALUE OF VARIABLES AT OPTIMUM EFFICIENCY

SHAFT REVS PER MINUTE = 160.0

DIAMETER = 5.027 M.

P/D RATIO = .800

F. A. P. = .859
NUMBER OF RLA ES = 6.

DELIVERED HORSEPOWER = 10489.5
ROOT THICKNESS = .2387 M.

OPTIMUM EFFICIENCY = .638669

VALUE OF CONSTRAINTS AT THE POINT :
-2.28855 -2.27065 AT THE POINT : -2.2721

VALUE OF CONSTRAINTS AT THE POINT :
-3.37279 -3.30000 -5.5875

VALUE OF CONSTRAINTS AT THE POINT :
-1.19125 -2.86364

EVALUATION OF THE PROCESS OF DIRECT SEARCH OPTIMISATION
NUMBER OF TEST STEPS : 22
NUMBER OF STEP IN THE DIRECTION : 8
NUMBER OF FUNCTION EVALUATION : 67

IELIP = 0
DELIVERED HORSEPOWER = 10489.5
DELIVERED SHAFT HORSEPOWER = 10813.9
ROOT THICKNESS (AT 2500 R/MIN) = .38.7MM

EXPAND-U AREA PARTICULARS

 DIMENSIONS ARE IN MILLIMETRES

RADIUS RATIO

 CHORD LENGTH

 MAXIMUM THICKNESS

 LEADING EDGE TO
 MAXIMUM ORDINATE

.2	7.7.8	1195.8	210.9	418.5
.3	8.0.1	1354.1	187.1	473.9
.4	8.6.5	1475.0	163.2	517.7
.5	9.7.4	1548.4	139.3	549.7
.6	8.2.8	1573.6	115.5	612.1
.7	8.8.3	1542.7	91.6	683.4
.8	6.6.3	1417.5	67.7	679.9
.9	3.9.5	1138.3	43.9	569.1
1.0	0.0	0.0	20.0	0.0

THICKNESS ORDINATES & DISTANCES FROM MAX. ORDINATE

RADIUS RATIO	TRAILING EDGE					LEADING EDGE								
	100%	80%	60%	40%	20%	MAX	20%	40%	60%	80%	90%	95%	100%	
.2	RACK ORDINATE	69.9	113.2	152.9	182.9	203.4	210.9	206.7	193.6	170.5	136.3	109.5	92.4	75.1
	FACE ORDINATE	55.6	38.7	23.7	11.6	3.4	0.0	1.0	6.4	17.0	35.5	49.6	59.5	75.1
	DIST. FROM MAX.	777.3	621.8	466.4	310.9	155.5	0.0	83.7	167.4	251.1	336.8	376.7	397.6	418.5
.3	RACK ORDINATE	52.9	94.5	131.6	160.3	180.4	187.1	182.9	169.6	150.1	118.2	92.7	76.3	54.7
	FACE ORDINATE	40.2	23.2	10.9	3.5	.6	0.0	.5	2.8	9.4	22.3	32.9	40.9	54.7
	DIST. FROM MAX.	880.2	704.2	528.1	352.1	176.0	0.0	94.8	189.6	284.4	379.2	426.6	450.3	473.9

.4	HACK ORIGINATE	33.4	73.9	111.0	139.8	157.8	163.2	158.7	146.3	127.0	95.6	70.6	55.5	35.6
	FACE ORIGINATE	22.3	9.6	3.3	.7	0.0	0.0	0.0	.5	3.1	10.4	17.8	23.9	35.6
	DIST. FROM MAX.	957.3	765.8	574.4	382.9	191.5	0.0	103.5	207.1	310.6	414.2	466.0	491.8	517.7
.5	HACK ORIGINATE	16.3	58.3	93.6	119.3	134.7	139.3	135.3	123.7	104.7	73.2	49.5	35.2	17.8
	FACE ORIGINATE	6.8	2.5	.5	0.0	0.0	0.0	0.0	0.0	.5	2.9	7.0	10.8	17.8
	DIST. FROM MAX.	998.7	799.0	598.2	399.5	199.7	0.0	109.9	219.9	329.8	439.7	494.7	522.2	549.7
.6	HACK ORIGINATE	7.9	46.4	76.9	98.5	111.3	115.5	111.9	101.5	83.1	53.4	32.2	19.1	4.4
	FACE ORIGINATE	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	.1	.8	2.0	4.4
	DIST. FROM MAX.	961.5	769.2	576.9	384.6	192.3	0.0	122.4	244.9	367.3	489.7	550.9	581.5	612.1
.7	HACK ORIGINATE	6.2	37.0	60.9	77.9	88.2	91.6	88.6	79.3	62.7	37.9	21.4	11.4	0.0
	FACE ORIGINATE	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	DIST. FROM MAX.	859.3	687.4	515.6	343.7	171.9	0.0	136.7	273.4	410.0	546.7	615.1	649.2	683.4
.8	HACK ORIGINATE	4.6	27.3	45.0	57.6	65.2	67.7	65.3	57.7	44.3	25.5	13.7	7.1	0.0
	FACE ORIGINATE	7.0	0.7	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	DIST. FROM MAX.	738.5	590.8	443.1	295.4	147.7	0.0	135.8	271.6	407.4	543.2	611.1	645.0	679.0
.9	HACK ORIGINATE	3.0	17.7	29.2	37.3	42.2	43.9	42.1	36.9	28.1	15.8	8.3	4.3	0.0
	FACE ORIGINATE	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	DIST. FROM MAX.	569.1	455.3	341.5	227.7	113.8	0.0	113.8	227.7	341.5	455.3	512.2	540.7	569.1
1.0	HACK ORIGINATE	0.0	0.0	0.0	0.0	0.0	20.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	FACE ORIGINATE	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	DIST. FROM MAX.	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

END OF PROGRAM PROPOPT