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SCHOOL OF MECHANICAL AND INDUSTRIAL ENGINEERING
Incorporating Aeronautical Engineering, Naval Architecture and Nuclear Engineering

SHORT COURSE

ELEMENTS OF SAILING YACHT DESIGN

by

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Maritime Research Institute [MARIN]
Wageningen, The Netherlands

1. **Introduction**
2. **Basic Objectives And Design Problems**
   - 2.1 Hydrodynamic resistance and its components
   - 2.2 Hydrodynamic side force [lift]
   - 2.3 Designing to a rule or rating formula
   - 2.4 Wind conditions - influence on design
   - 2.5 Sea conditions - influence on design
   - 2.6 Use of a velocity prediction program [UPP] in design
3. **Design Of Canoe Body**
   - 3.1 General design methodology
   - 3.2 Choice of overall and waterline length
   - 3.3 Initial estimate of displacement
   - 3.4 Initial estimate of beam and draft
   - 3.5 Estimate of required stability characteristics
   - 3.6 Initial estimate of sail dimensions
   - 3.7 Required prismatic coefficient
   - 3.8 Required LCB location
   - 3.9 Finalization of linesplan
4. **Design Of Keel And Rudder**
   - 4.1 Required lateral area of keel and rudder
   - 4.2 Significance of aspect ratio
   - 4.3 Taper ratio and sweep angle
   - 4.4 Application of winglets
   - 4.5 Section shapes of keels, rudders and winglets
5. **Detailed Design Exercise**

We are pleased to announce that Dr. van Oossanen has agreed to give a Short Course of approximately 20 hours' duration to the final-year students in Naval Architecture. Dr. van Oossanen has an international reputation in hydrodynamics and was responsible for much of the research on the design of the twelve-metre yacht Australia II. These lectures are supported by the Waters Bequest to the Royal Institution of Naval Architects [Australian Division]. The first two-hour class will take place on August 10, 1989 in room ME301 at 9.00 am. The schedule for the remaining lectures will then be announced.

All other interested persons are welcome to attend.

Further details: Assoc. Prof. L.J. Doctors
697-4098 [office]
697-4093 [messages]

SHORT COURSE

"ELEMENTS OF SAILING YACHT DESIGN"

- CONTENTS

- RESTRICTED TO DESIGN OF HULL, KEEL AND RUDDER IN THE CONTEXT OF OPTIMIZING PERFORMANCE

- LITTLE OR NO SUBJECT MATTER ON STRUCTURAL DESIGN OR SAIL DESIGN

- COURSE ASSUMES BASIC KNOWLEDGE OF NAVAL ARCHITECTURE, MATHEMATICS AND SOME KNOWLEDGE OF FLUID MECHANICS

- WHERE POSSIBLE PRACTICAL/EMPIRICAL FORMULAS WILL BE USED TO RENDER APPLICATION OF SUBJECT MATTER

- LITERATURE : TO BE DISTRIBUTED DURING THE COURSE

- SCHEDULE FOR REMAINING LECTURES

OVERALL OBJECTIVE IN (RACING) SAILING YACHT DESIGN

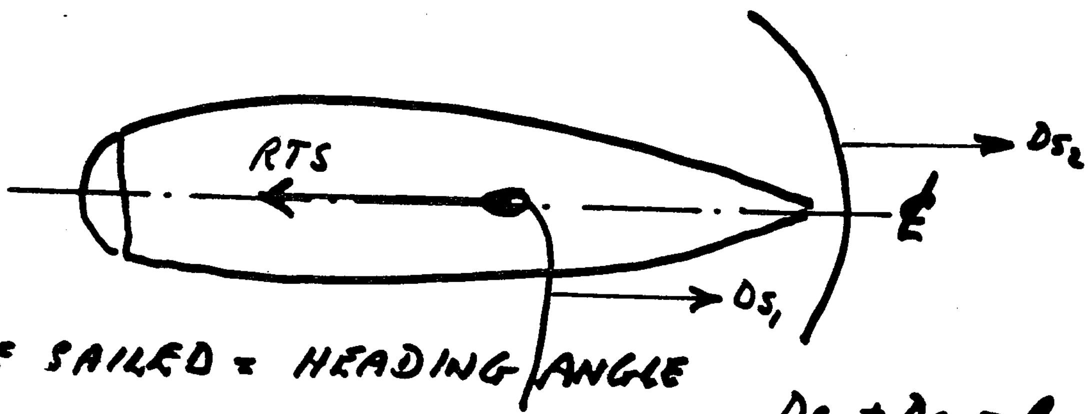
- TO FIND THE FASTEST COMBINATION OF HULL AND SAILS FOR THE EXPECTED WIND AND SEA CONDITIONS

- MORE SPECIFICALLY:
  TO DESIGN A HULL WITH THE LOWEST POSSIBLE RESISTANCE, THE HIGHEST POSSIBLE SIDE FORCE AND SAILS, WHICH WHEN INTEGRATED WITH THE HULL CHARACTERISTICS, PRODUCE THE GREATEST THRUST

- FOR CRUISING YACHTS, THE SAME HOLDS TRUE. IN THIS CASE DISPLACEMENT WILL NOT BE SELECTED FROM SPEED CONSIDERATIONS HOWEVER

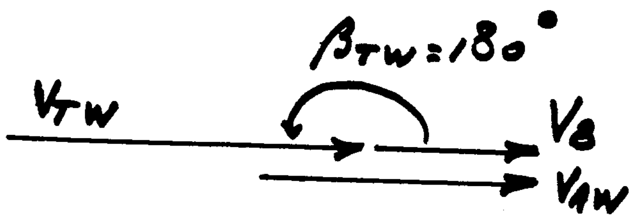
BASIC CONSIDERATIONS

• FORCES AND GEOMETRY "RUNNING"



COURSE SAILED = HEADING ANGLE

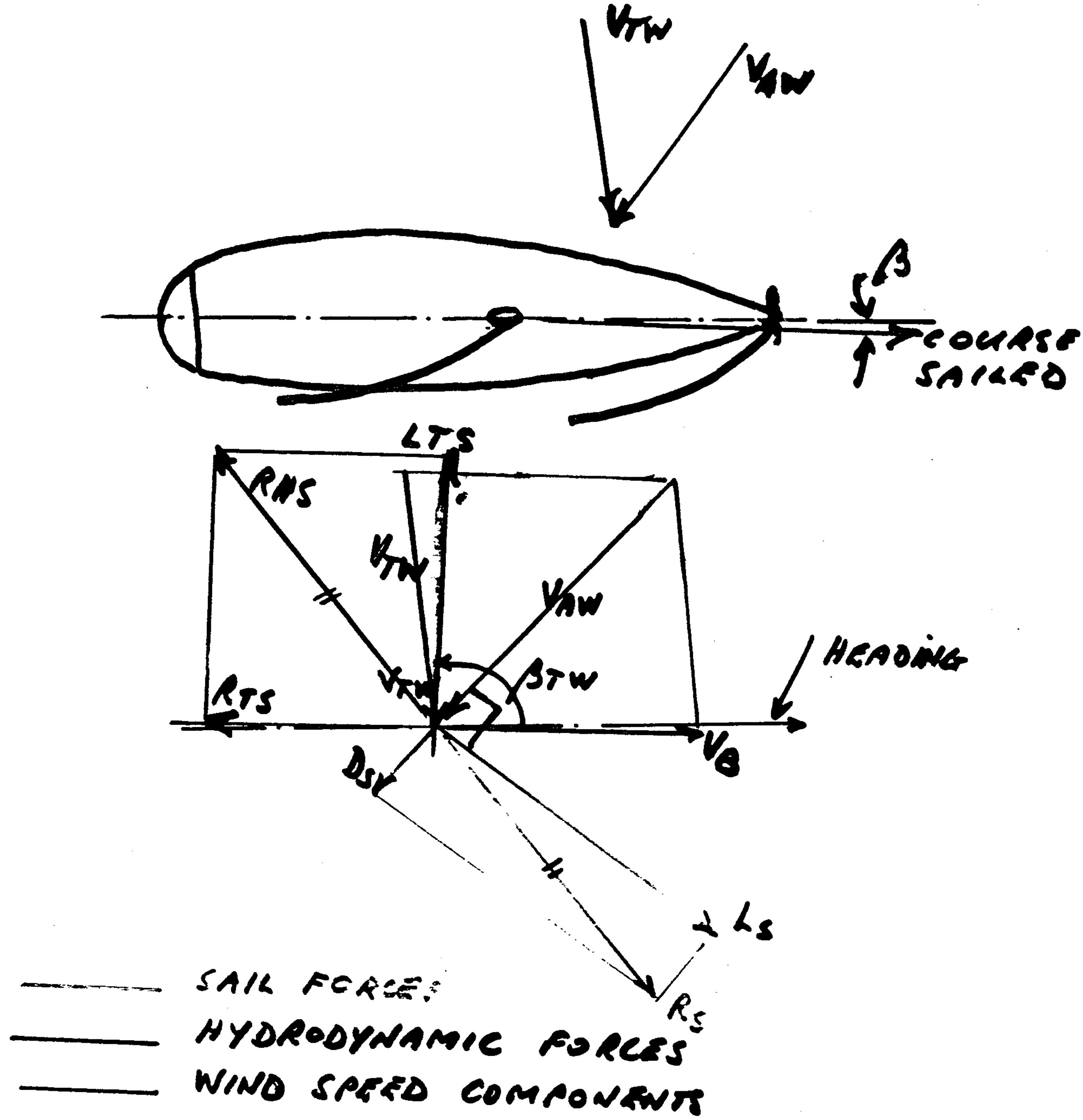
$D_{S_1} + D_{S_2} = R_{TS}$
(EQUILIBRIUM)



$$V_{AW} = V_{TW} - V_B$$

- SAILS ARE IN "DRAG MODE": NO LIFT PRODUCED
- APPARENT WIND SPEED V_{AW} IS LESS THAN TRUE WIND SPEED
- BOAT SPEED IS MAXIMIZED BY MINIMIZING R_{TS} AND MAXIMIZING SAIL AREA

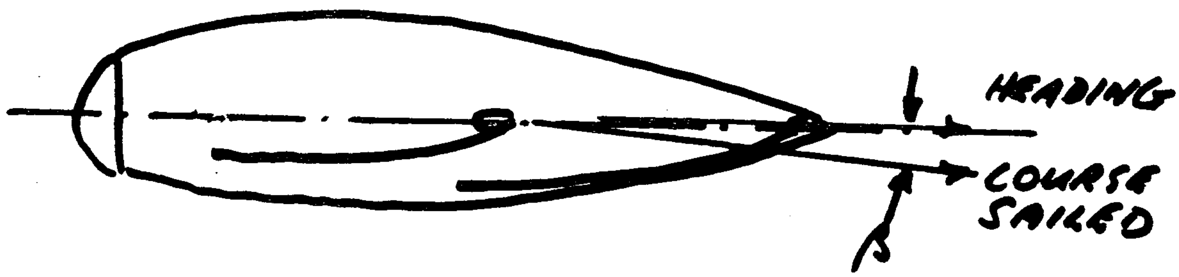
FORCES AND GEOMETRY "REACHING"

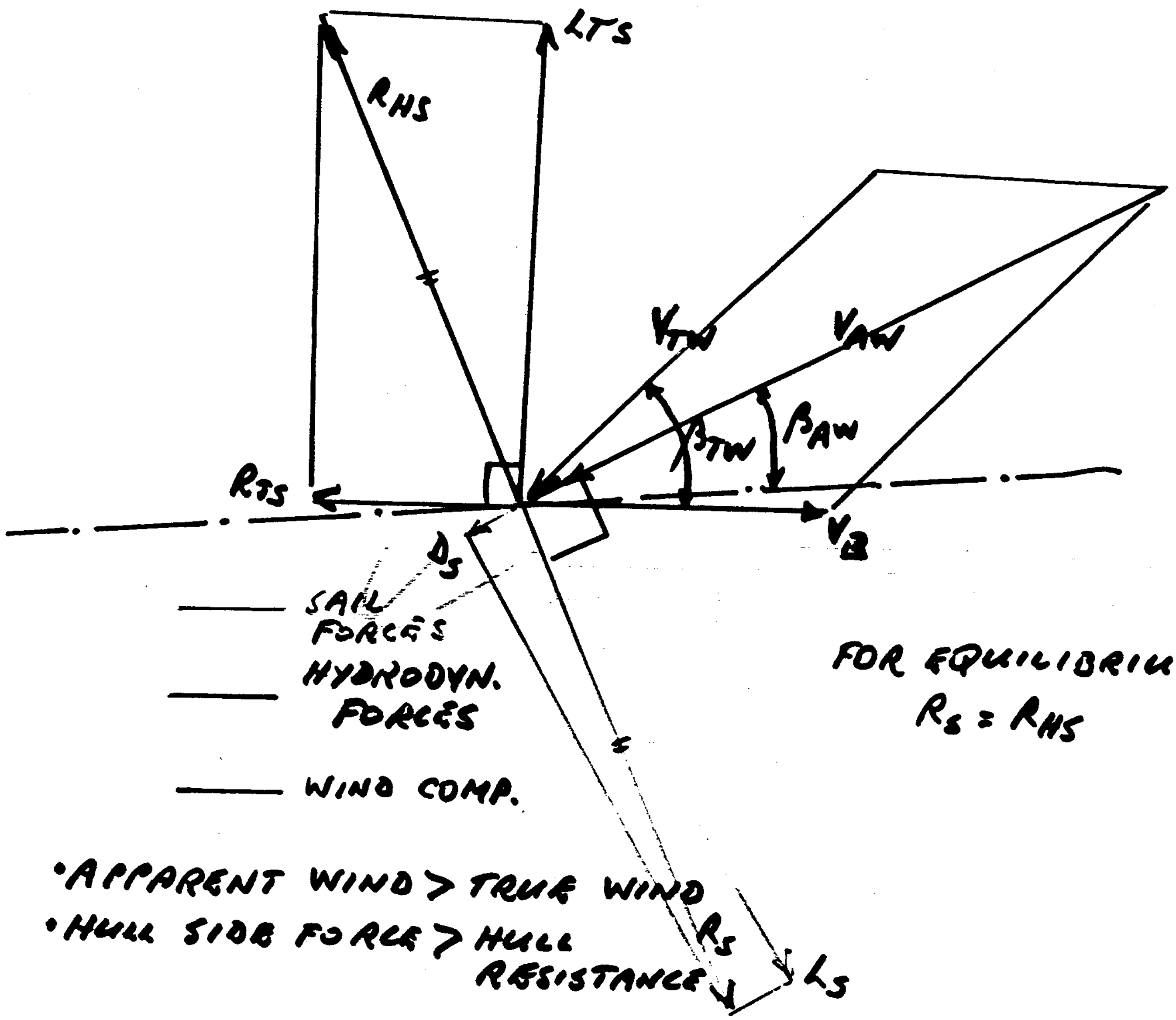


- FOR EQUILIBRIUM: $R_S = R_{HS}$
- SAILS ARE IN "LIFT MODE"
- HULL MAKES LEEWAY (β = LEEWAY ANGLE)
- APPARENT WIND SPEED ≈ TRUE WIND SPEED
- HULL SIDE FORCE ≈ HULL RESISTANCE
- FOR OPTIMUM PERFORMANCE: MINIMIZE R_{TS}, CHOOSE SAIL AREA IN ACCORDANCE WITH STABILITY

• FORCES AND GEOMETRY "BEATING"

• THE MOST DIFFICULT POINT OF SAIL FROM THE POINT OF VIEW OF OBTAINING OPTIMUM PERFORMANCE





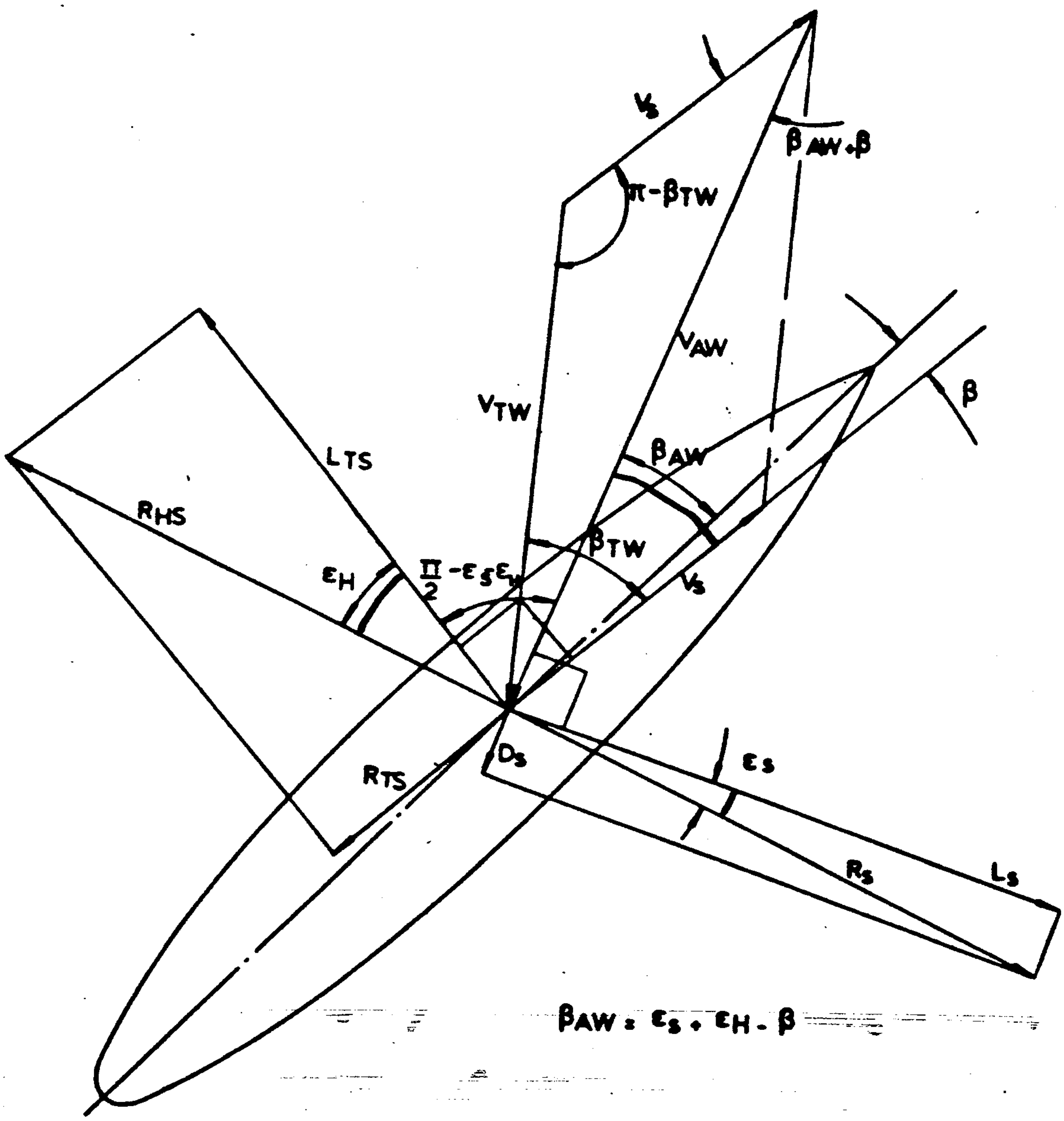
FOR EQUILIBRIUM:
$R_S = R_{HS}$

• APPARENT WIND > TRUE WIND
• HULL SIDE FORCE > HULL RESISTANCE

(5B)

L_{TS} = TOTAL HYDRODYNAMIC SIDE FORCE

R_{TS} = TOTAL HYDRODYNAMIC RESISTANCE



$\beta_{AW} = \varepsilon_S + \varepsilon_H - \beta$

FORCE EQUILIBRIUM

FOR EQUILIBRIUM:

$$\boxed{\beta_{AW} + \beta = \varepsilon_S + \varepsilon_H}$$ (F.W. LANCHESTER 1907)

$\varepsilon_S = \arctan \frac{D_S}{L_S}$ "SAIL DRAG ANGLE"

$\varepsilon_H = \arctan \frac{R_{TS}}{LT_S}$ "HULL DRAG ANGLE"

- A SAILING YACHT CANNOT SAIL CLOSER TO THE WIND THAN THE SUM OF THE SAIL AND HULL "DRAG" (OR "GLIDING") ANGLES

- NOTE: β_{AW} IS HERE DEFINED FROM ℄ AND $(\beta_{AW} + \beta)$ IS THE APPARENT WIND ANGLE RELATIVE TO THE COURSE SAILED

- "CLOSE-WINDED" YACHTS: $\beta_{AW} + \beta \approx 20°-21°$ WHERE $\varepsilon_S \approx 7°$; $\varepsilon_H \approx 13° - 14°$

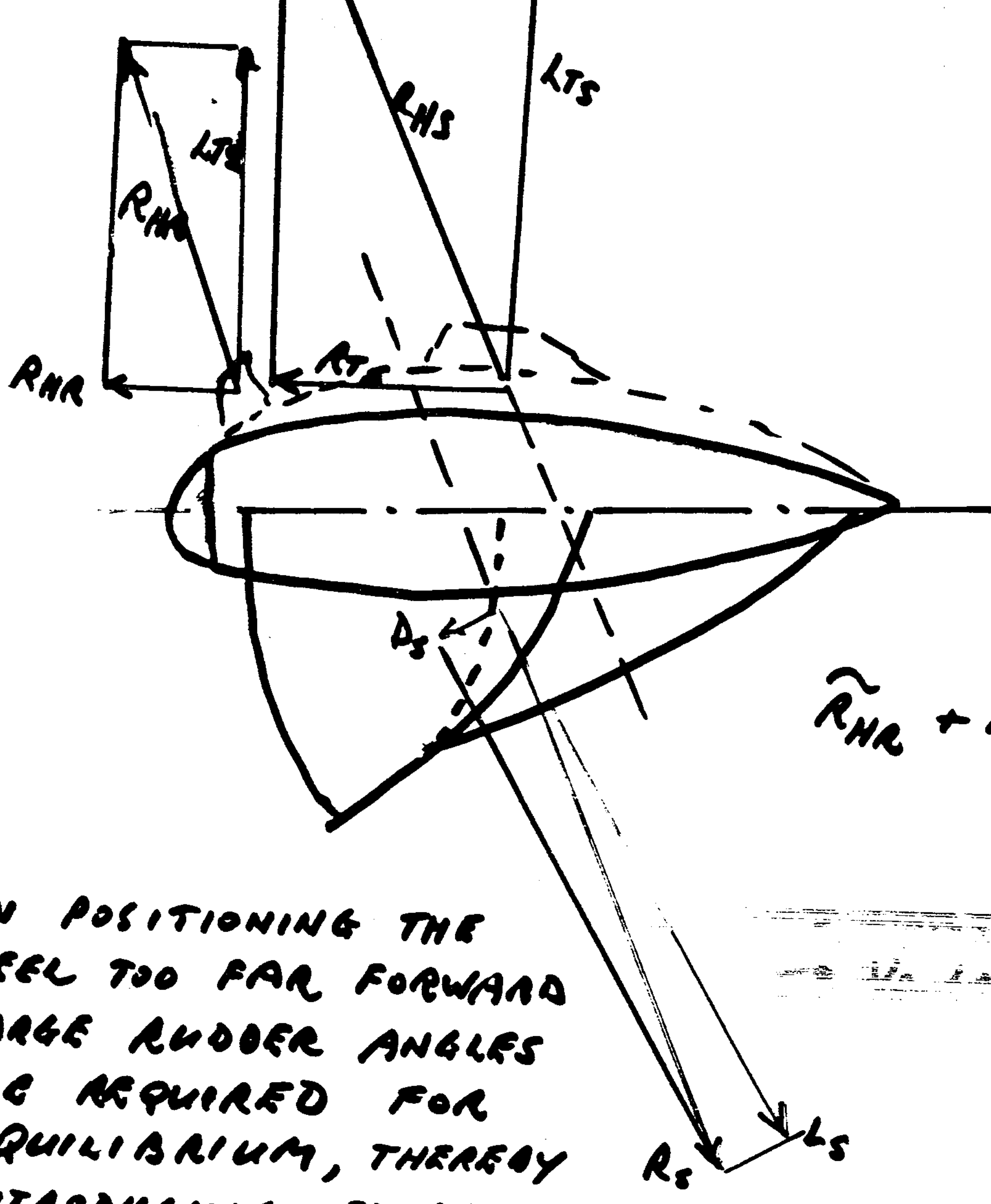
$\varepsilon_S = 7°$ IMPLIES $L_S = 8.14 * D_S$

$\varepsilon_H = 13°$ IMPLIES $L_{TS} = 4.33 * R_{TS}$

- THUS AN INCREASE IN SAIL LIFT-DRAG RATIO AND/OR HULL SIDE FORCE-RESISTANCE RATIO IMPROVES THE WINWARD ABILITY OF A SAILING CRAFT.

FOR OPTIMUM PERFORMANCE BEATING:

- MINIMIZE HYDRODYNAMIC RESISTANCE
- MAXIMIZE HYDRODYNAMIC SIDE FORCE
- MAXIMIZE STABILITY
- CHOOSE CORRECT RELATIVE POSITION OF SAIL CENTRE OF EFFORT AND KEEL CENTRE OF SIDE FORCE

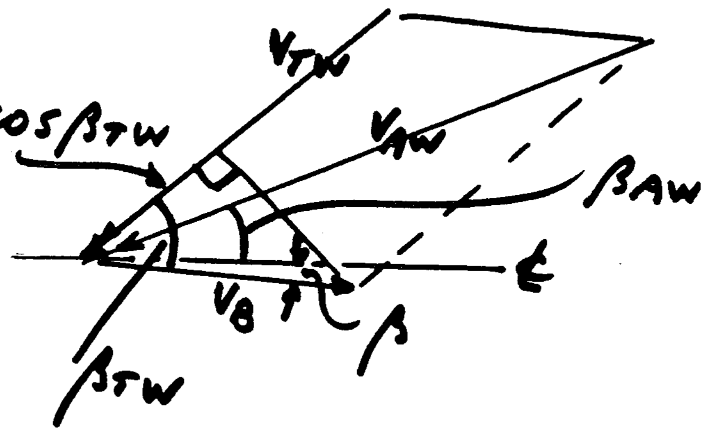


$$\tilde{R}_{HR} + \tilde{R}_{HS} = \tilde{R}_S$$

ON POSITIONING THE KEEL TOO FAR FORWARD LARGE RUDDER ANGLES ARE REQUIRED FOR EQUILIBRIUM, THEREBY INTRODUCING EXTRA HYDRODYNAMIC RESISTANCE

SPEED TO WINDWARD IS MEASURED BY THE SO-CALLED SPEED MADE GOOD VMG

$$VMG = V_B \cos\beta_{TW}$$



MAXIMUM VMG : • HIGH V_B (LOW RESISTANCE)

• SMALL β_{TW}, i.e. SMALL β_{AW} AND SMALL β

NOTE : A LARGE KEEL → HIGH SIDEFORCE
→ SMALL ε_H
→ LARGE WETTED AREA
→ V_B LOW, β_{TW} SMALL
SAILS "HIGH" & "SLOW"

A SMALL KEEL → LOW SIDE FORCE
→ LARGE ε_H
→ SMALL WETTED AREA
→ V_B HIGH, β_{TW} LARGE
SAILS "LOW" & FAST

HYDRODYNAMIC RESISTANCE AND ITS COMPONENTS

FRICTIONAL RESISTANCE (FLAT PLATE EQUIVALENT

- DEPENDENT ON WETTED AREA AND STATE OF BOUNDARY LAYER (REYNOLDS NO.)

$$R_F = \frac{1}{2} \rho V_B^2 S_H C_F$$

ρ = DENSITY OF WATER
1000 kg/m^3 FRESH WATER } 15°C
1025.9 kg/m^3 SALT WATER }

V_B = SPEED OF YACHT m/sec

S_H = WETTED SURFACE

C_F = FRICTIONAL RESISTANCE COEF.

$$C_F = C_{F_{TURB}} - \frac{c}{R_N} \quad \text{(PRANDTL)}$$

$$= \frac{0.074}{(R_N)^{0.2}} - \frac{1700}{R_N} \quad (\text{''})$$

NOW $$C_F = \frac{0.075}{(\log_{10} R_N - 2)^2} - \frac{1800}{R_N}$$

WHERE $$R_N = \frac{V_B . L_{WL}}{\nu} > 5 \times 10^5$$

15°C { $\nu = 1.191 \times 10^{-6}$ m^2/sec SEA W.
$\nu = 1.142 \times 10^{-6}$ m^2/sec FRESH W.

FRICTIONAL RESISTANCE TO BE DETERMINED SEPARATELY FOR CANOE BODY, KEEL AND RUDDER:

FOR CANOE BODY:

- $S_H \approx \left(1.97 + 0.171 \frac{B_{WL}}{T_C}\right)\sqrt{\nabla_C L_{WL}}$

  B_{WL}: MAX. BEAM ON L_{WL}
  T_C: MAX. DRAUGHT OF CANOE BODY
  ∇_C: VOLUME OF DISPLACEMENT OF CANOE BODY
  L_{WL}: LOAD WATERLINE LENGTH

- $R_{N_C} = \frac{V_B \cdot L_{WL}}{\nu}$ (OR $0.7 L_{WL}$)

FOR KEEL:

- $S_K \cong 2 \bar{c}_K \cdot span$ (WETTED SURFACE)
- $R_{N_K} = \frac{V_B \bar{c}_K}{\nu}$

FOR RUDDER:

- $S_R \cong 2 \bar{c}_R \cdot span$

  $R_{N_R} = \frac{V_B \cdot \bar{c}_R}{\nu}$ (OR $\frac{0.8 V_B \bar{c}_R}{\nu}$)

VISCOUS RESISTANCE OF CANOE BODY

- HULL (CANOE BODY) IS NOT A FLAT PLATE
- BOUNDARY LAYER UNDERGOES ACCELERATION AND DECELERATION DUE TO 3D FORM
- SOME (NOMINAL) BOUNDARY LAYER SEPARATION WILL OCCUR:

$$R_V = R_F(1+k)$$

$1+k$ = FORM FACTOR
$\cong 1.1$ FOR GOOD SAILING YACHT

VISCOUS RESISTANCE OF KEEL/RUDDER

$$R_V = R_F\left(1 + 1.2(t/c)_k + 70(t/c)_k^4\right)$$

NACA 63, 64 AND 65

$$R_V = R_F\left(1 + 2(t/c)_k + 60(t/c)_k^4\right)$$

NACA 4-DIGIT

t/c = average thickness-chord ratio of keel (k) or rudder (r)

MINIMIZE THE FORM FACTOR BY ADOPTING SLENDER WATERLINES IN LOW B_{WL}/T_C HULLS AND SLENDER/STRAIGHT BUTTOCKS IN HIGH B_{WL}/T_C FORMS

PRESSURE DRAG CAUSED BY IMMERSED TRANSOM

- IS COMPONENT OF THE VISCOUS RESISTANCE

$$R_{TR} = \frac{1}{2} \rho V_B^2 A_{TR} C_{TR}$$

A_{TR} : IMMERSED TRANSOM AREA

$C_{TR} = 0.2(1-0.2 F_{NT}) \qquad F_{NT} < 5$

$C_{TR} = 0 \qquad F_{NT} > 5$

$$F_{NT} = \frac{V_B}{\sqrt{g \cdot B_T}}$$

with $B_T \cong \frac{1}{2}\left(B_{WL} + \frac{A_{WP}}{L_{WL}}\right)$

HENCE :

$$R_V = R_F(1+k)_C + R_F(1+k)_K + R_F(1+k)_R + R_{TR}$$

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WAVE RESISTANCE (R_W)

- CAUSED BY PRESSURE VARIATIONS ALONG THE HULL
  - WAVE PEAK IN AREAS OF HIGH PRESSURE (BOW IN PARTICULAR)
  - WAVE TROUGH AMIDSHIPS

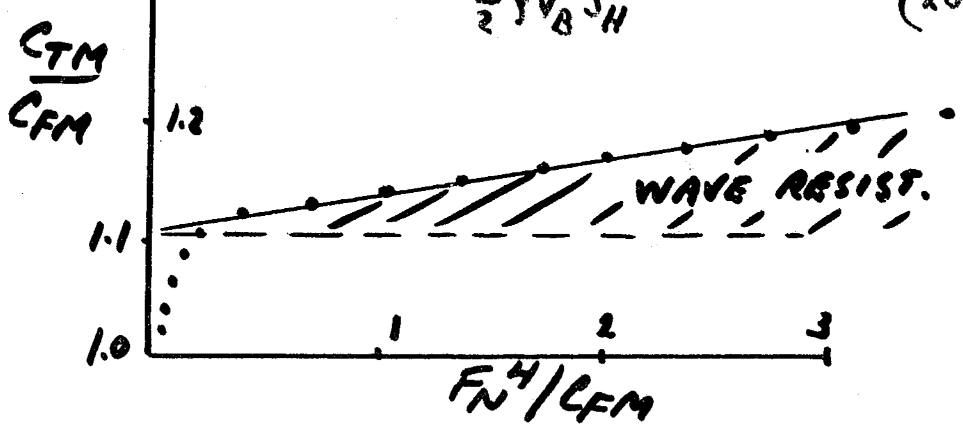
- WATER SURFACE IS A SURFACE OF CONSTANT PRESSURE
- IS DEPENDENT ON FORM AND FROUDE NO (F_N)
- R_W CANNOT BE DETERMINED FROM A SIMPLE MATHEMATICAL RELATIONSHIP. IT IS USUALLY DETERMINED FROM MODEL TESTS, VIZ:

$$R_{W_M} = R_{T_M} - R_{V_M}$$

R_{T_M} = MEASURED TOTAL RESISTANCE ON MODEL SCALE

R_{V_M} = CALCULATED VISCOUS RESISTANCE FOR MODEL WITH $1+k$ DETERMINED FROM PROHASKA'S TECHNIQUE

$$C_{TM} = \frac{R_{TM}}{\frac{1}{2}\rho V_B^2 S_H} \; ; \quad C_{FM} = \frac{0.075}{(\log R_{N_m} - 2)^2}$$



THEN $R_{W_S} = R_{WM} (\lambda)^3 \frac{\rho_{SALT}}{\rho_{FRESH}}$

λ = SCALE RATIO

AND $R_{TS} = R_{VS} + R_{WS}$

- IF $1+k$ CANNOT BE DETERMINED ACCURATELY FROM LOW-SPEED TESTING, THEN THE RESIDUAL RESISTANCE IS CALCULATED INSTEAD:

$$R_{RM} = R_{TM} - R_{F_M}$$

THEN $R_{R_S} = R_{RM} (\lambda)^3 \frac{\rho_{SALT}}{\rho_{FRESH}}$

- FOR ESTIMATING RWAVE OR R_R USE CAN BE MADE OF SYSTEMATIC MODEL SERIES RESULTS. REGRESSION ANALYSIS TECHNIQUES ARE THEN USED FOR DERIVING GENERAL RESISTANCE FORMULATIONS.

  - D.U.T. 22 MODEL SERIES
  - VAN OORTMERSSEN FORMULES
  - HOLTROP (1980) "
  - IMS FORMULA

ADDITIONAL RESISTANCE DUE TO HEEL

- THIS IS MAINLY OF WAVE RESISTANCE IN ORIGIN. ADDITIONAL VISCOUS RESISTANCE IS ~ OFFSETT BY REDUCED WETTED AREA

$R_{WH} \approx R_W / \cos^2\theta \; ; \; R_{RH} \approx R_R / \cos^2\theta$

DERIVATION OF THEORETICAL R_W EXPRESSION

FOR YACHTS, PROHASKA'S ANALYSIS USUALLY REVEALS THAT:

$$C_{TM} = C_{FM}\left(1 + k + y\frac{F_N^4}{C_{FM}}\right)$$

$$\text{i.e. } C_{TM} = C_{FM}(1+k) + yF_N^4$$

$$= C_{VM} + C_W$$

where $C_W = yF_N^4$

Since $R_W = \frac{1}{2}\rho V_B^2 C_W S_H$

OR $R_W = \frac{1}{2}\rho V_B^2 y F_N^4 . S_H$

WITH $F_N = \dfrac{V_B}{\sqrt{gL_{WL}}}$

AND $S_H = \left(a + b\dfrac{B_{WL}}{T_c}\right)\sqrt{\nabla_c L_{WL}}$

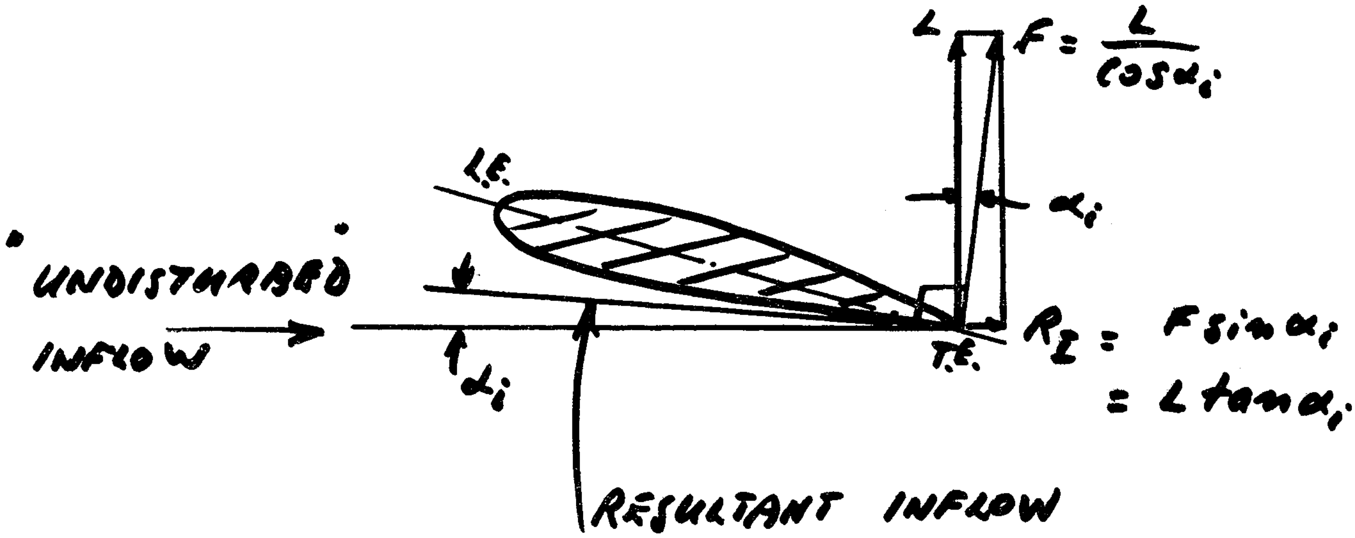
$$R_W = \frac{1}{2}\rho V_B^2 y \frac{V_B^4}{g^2 L_{WL}^2} . \left(a + b\frac{B_{WL}}{T_c}\right)\sqrt{\nabla_c L_{WL}}$$

$$= \frac{1}{2}\rho V_B^6 \left(ay + by\frac{B_{WL}}{T_c}\right)\frac{1}{g^2}\frac{\nabla_c^{1/2}}{L_{WL}^{3/2}}$$

$$R_W = \frac{1}{2}\rho\left(\frac{ay}{g^2} + \frac{by}{g^2}\frac{B_{WL}}{T_c}\right)V_B^6 . \frac{1}{\left(\sqrt{\frac{L_{WL}}{\nabla_c}}\right)^3}$$

INDUCED RESISTANCE

- IS A COMPONENT OF LIFT (SIDE FORCE)



- α_i IS EXPERIENCED AS AN "UPWASH"

FROM AERODYNAMICS: $\alpha_i = \frac{C_L}{\pi AR}$; C_L = LIFT COEFFICIENT

AR = ASPECT RATIO $= \frac{SPAN^2}{AREA}$

with $R_I = L\tan\alpha_i \approx L\alpha_i$ (α_i = SMALL)

$$= \frac{L . C_L}{\pi AR}$$, where $L = \frac{1}{2}\rho V_B^2 A_L C_L$

ie $R_I = \frac{\frac{1}{2}\rho V_B^2 A_L C_L^2}{\pi AR}$

WITH $C_L = \frac{2\pi AR}{2 + \sqrt{AR + 4}} \beta$ * (WHICKER & FEHLNER)

β = LEEWAY ANGLE

* NEGLECTING SWEEP + VISCOUS EFFECTS FOR SMALL β-VALUES

$$R_I = \tfrac{1}{2}\rho V_B^2 A_L \frac{(2\pi AR\beta)^2}{(2+\sqrt{AR+4})^2} \cdot \frac{1}{\pi AR}$$

$$R_I = \frac{2\rho V_B^2 A_L \pi AR\beta^2}{(2+\sqrt{AR+4})^2}$$

A_L = LATERAL AREA OF KEEL OR RUDDER

NOTE 1) R_I DECREASES ON INCREASING AR

2) $$\frac{L}{R_I} = \frac{\tfrac{1}{2}\rho V_B^2 A_L 2\pi AR\beta(2+\sqrt{AR+4})}{2\rho V_B^2 A_L \pi AR\beta^2}$$

i.e. $$\frac{L}{R_I} = \frac{2+\sqrt{AR+4}}{2\beta}$$

NOTE THAT L/R_I INCREASES WITH INCREASING AR AND DECREASES WITH INCREASING LEEWAY!

ADDED RESISTANCE IN WAVES

- AS WIND VELOCITY INCREASES WAVE HEIGHT INCREASES.
- WHEN BEATING, YACHT WILL PITCH RESULTING IN IMPORTANT RESISTANCE INCREASE, DEPENDING ON RATIO OF WAVE LENGTH TO LWL.
- IN SERIOUS DESIGN WORK (IOR, AMERICAS CUP, WHITBREAD, ETC) THIS MUST BE ACCOUNTED FOR.

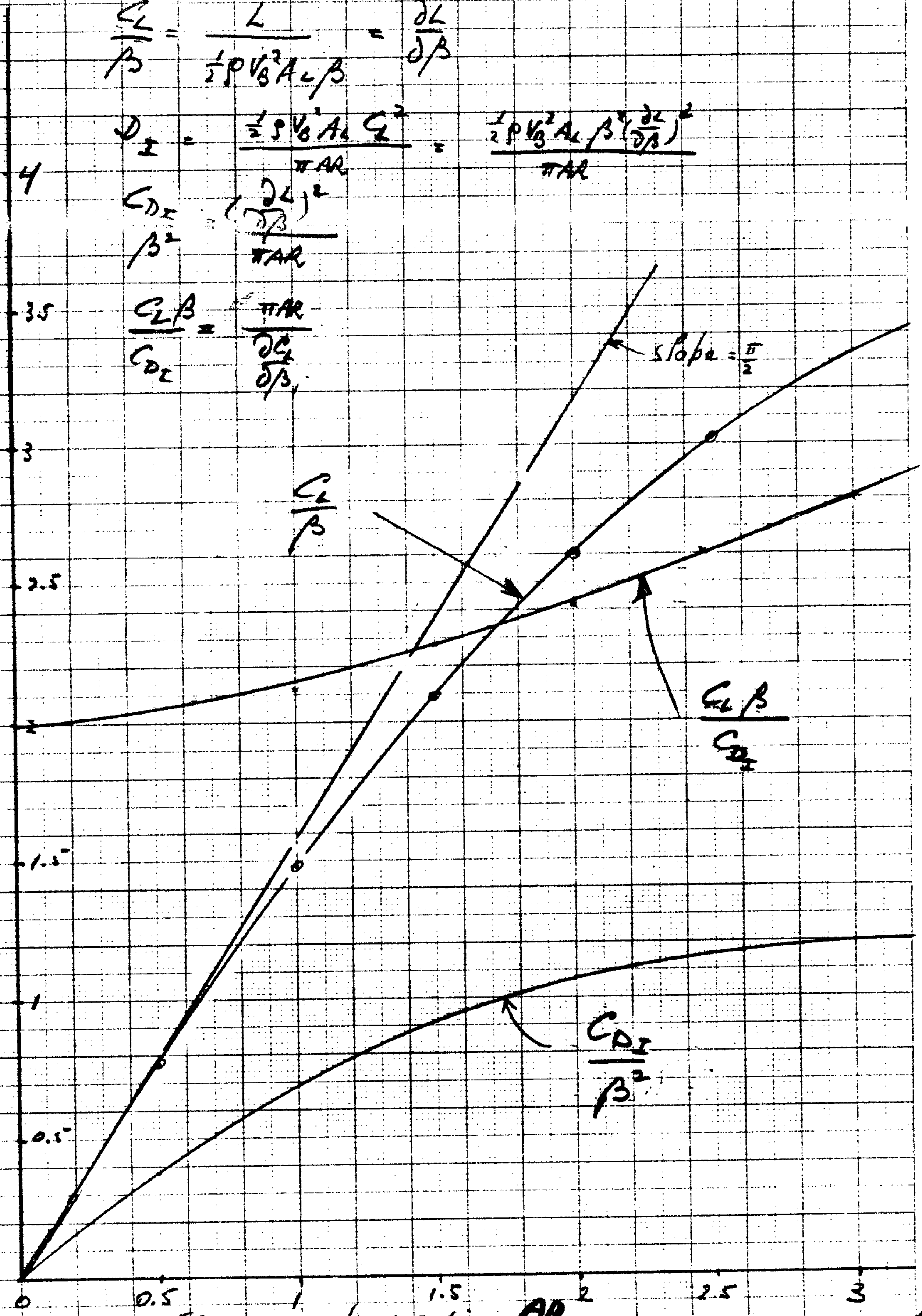
$$L = \frac{1}{2}\rho V_B^2 A_L \beta \cdot \frac{\partial L}{\partial \beta}, \text{ with } \frac{\partial L}{\partial \beta} \cong \frac{2\pi AR}{2 + \sqrt{AR + 4}}$$

$$\frac{C_L}{\beta} = \frac{L}{\frac{1}{2}\rho V_B^2 A_L \beta} = \frac{\partial L}{\partial \beta}$$

$$D_I = \frac{\frac{1}{2}\rho V_B^2 A_L C_L^2}{\pi AR} = \frac{\frac{1}{2}\rho V_B^2 A_L \beta^2 \left(\frac{\partial L}{\partial \beta}\right)^2}{\pi AR}$$

$$\frac{C_{D_I}}{\beta^2} = \frac{\left(\frac{\partial L}{\partial \beta}\right)^2}{\pi AR}$$

$$\frac{C_L \beta}{C_{D_I}} = \frac{\pi AR}{\frac{\partial C_L}{\partial \beta}}$$



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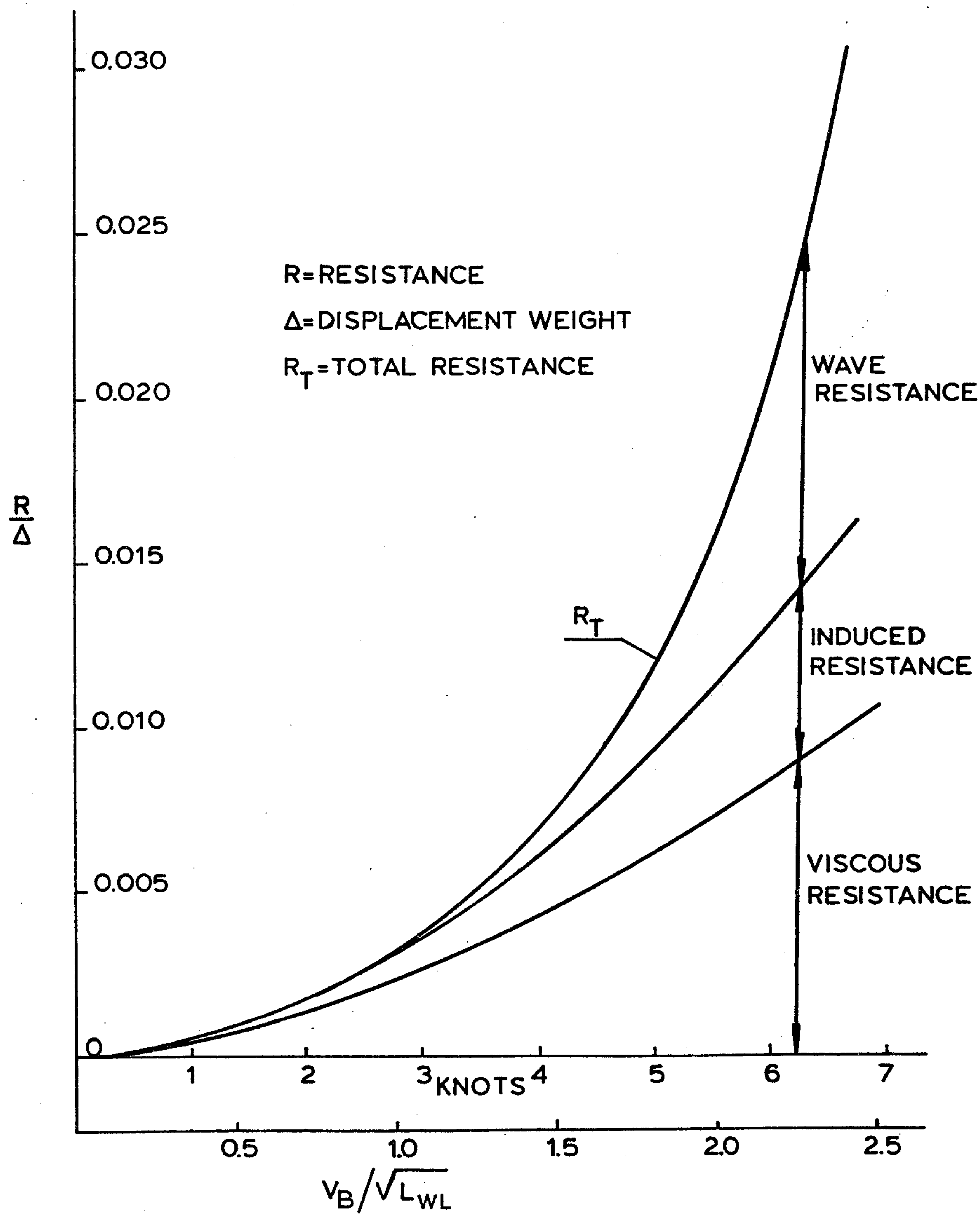
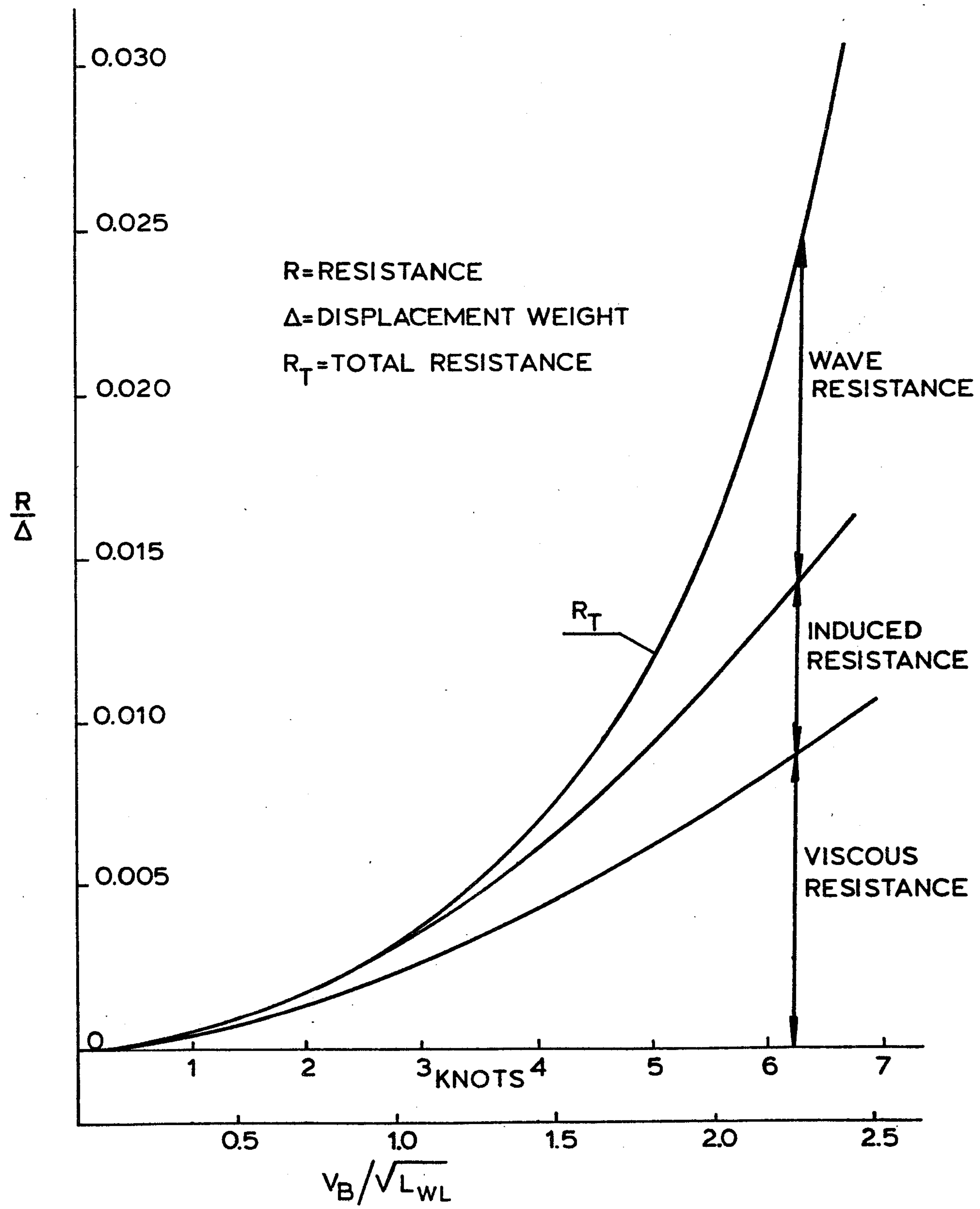
C_L/C_{D_I} on AR


0.030
0.025
0.020
0.015
0.010
0.005
0
R/Δ
R=RESISTANCE
Δ=DISPLACEMENT WEIGHT
R_T=TOTAL RESISTANCE
R_T
WAVE RESISTANCE
INDUCED RESISTANCE
VISCOUS RESISTANCE
1
2
3
4
5
6
7
KNOTS
0.5
1.0
1.5
2.0
2.5
$V_B/\sqrt{L_{WL}}$

1.

S.S. "ANTIOPE"



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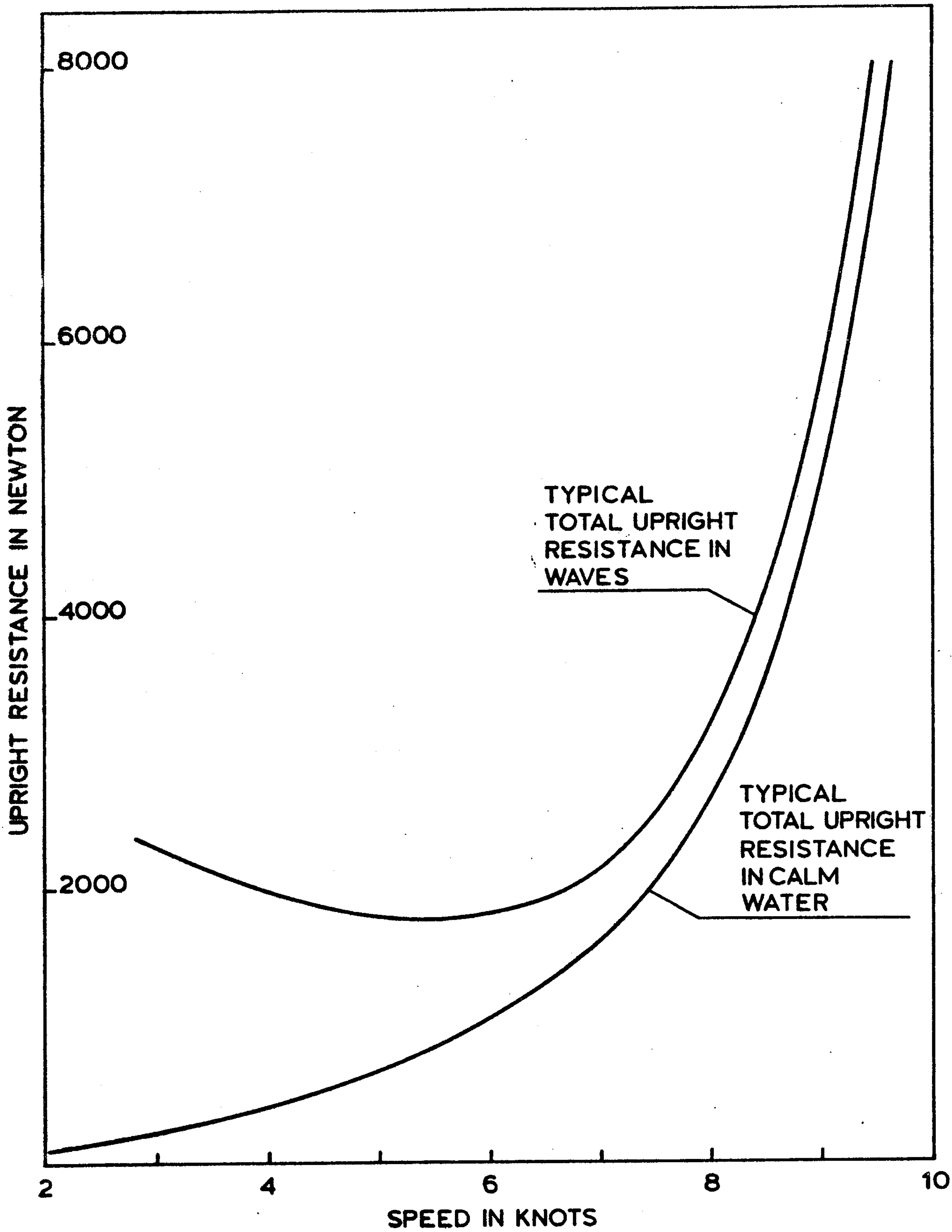
5.5m "ANTIOPE"



2.

1AM NEWPORT

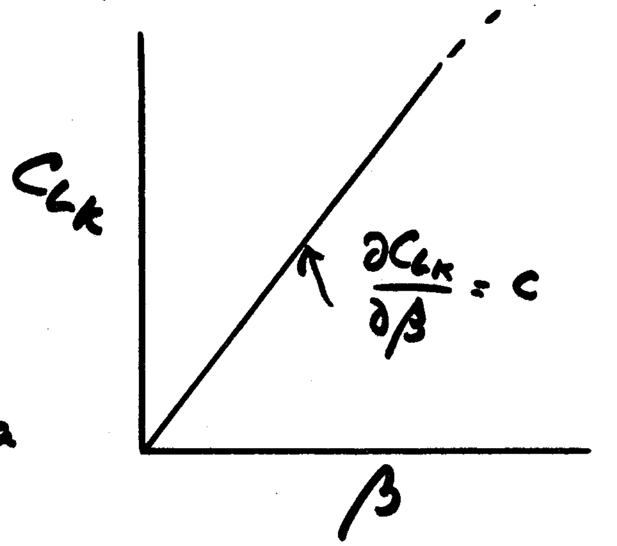
HYDRODYNAMIC SIDEFORCE OR LIFT

SIDE FORCE OF KEEL

$$L_K = \frac{1}{2}\rho V_B^2 \frac{\partial C_{LK}}{\partial \beta} \beta \cdot A_{LK} \cos\theta$$

L_K = lift (side force) of keel
ρ = density of water
V_B = boat speed
β = leeway angle
$\frac{\partial C_{LK}}{\partial \beta}$ = lift-curve slope (gradient)

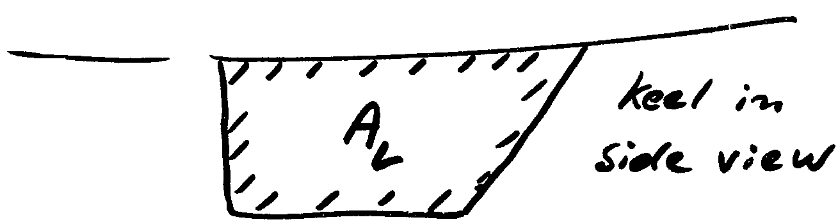
for small β-angles
$\frac{\partial C_{LK}}{\partial \beta}$ is a constant c



A_{LK} = lateral area of keel

θ = heel angle of yacht

NOTE: $C_{LK} = \frac{L_K}{\frac{1}{2}\rho V_B^2 A_L}$



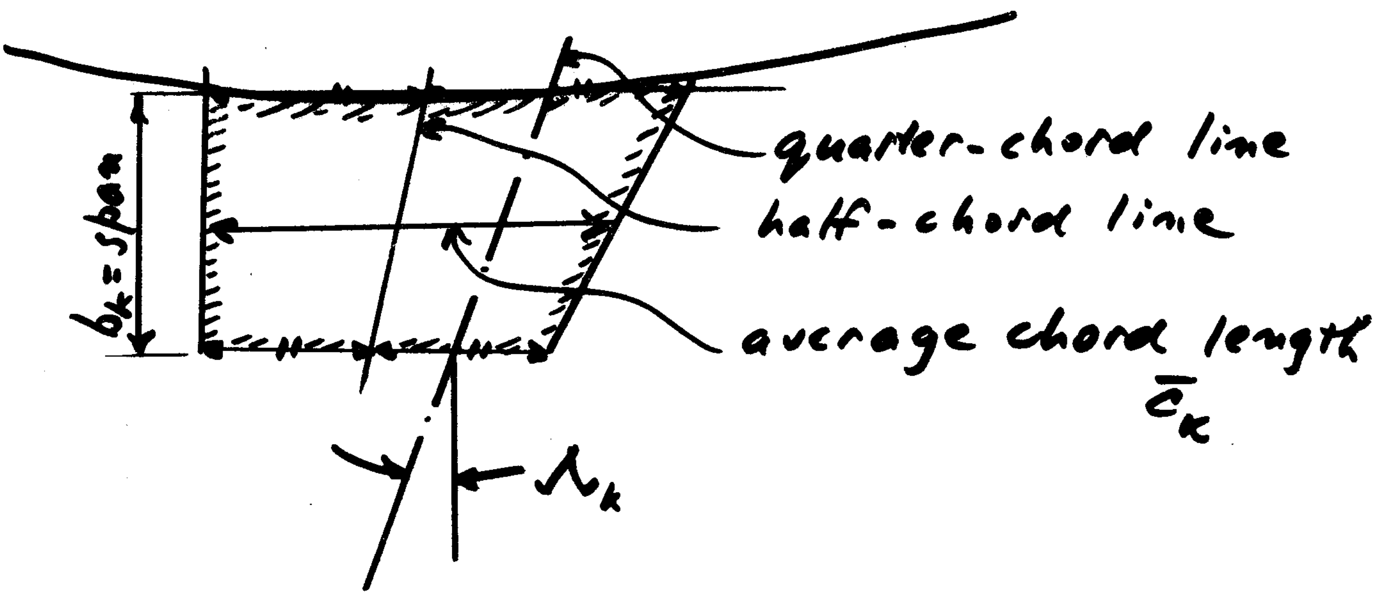
ACCORDING TO WHICKER + FEHLNER:

$$\frac{\partial C_{L_K}}{\partial \beta} = \frac{2\pi a_0 AR_K}{2a_0 + \cos\Lambda_K \sqrt{AR_K^2/\cos^4\Lambda_K + 4}}$$

where a_0 : lift-curve slope of equivalent 2D foil (i.e. with "$AR = \infty$")

AR_K : effective aspect ratio

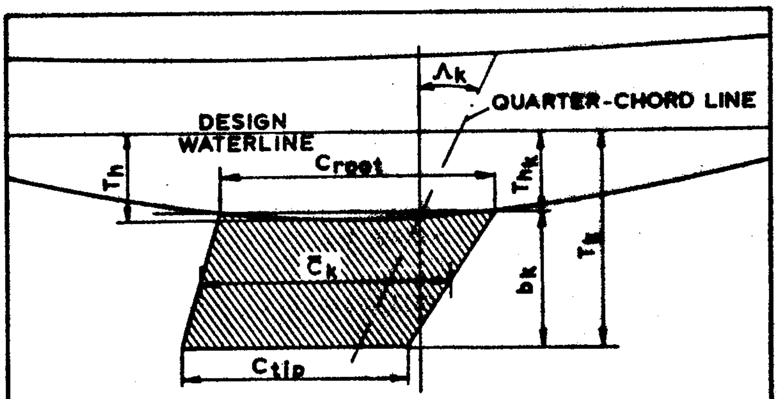
Λ : sweep angle of "quarter-chord" line of keel



the effective aspect ratio $AR_K = \frac{2 \cdot span}{av.\ chord} \cdot \cos\theta$

i.e. $AR_K = \frac{2 \cdot (span)^2}{A_L} \cos\theta = \frac{2 b_K \cos\theta}{\bar{c}_K}$

where A_L = lateral area $(= \bar{c}_K \cdot b_K)$



PROJECTED (UPRIGHT) LATERAL AREA = A_k

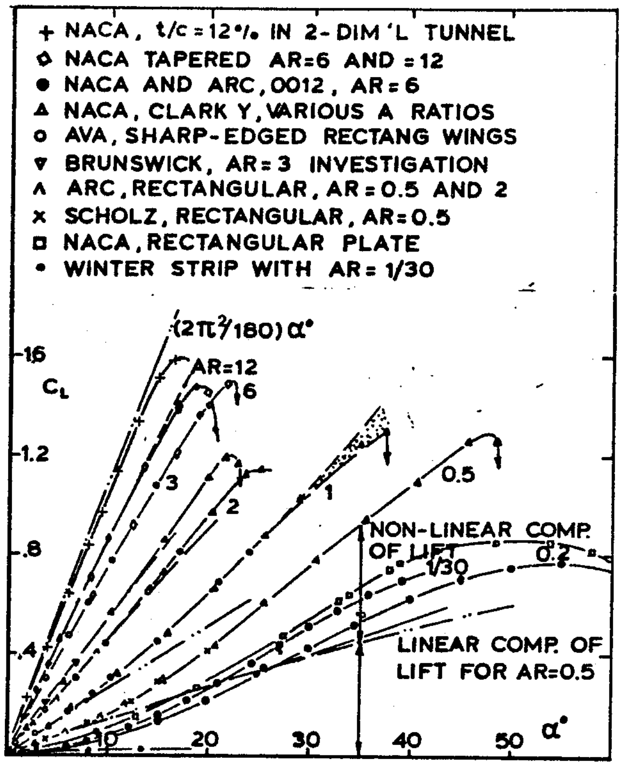
TAPER RATIO = $\frac{C_{tip}}{C_{root}}$

SWEEP ANGLE = Λ_k

SPAN = b_k

LATERAL AREA = $\bar{C}_k \cdot b_k$

DEFINITION OF TAPER, SWEEP & SPAN



EFFECT OF AR ON C_L/β

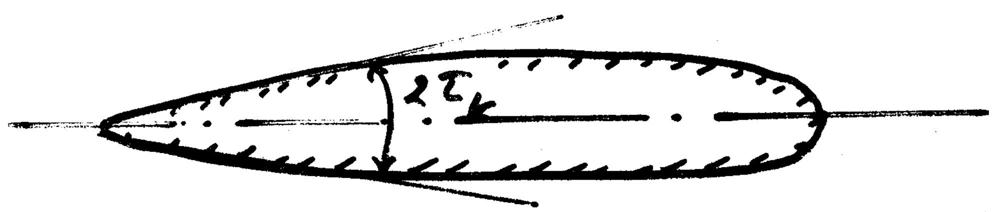
the lift-curve slope of equivalent 2D foil is approx = 2π (2SA)

<u>In actual fact:</u>

$$a_0 = 2\pi\left(1 + 0.82\,(t/c)_k - \tan\tau_k\left(\frac{0.117}{(t/c)_k} + 3.2\,(t/c)_k + 3.9\,(t/c)_k^2\right)\right)$$

where $(t/c)_k$ = average thickness-chord ratio of keel

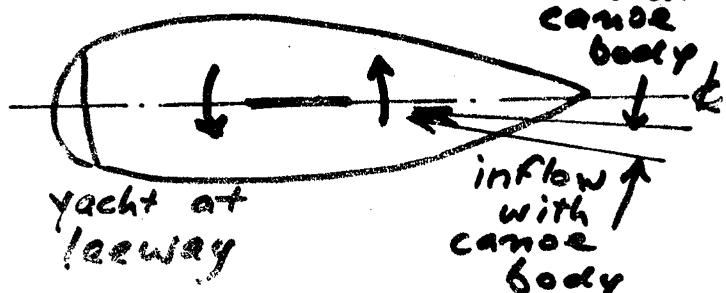
τ_k = half trailing-edge angle of average section of keel



EFFECT OF CANOE BODY ON SIDE FORCE OF KEEL

THE PRESENCE OF THE CANOE BODY, TO WHICH THE KEEL IS ATTACHED, CONSIDERABLY INCREASES THE SIDE FORCE ON THE KEEL:

- "upwash" ↑ and "downwash" ↓ induced by canoe body



additional sideforce on keel is approximately:

$$\Delta L_K = \sigma L_K$$

where σ = canoe draught to total draught

i.e. $\sigma = \dfrac{T_c}{T_k}$

where T_c = draught of canoe body
T_k = draught to bottom of keel

Hence, total side force on keel:

$$L_{K_T} = L_k + \Delta L_k = L_k + \sigma L_k$$

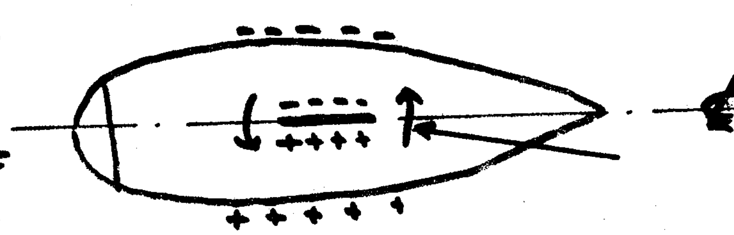
$$= (1+\sigma) L_k$$

or $$L_{K_T} = \left(1 + \frac{T_c}{T_k}\right) L_k$$

ADDITIONAL SIDE FORCE ON CANOE BODY INDUCED BY KEEL

THE PRESENCE OF THE KEEL ALSO INDUCES ADDITIONAL SIDE FORCE ON THE CANOE BODY:

AREA OF LOW PRESSURE
ON KEEL INDUCES
LOW PRESSURE
ON HULL ABOVE
KEEL DUE TO
INCREASE IN VELOCITY, ETC



additional side force on canoe body induced by keel:

$$\Delta L_H = \sigma(1+\sigma)L_k \quad \text{(see Flax)}$$

where L_H = side force on canoe body

ΔL_H = " " " " " induced by keel

$$\sigma = \frac{T_c}{T_k}$$

The <u>total side force on, and induced by, the keel</u> is:

$$L_{T_k} = (1+\sigma)L_k + \sigma(1+\sigma)L_k$$

or $$L_{T_k} = (1+\sigma)^2 L_k$$

i.e.
$$\boxed{L_{T_k} = \frac{1}{2}\rho V_B^2 \frac{\partial C_{L_k}}{\partial \beta} \cdot \beta\left(1+\frac{T_c}{T_k}\right)^2 A_{L_k}\cos\theta}$$

where
$$\boxed{\frac{\partial C_{L_k}}{\partial \beta} = \frac{2\pi a_0 AR_k}{2a_0 + \cos\Lambda_k\sqrt{AR_k^2/\cos^4\Lambda_k + 4}}}$$

and
$$\boxed{AR_k = \frac{2 b_k \cos\theta}{\bar{c}_k}}$$

and $a_0 \approx 2\pi$ (for preliminary design)

SIDE FORCE ON, AND INDUCED BY, RUDDER

IS DEALT WITH IN SIMILAR FASHION, i.e.

$$L_{T_R} = \frac{1}{2}\rho V_B^2 \frac{\partial C_{L_R}}{\partial \beta} . \beta \left(1 + \frac{T_{C_R}}{T_R}\right)^2 A_{L_R} \cos\Theta$$

where $\frac{\partial C_{L_R}}{\partial \beta}$ = lift-curve slope of rudder with respect to β (leeway)

β = leeway angle

T_{C_R} = draught of canoe body to top of rudder

T_R = draught to bottom of rudder

A_{L_R} = lateral area of rudder

Θ = heel angle

Here also:

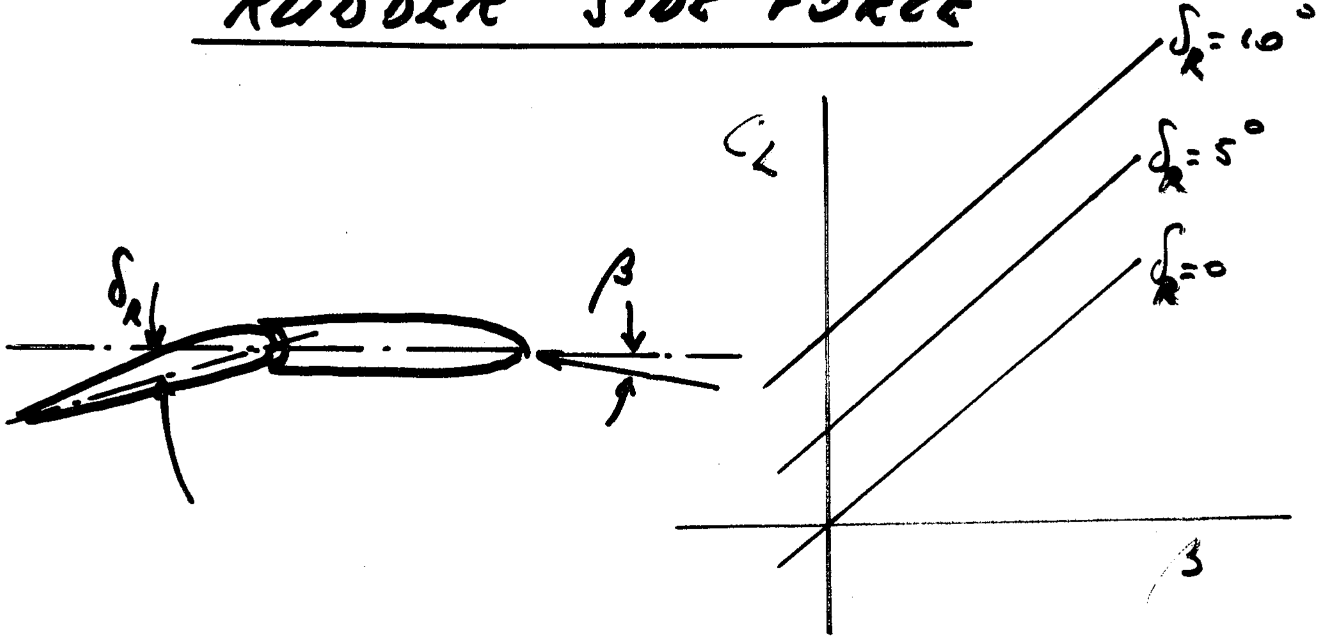
$$\frac{\partial C_{L_R}}{\partial \beta} = \frac{2\pi a_0 AR_R}{2a_0 + \cos\Lambda_R \sqrt{AR_R^2/\cos^4\Lambda_R + 4}}$$

where

$$AR_R = \frac{2 b_R \cos\Theta}{\bar{c}_R}$$

$a_0 \approx 2\pi$

EFFECT OF RUDDER ANGLE ON RUDDER SIDE FORCE



FOR A RUDDER-SKEG ARRANGEMENT, THE RUDDER ANGLE δ IN EFFECT SHIFTS THE LIFT CURVE VERTICALLY UPWARD

i.e. $$C_{L_R} = \frac{\partial C_{L_R}}{\partial \beta} \cdot \beta + \frac{\partial C_L}{\partial \delta_R} \cdot \delta_R$$

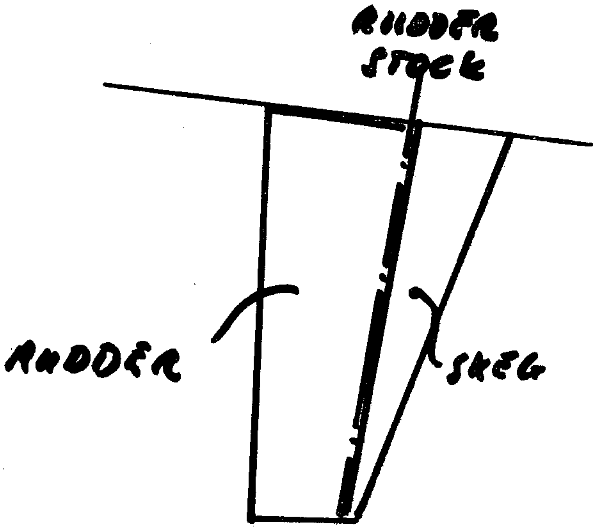
where $\frac{\partial C_{L_R}}{\partial \beta}$ = lift-curve slope of rudder w.r.t β only (at $\delta_R = 0$)

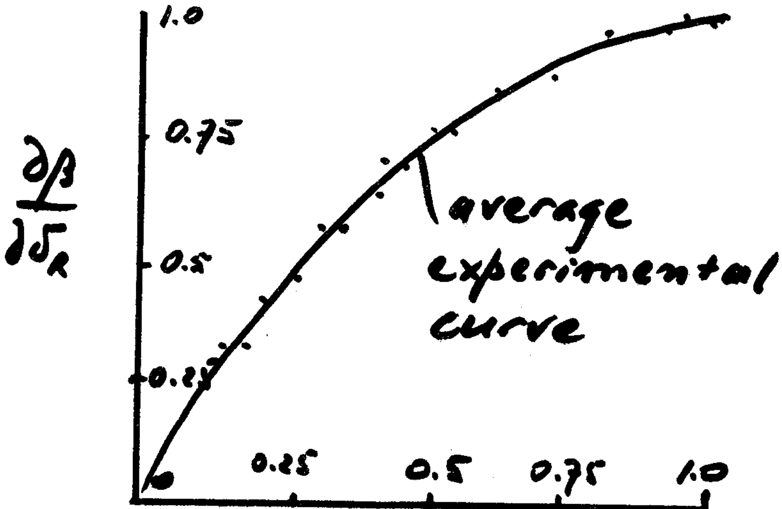
and $\frac{\partial C_{L_R}}{\partial \delta_R}$ = lift-curve slope of rudder w.r.t. δ_R only (at $\beta = 0$)

C_{L_R} can also be written as:

$$\boxed{C_{L_R} = \frac{\partial C_{L_R}}{\partial \beta}\left(\beta + \frac{\partial \beta}{\partial \delta_R} \delta_R\right)}$$

IN AERODYNAMICS $\frac{\partial \beta}{\partial \delta}$ is called "flap-effectiveness ratio"





flap chord / total chord

or $\frac{\text{moveable rudder area}}{\text{total rudder-skeg area}}$

typical values:

$\frac{A_R}{A_{RS}}$	$\frac{\partial\beta}{\partial\delta_R}$
0	0
0.1	0.27
0.2	0.45
0.3	0.55
0.4	0.65
0.5	0.73
0.75	0.89
1.00	1.00

A_R = moveable rudder area

A_{RS} = total rudder-skeg area

if skeg area = 0 (spade rudder)

then $A_R/A_{RS} = 1 \longrightarrow \frac{\partial\beta}{\partial\delta_R} = 1$

in that case $C_{L_R} = \frac{\partial C_{L_R}}{\partial\beta}(\beta + \delta_R)$

SIDE FORCE OF KEEL - TRIM TAB ARRANGEMENT

AS FOR THE RUDDER-SKEG CONFIGURATION:

$$C_{L_K} = \frac{\partial C_{L_K}}{\partial \beta}\left(\beta + \frac{\partial \beta}{\partial \delta_{tt}} \cdot \delta_{tt}\right)$$

where $\frac{\partial \beta}{\partial \delta_{tt}}$ = effectiveness of trim tab

δ_{tt} = trim tab angle

It follows that for a keel - trim tab:

$$L_{T_K} = \frac{1}{2}\rho V_B^2 \frac{\partial C_{L_K}}{\partial \beta}\left(\beta + \frac{\partial \beta}{\partial \delta_{tt}} \cdot \delta_{tt}\right)\left(1 + \frac{T_c}{T_K}\right)^2 A_{L_K} \cos\Theta$$

SIDE FORCE OF CANOE BODY

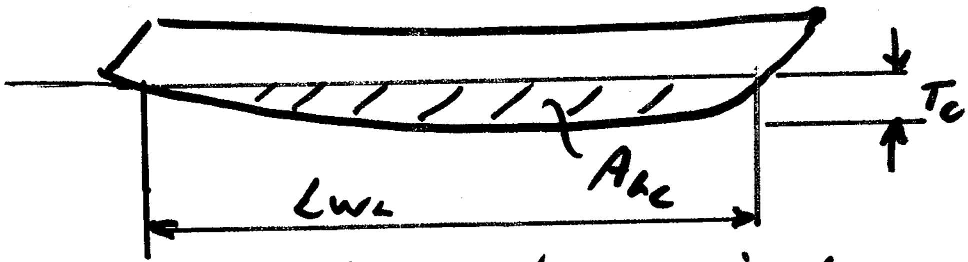
Without keel and/or rudder the canoe body producess little side force

Again we can write

$$L_{T_C} = \frac{1}{2}\rho V_B^2 \frac{\partial C_{L_C}}{\partial \beta} \cdot \beta \cdot A_{L_C} \cdot \cos\Theta$$

where $\frac{\partial C_{L_C}}{\partial \beta}$ = lift curve slope of canoe body w.r.t β

A_{L_C} = lateral area of canoe body



for such a low aspect ratio body,
(where $AR_c = T_c^2 / A_{Lc}$)

$$\frac{\partial C_{Lc}}{\partial \beta} \cong \frac{\pi}{2} AR_c + 1.8 \beta$$

here $\frac{\partial C_{Lc}}{\partial \beta}$ is dependent on β, because of low aspect ratio

in fact a good fit to experimental data is:

$$L_{T_c} = \frac{1}{2} \rho V_S^2 \left(\frac{\pi}{2} \cdot T_c^2 + 1.8\, T_c\, L_{WL} \cdot \frac{3}{4} \beta \right) \beta$$

(this is a small component only)

AERODYNAMIC SAIL MODELS

- GIMCRACK SAIL FORCE COEFFICIENTS WERE DERIVED BY KEN DAVIDSON 50 YEARS AGO
- MIT/DUT HAVE DEVELOPED SAIL FORCE COEFFICIENTS BASED ON "BAY-BEA" AND "STANDFAST 40" TESTS
- FOR MHS AND IMS "HANDICAPPING" A MORE COMPLETE (EMPIRICAL) SAIL FORCE MODEL HAS BEEN DEVELOPED
- MILGRAM HAS DEVELOPED A "LIFTING LINE" METHOD FOR CALCULATING AERODYNAMIC FORCES

- MOST REALISTIC (ACCURATE) RESULTS ARE OBTAINED USING THE NEWEST IMS MODEL. THESE YIELD "OPTIMISTIC" RESULTS, REQUIRING CALIBRATION BEFORE SERIOUS DESIGN APPLICATION

- REASONABLE RESULTS FOR FRACTIONAL RIGS ARE OBTAINED WITH FOLLOWING FORMULAS

APPARENT WIND SPEED FOLLOWS FROM:

$$V_{AW}^2 = (V_{TW} \sin \beta_{TW})^2 + (V_{TW} \cos \beta_{TW} + V_B)^2$$

AND APPARENT WIND ANGLE FROM:

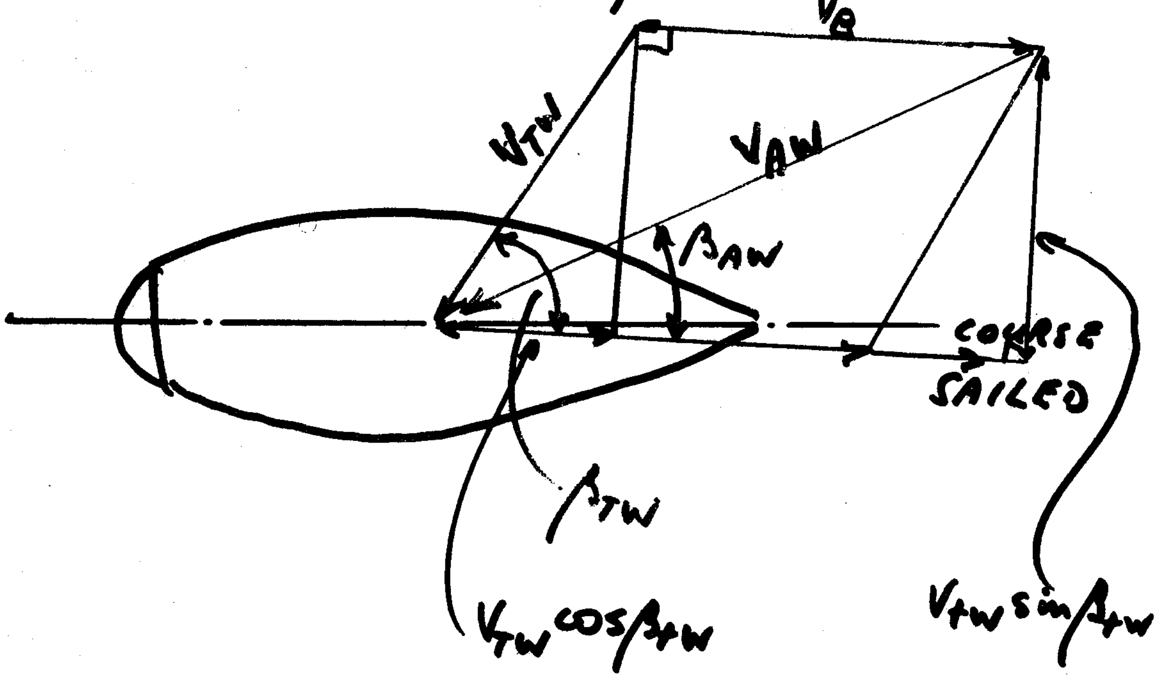
$$\beta_{AW} = \tan^{-1}\left(\frac{V_{TW} \sin \beta_{TW}}{V_{AW}}\right)$$

where V_{tw} = true wind speed

β_{tw} = true wind angle

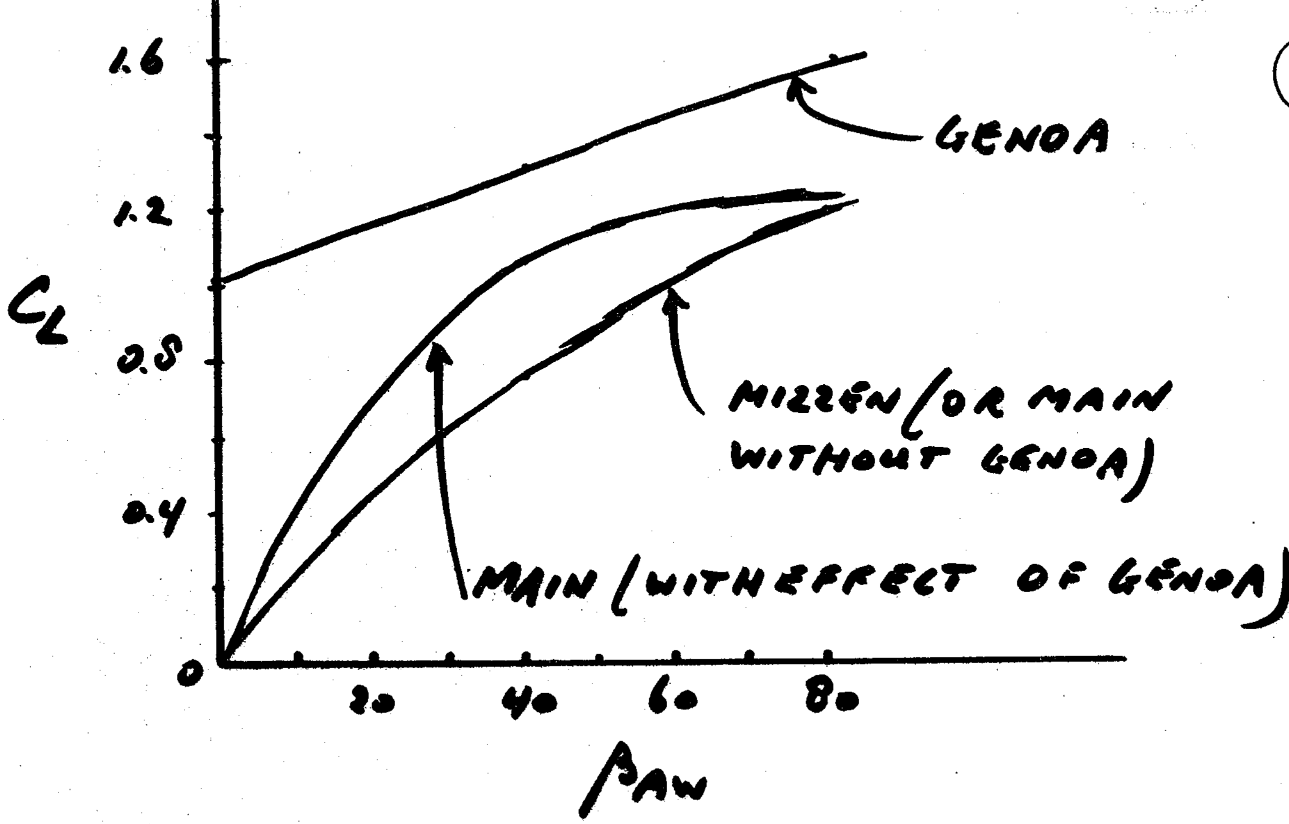
V_B = speed of yacht

NOTE: DEFINITION OF β_{AW} DIFFERS TO EARLIER USAGE OF β_{AW}

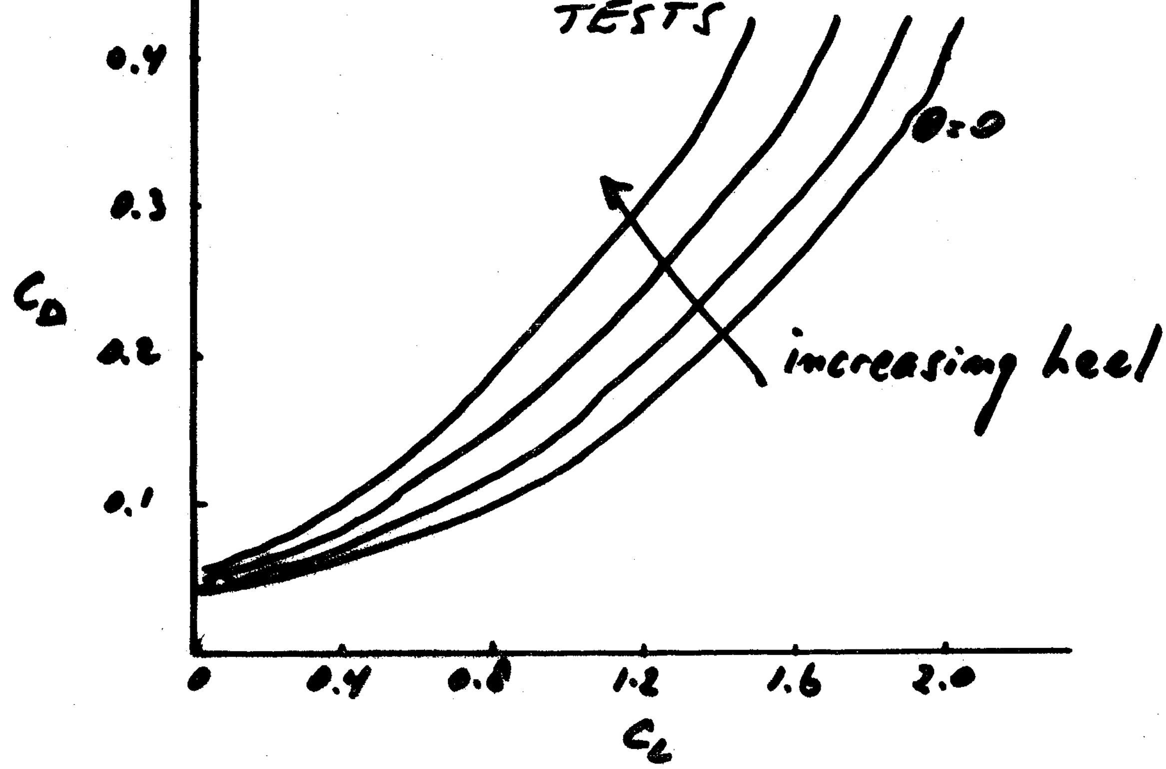


HERE $\beta_{AW} = \beta_{AW_{\text{AS MEASURED}}} + \beta$

β = LEEWAY ANGLE



RESULTS OF WIND TUNNEL TESTS



FOR $\beta_{AW} < 80°$

(with β_{AW} relative to track of yacht - including leeway)

IF $\beta_{AW} < 80°$ THEN

$$CLS = \left(0.041\beta_{AW}^2 + 0.344\beta_{AW} + 1.04\right) GEA + \left(2.357\beta_{AW} - 1.073\beta_{AW}^2\right) MSA + 1.2\sin\beta_{AW} . MIA$$

$$CDS = \left[0.044 + \frac{0.089\, CLS^2/SAT^2}{1-\theta}\right] . SAT$$

where:

- MSA = mainsail area
- GEA = Genoa area
- MIA = Mizzen area
- SPA = SPINNAKER AREA
- θ = HEEL ANGLE
- SAT = MSA + GEA + MIA

(all angles in radians)

NOTE: CLS and CDS are in $[L^2]$

TOTAL SAIL FORCE FOLLOWS FROM:

$$R_S = \sqrt{CLS^2 + CDS^2} . \frac{1}{2}\rho_A V_{AW}^2$$

DRIVING FORCE COMPONENT:

$$R_{SD} = \frac{1}{2}\rho_A V_{AW}^2 \left[CLS \sin\beta_{AW} - CDS\cos\beta_{AW}\right]\cos\theta$$

HEELING FORCE COMPONENT FROM:

$$R_{SH} = \frac{1}{2}\rho_A V_{AW}^2 \left[CLS\cos\beta_{AW} + CDS\sin\beta_{AW}\right]$$

SIMILAR RELATIONS CAN BE DERIVED FOR $\beta_{AW} > 80°$

USE OF A VELOCITY PREDICTION PROGRAM IN PRELIMINARY DESIGN

IN A VELOCITY PREDICTION PROGRAM (VPP) A MATHEMATICAL MODEL FOR HYDRODYNAMIC RESISTANCE AND SIDE FORCE IS USED TO CALCULATE SPEED-POTENTIAL OF YACHT

BASICALLY 3 ~~EQUATIONS~~ ARE SOLVED:

1) SAIL DRIVING FORCE = HYDRODYNAMIC RESISTANCE

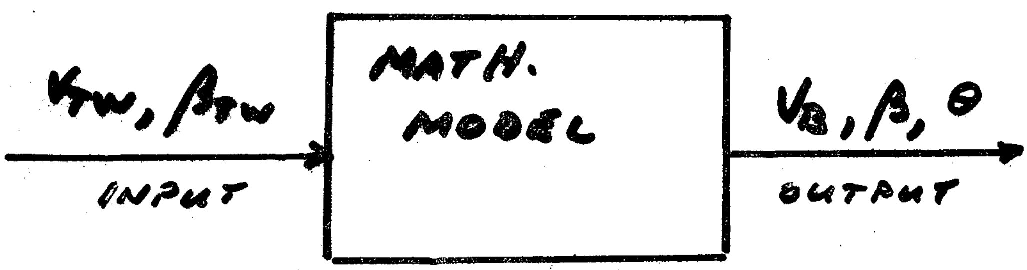
2) SAIL TRANSVERSE FORCE = HYDRODYNAMIC SIDE FORCE

3) SAIL OVERTURNING MOMENT = RIGHTING MOMENT

L.H.S = f(SAIL PLAN, V_{TW}, β_{TW}, V_B, β, θ)

R.H.S. = f(CANOE BODY, KEEL, RUDDER, V_B, β, θ)

THUS FOR A GIVEN SAIL PLAN, CANOE BODY, KEEL AND RUDDER



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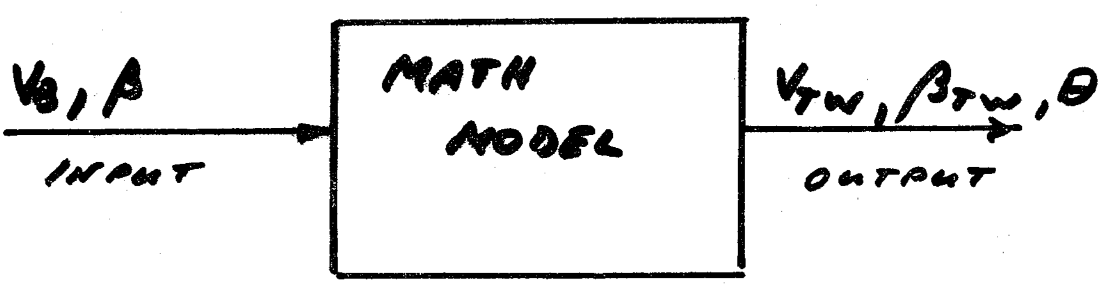


- **SOLUTION OF DRIVING FORCE EQUATION:**
  (IN CALM WATER)

$R_{SD} = R_V + R_W + R_I$, where :

$R_{SD} = f(V_{AW}, \beta_{AW}, \beta, \theta)$

$R_V = f(V_B, S_H, 1+k, L_{WL}, \bar{c}_{K,R}, span_{K,R}, \bar{t}_{K,R})$

$R_W = f(V_B, L_{WL}, \theta, L/\nabla^{1/3}, C_p, LCB, L/B, ETC)$

$R_I = f(V_B, AR_{K,R}, \beta, A_{K,R}, L_{WL}, T_c)$

<u>here V_B, β, θ are unknown</u>

- **SOLUTION OF SIDE FORCE EQUATION**

$R_{SH} = L_{T_K} + L_{T_R} + L_{T_c}$, where :

$R_{SH} = f(V_{AW}, \beta_{AW}, \beta, \theta)$

$L_{T_K} = f(V_B, \bar{c}_K, span_K, \bar{t}_K, \Lambda_K, \tau_K, \beta, \theta)$

L_{T_R} "

$L_{T_c} = f(V_B, L_{WL}, T_c, \beta, \theta)$

<u>again V_B, θ, β are unknown</u>

SOLUTION OF OVERTURNING MOMENT EQUATION

$$R_{SH} \cdot H = GM \cdot \Delta \sin\theta \qquad (\text{small } \theta)$$

$$R_{SH} \cdot H = GZ \cdot \Delta \qquad (\text{large } \theta)$$

$$R_{SH} = \tfrac{1}{2} \rho_A V_{AW}^2 \left[CLS \cos\beta_{AW} + CDS \sin\beta_{AW} \right]$$

where $CLS, CDS = [L^2]$

H = vertical distance between centre of effort of sail plan and centre of lateral resistance (effective values)

(see fig. 38 + 39)

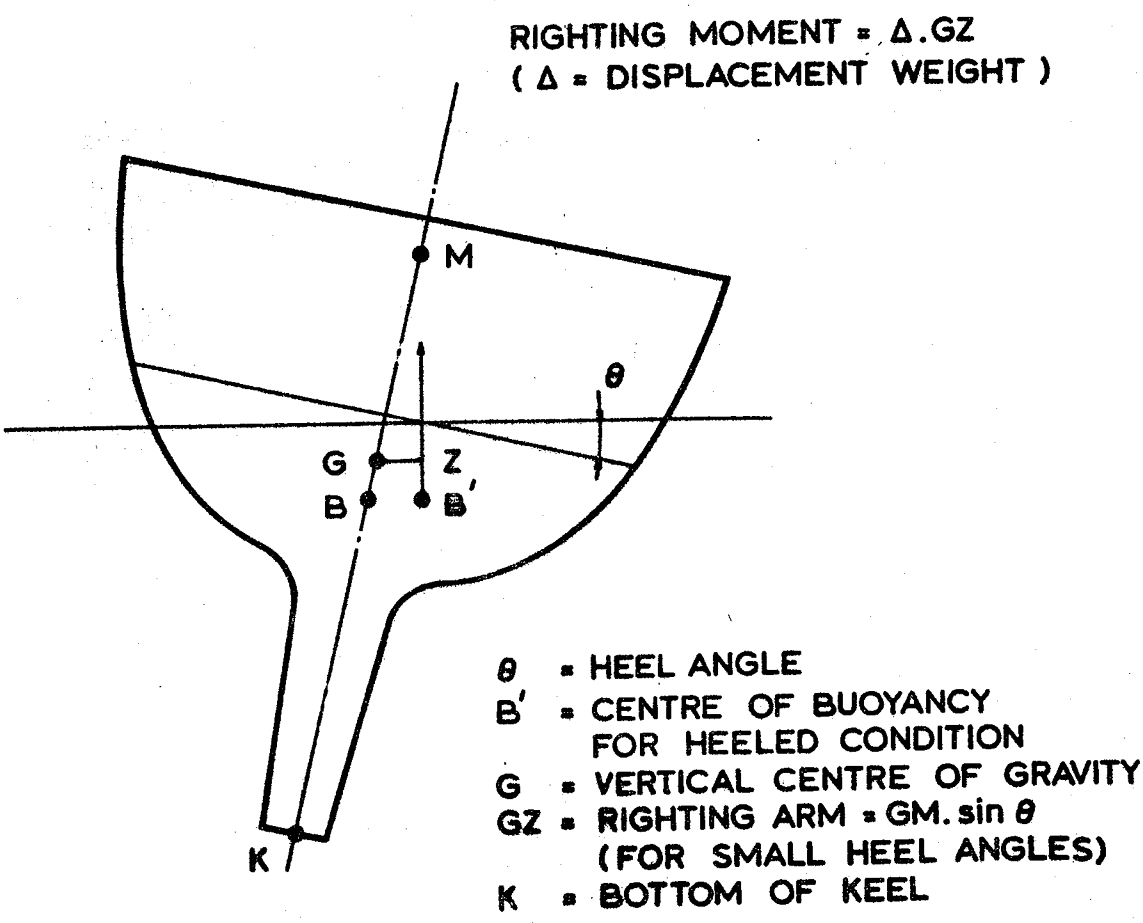
thus:

$$R_{SH} = f(V_{AW}, \beta_{AW}, MSA, GEA, MIA, SPA, V_B, \beta)$$

$GM, GZ = f(\text{hull form}, \theta)$

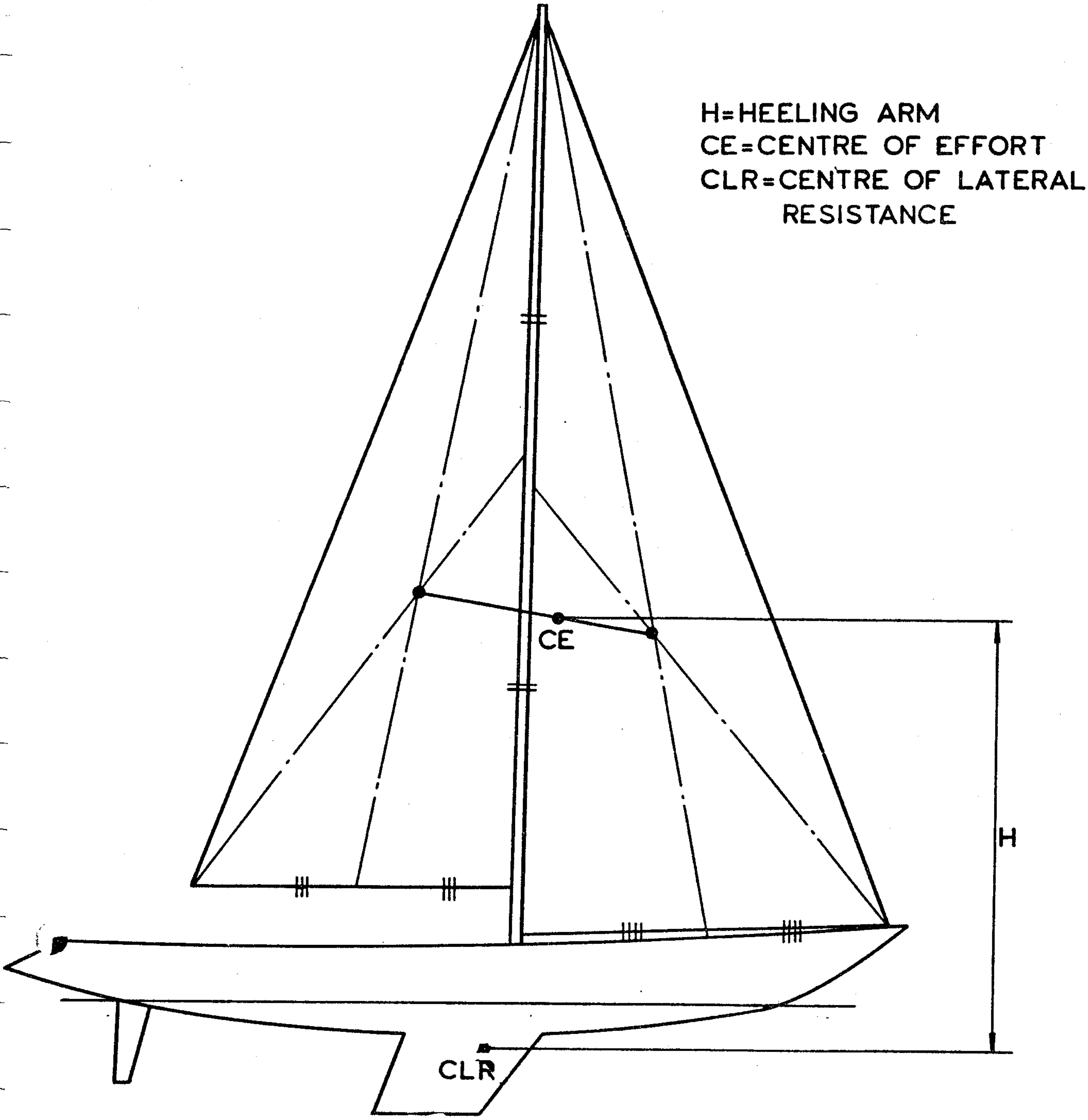
here also V_B, β, θ are unknown.

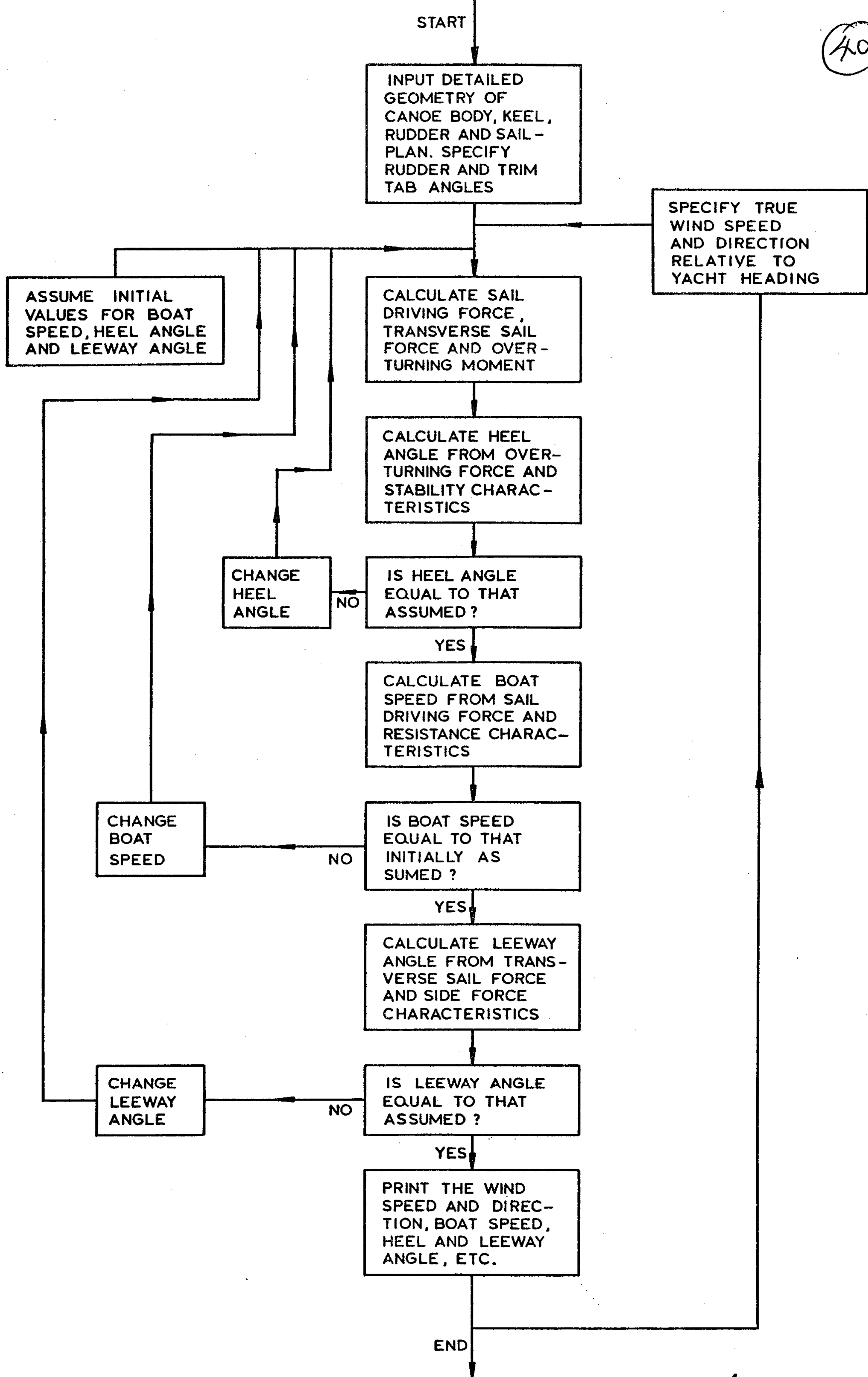
A "SMART" METHOD OF SOLUTION NEEDS TO BE ADOPTED TO FIND V_B, β, θ FROM THESE 3 EQUATIONS



15.

DEFINITION OF GZ


H=HEELING ARM
CE=CENTRE OF EFFORT
CLR=CENTRE OF LATERAL RESISTANCE
CE
H
CLR



(40)

3.

VPP SET-UP

DESIGNING TO A RULE OR RATING FORMULA

DESIGNERS ARE OFTEN FACED WITH THE PROBLEM TO OPTIMIZE TO A SPECIFIC RULE OR FORMULA

FOR EXAMPLE:

1. $$R = \frac{L \times \text{SAIL AREA}}{6000}$$ (DIXON KEMP) 1886

IN ORDER TO REDUCE THE VALUE OF THIS RATING FORMULA, BEAM WAS GREATLY INCREASED LEADING TO THE SO-CALLED "SKIMMING DISHES

IN THE USA A SIMILAR FORMULA WAS INTRODUCED IN 1883, THE SO-CALLED "SEAWANHAKA RULE"

$$\text{RATING (ft)} = \frac{L + \sqrt{SA}}{2}$$

2 IN 1896 A NEW RULE WAS INTRODUCED BY R.E. FROUDE, VIZ:

$$\text{RATING (ft)} = \frac{L + B - \frac{3}{4}G + \frac{1}{2}\sqrt{S}}{2}$$

G = skin girth midships

this failed to stop the "skimming dishes"

3. IN 1901 THIS RULE WAS CHANGED INTO:

$$\text{RATING (ft)} = \frac{L + B - 3/4G - 4d + 1/2\sqrt{S}}{2.1}$$

d = difference between "skin" and "chain" girth midships

so as to avoid large hollows in light displacement hulls (skimming dishes)

4. IN THE USA N. HERRESHOFF SOLVED THE SKIMMING DISH PROBLEM BY DEFINING THE SO-CALLED UNIVERSAL RULE:

$$\text{RATING (ft)} = 0.2\,\frac{L \times \sqrt{S}}{\Delta^{1/3}}$$

i.e. A LIGHT DISPLACEMENT → LARGE RATING

THIS RULE WAS USED UNTIL 1924. THEREAFTER IT WAS ONLY USED FOR THE LARGE J-CLASS YACHTS, RATING 76 ft.

5. THE FIRST INTERNATIONAL RULE WAS ADOPTED IN 1906 BY 16 COUNTRIES (NOT THE USA)

$$R\,(\text{metres}) = \frac{L + B + 1/2G + 3d + 1/3\sqrt{S} - F}{2}$$

with organized races for R = 5, 6, 7, 8, 9, 10, 12, 15 and 23 metres

THE MOST POPULAR WERE THE 6, 8 and 12 metres

THIS RULE HAS BEEN MODIFIED, AS FOLLOWS

IN 1919:

$$R(m) = \frac{L - 1/4 G - 2d + \sqrt{S} - F}{2.5}$$

(BEAM WAS DROPPED BECAUSE A "PLANK ON EDGE" TENDENCY WAS DEVELOPING)

A MINIMUM BEAM REQUIREMENT WAS INTRODUCED ALSO
(FOR 12M 3.6m)

IN 1933 THIS RULE WAS CHANGED TO THE FORM IN WHICH IT IS USED TODAY FOR THE 12m, VIZ:

$$R(m) = \frac{L + 2d + \sqrt{S} - F}{2.37}$$

G WAS INCORPORATED INTO THE METHOD FOR MEASURING L. MORE IMPORTANTLY, FOR 12m, THE DISPLACEMENT WAS FIXED AS:

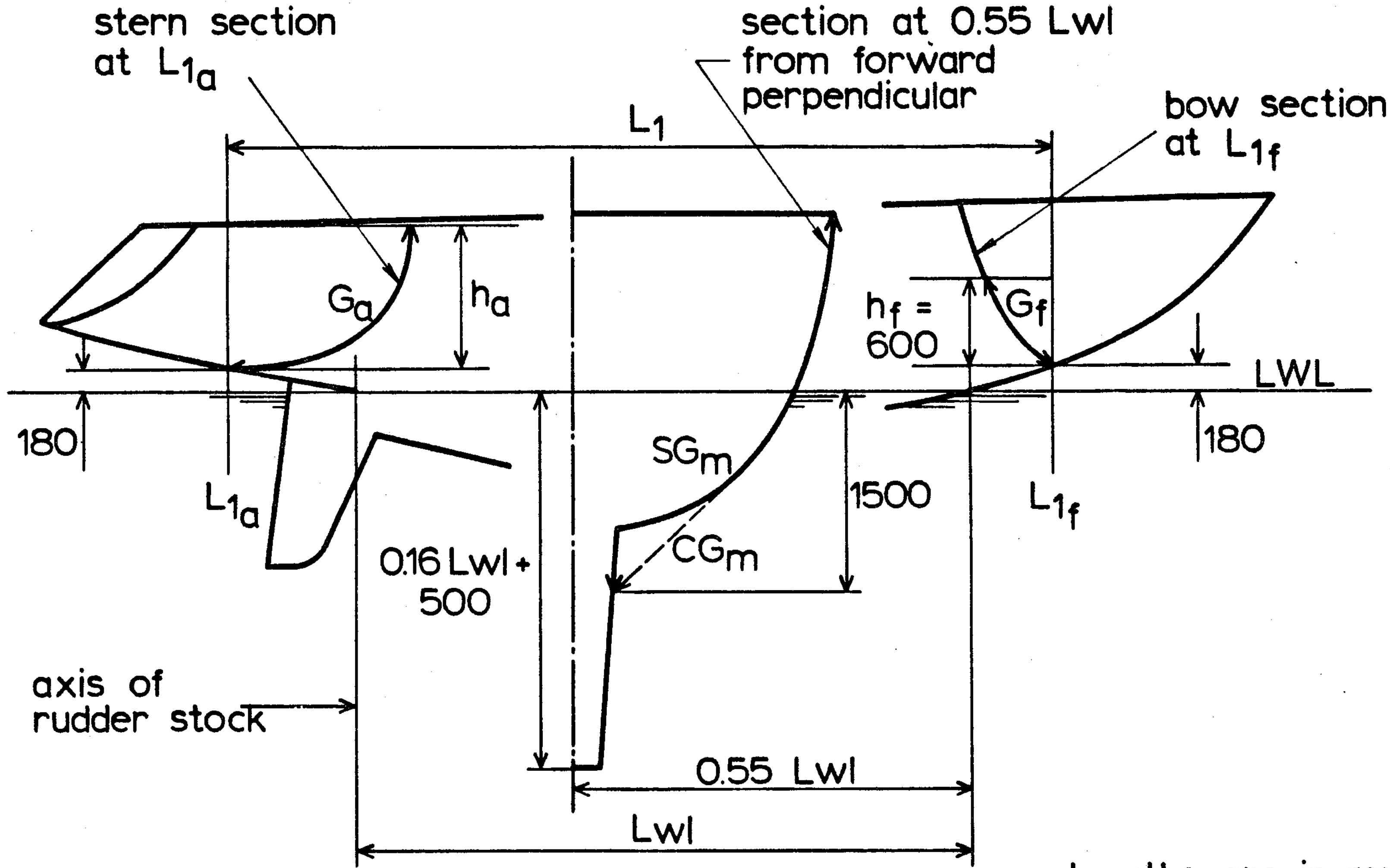
$$\nabla_{min} = (0.2 L_{WL} + 0.15)^3$$

and $T_{max} = 0.16 L_{WL} + 0.5$

$$L = L_1 + 1.5(2G_F - 1.2) + \frac{3}{3}(G_A - h_a)$$

L_1 = length hull 0.18m above WL

G_F = 1/2 girth at FGM (see fig 44)
G_A = 1/2 girth at AGM

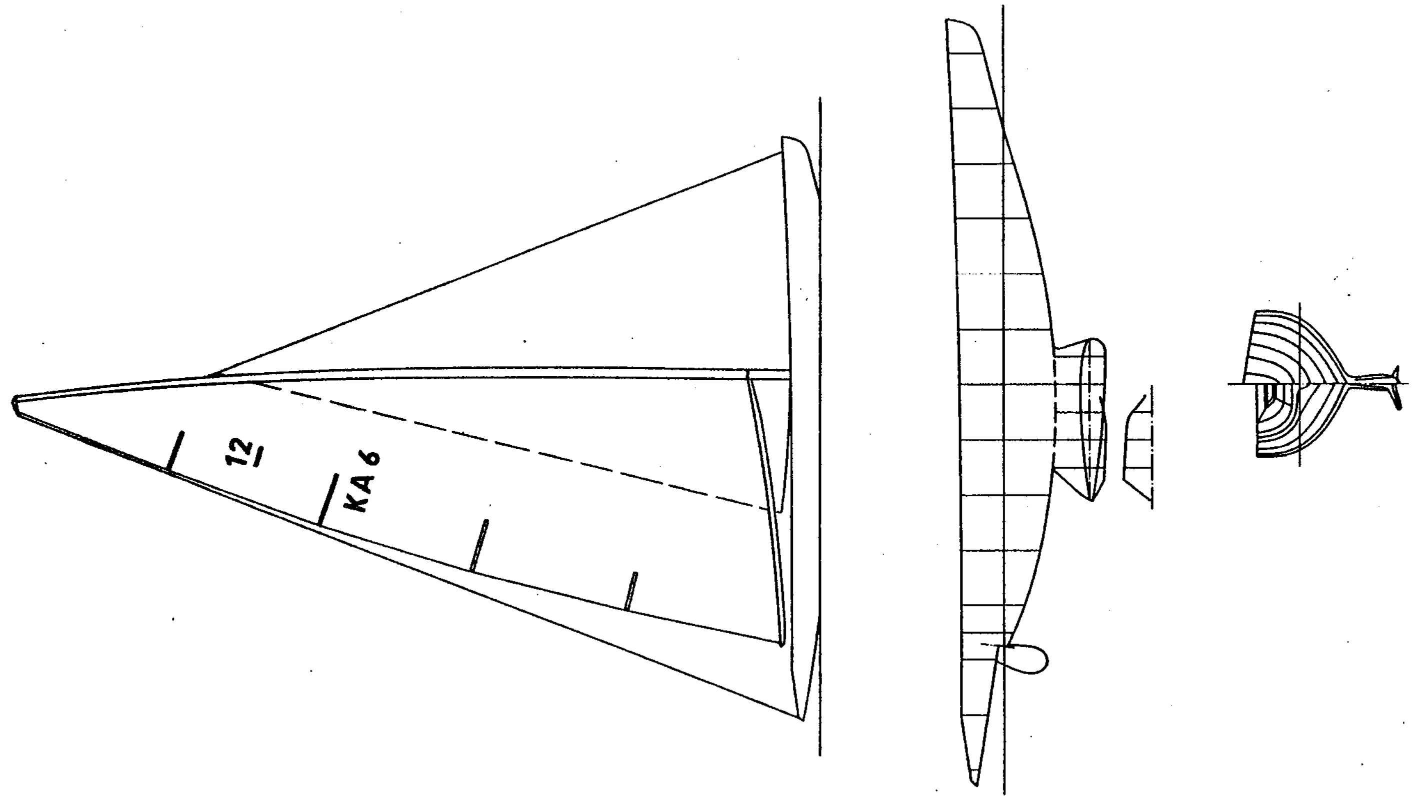


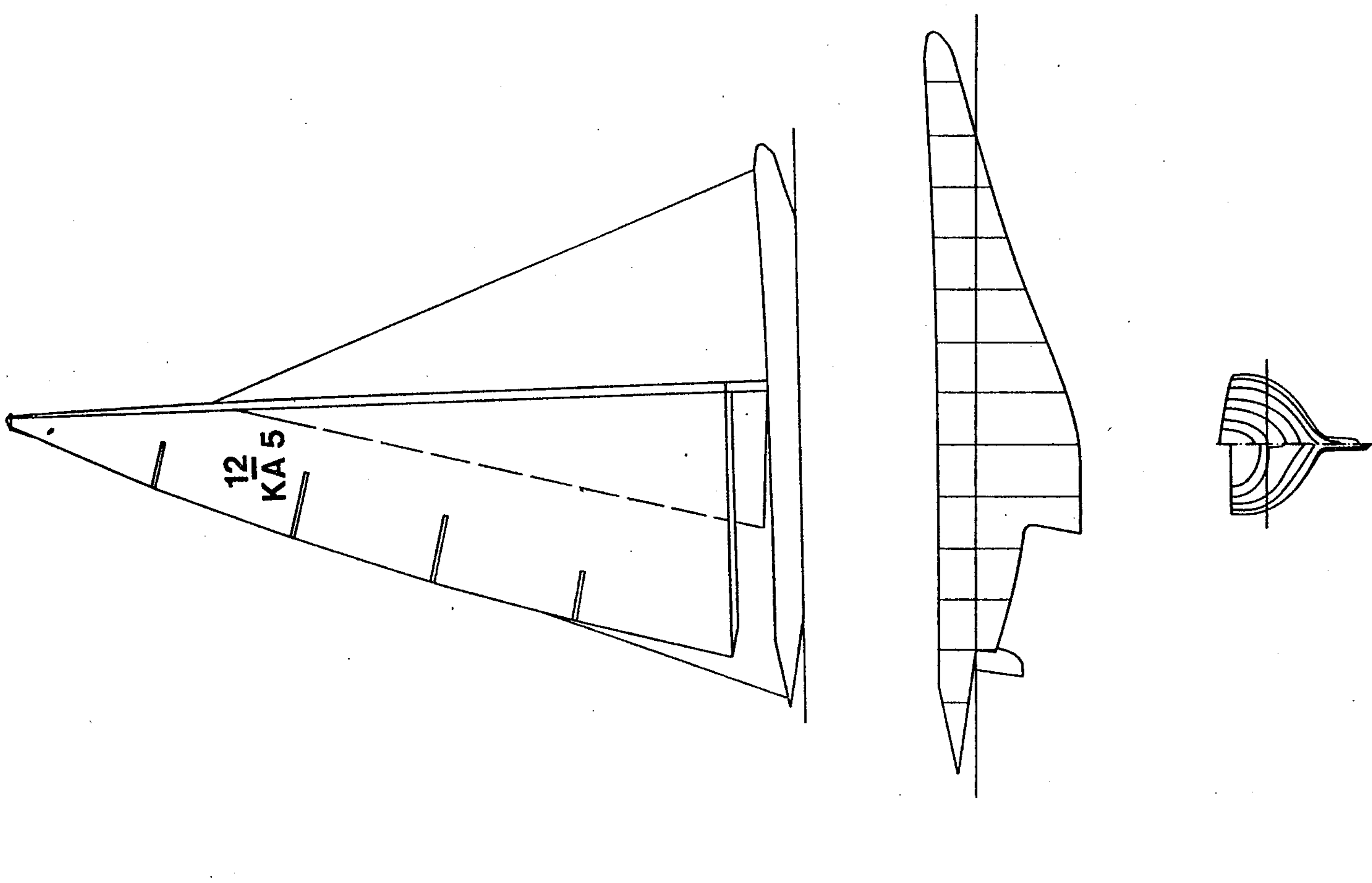
lengths are in mm

(44)

4.

12M PARAMETE





5.

12M EVOLUTION

6 THE INTERNATIONAL OFFSHORE RULE (IOR)

- ALTHOUGH NOW BECOMING LESS IMPORTANT IT HAS FOR YEARS BEEN THE MOST IMPORTANT RULE IN RACING SAILING YACHT DESIGN

- IN 1912 YACHTSMEN IN ENGLAND FORMED A "BOAT RACING ASSOCIATION" TO RACE SMALLER YACHTS THAN WERE ENCOURAGED BY THE YACHT RACING ASSOCIATION (THE LATER IYRU). IT DEVELOPED THE SO-CALLED BRA RULE:

$$\text{RATING (ft)} = \frac{L \times \sqrt{S}}{3\,\Delta^{1/3}} + \frac{L}{4} + \frac{\sqrt{S}}{4}$$

THE FIRST PART IS ESSENTIALLY THE HERRESHOF UNIVERSAL RULE. THE SECOND PART THE OLD SEAWANHAKA RULE

- BECAUSE Δ WAS DIFFICULT TO MEASURE, THE TERM $\sqrt{B \cdot D}$ WAS INTRODUCED. THE FORMULA:

$$R\,(ft) = \frac{0.2L \times \sqrt{S}}{\sqrt{B \cdot D}}$$

where D = DEPTH (MAX. DRAUGHT OF CANOE BODY)

- THEN IN 1934 THE CRUISING CLUB OF AMERICA ADOPTED THE CONCEPT OF ADOPTING A BASIC FORMULA TOGETHER WITH A WHOLE SERIES OF CORRECTIONS (DIFFERENCES) TO ACCOUNT FOR DEVIATIONS FROM A BASE YACHT. THIS LED TO THE <u>FIRST CCA RULE</u>:

$$\text{RATING (ft)} = 0.6\sqrt{S} \times \text{RIG ALLOWANCE} + 0.4(L \pm B \pm D \pm P \pm F)$$

- THE FINAL CCA RULE WAS:

$$\text{RATING (ft)} = (L \pm B \pm D \pm \Delta \pm \sqrt{S}) \times \times \text{STABILITY F} \times \text{PROP F}$$

- IN ENGLAND IN 1931 THE ROYAL OCEAN RACING CLUB SETTLED ON:

$$\text{RATING (ft)} = 0.15\,\frac{L \times \sqrt{S}}{B \times D} + 0.2(L + \sqrt{S})$$

± STABILITY ALLOWANCE
± PROP FACTOR
± DRAUGHT PENALTY

THESE 2 RULES REMAINED UNTIL 1970 WHEN THE IOR WAS ACCEPTED

$$R = \left[0.13\,\frac{L \times \sqrt{S}}{\sqrt{B \cdot D}} + 0.25L + 0.2\sqrt{S} + DC + FC\right] \cdot$$

$$\cdot\, EPF \cdot CGF \cdot MAF \cdot DLF \cdot SMF \cdot LRP \cdot CBF \cdot$$

$$\cdot\, TPF$$

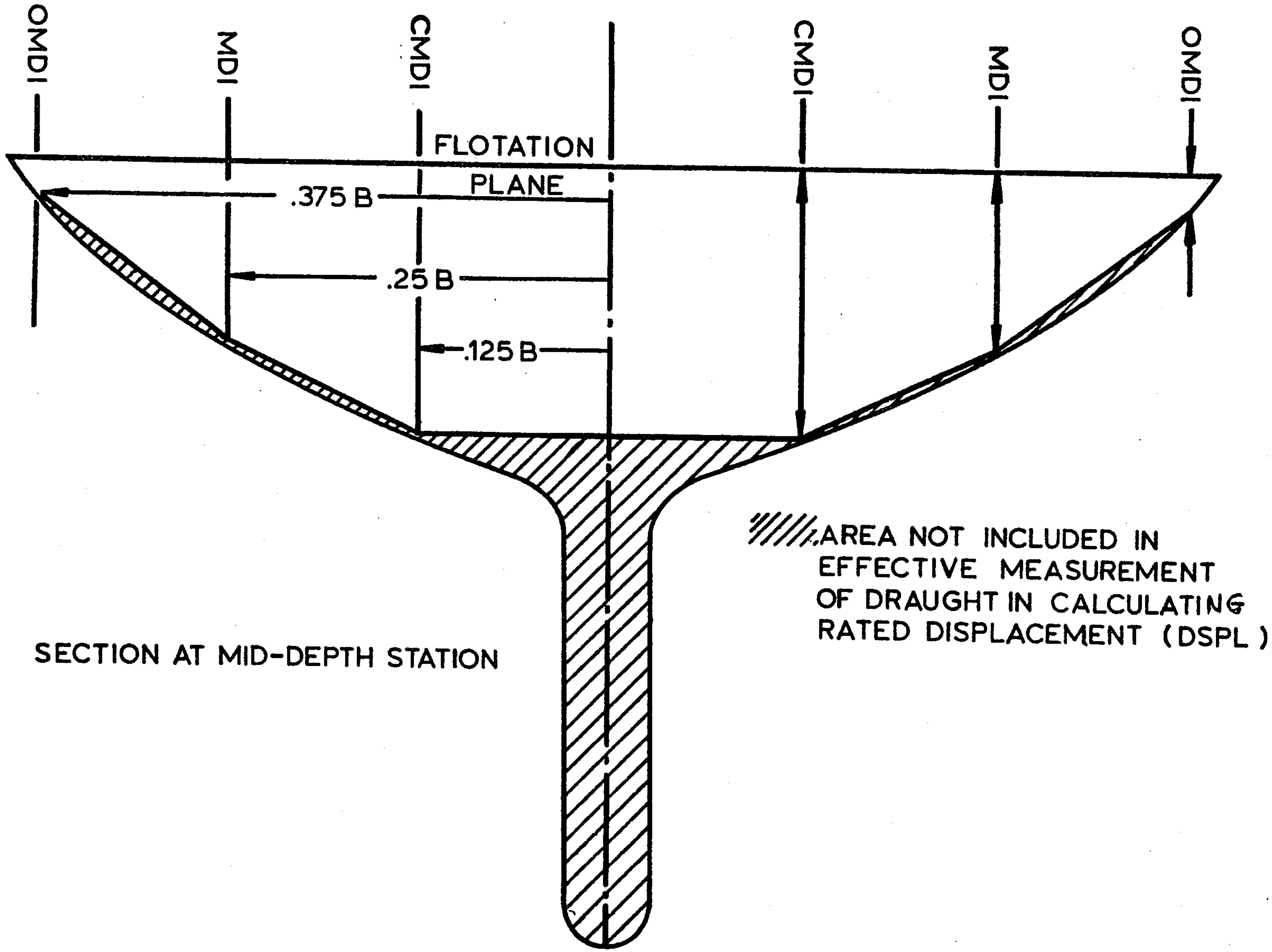
IN THE IOR RULE:

L = measured length
S = " sail area
B = " beam
D = " depth
DC: DRAUGHT CORRECTION FACTOR
FC: FREEBOARD " "
DLF: DISPLACEMENT-LENGTH FACTOR
EPF: ENGINE/PROPELLER DRAG FACTOR
CGF: VERTICAL CENTRE-OF-GRAVITY FACTOR
MAF: MOVEABLE APPENDAGE FACTOR
SMF: SPAR MATERIAL FACTOR
LRP: LOW RIGGING PENALTY FACTOR
CBF: CENTREBOARD FACTOR
TPF: TRIM PENALTY FACTOR

- IN OPTIMIZING PERFORMANCE WITHIN THIS RULE A BEAMY YACHT RESULTS, WITH TAPERED ENDS (LARGE B.D WHILE OBTAINING RELATIVELY LOW DISPLACEMENT)

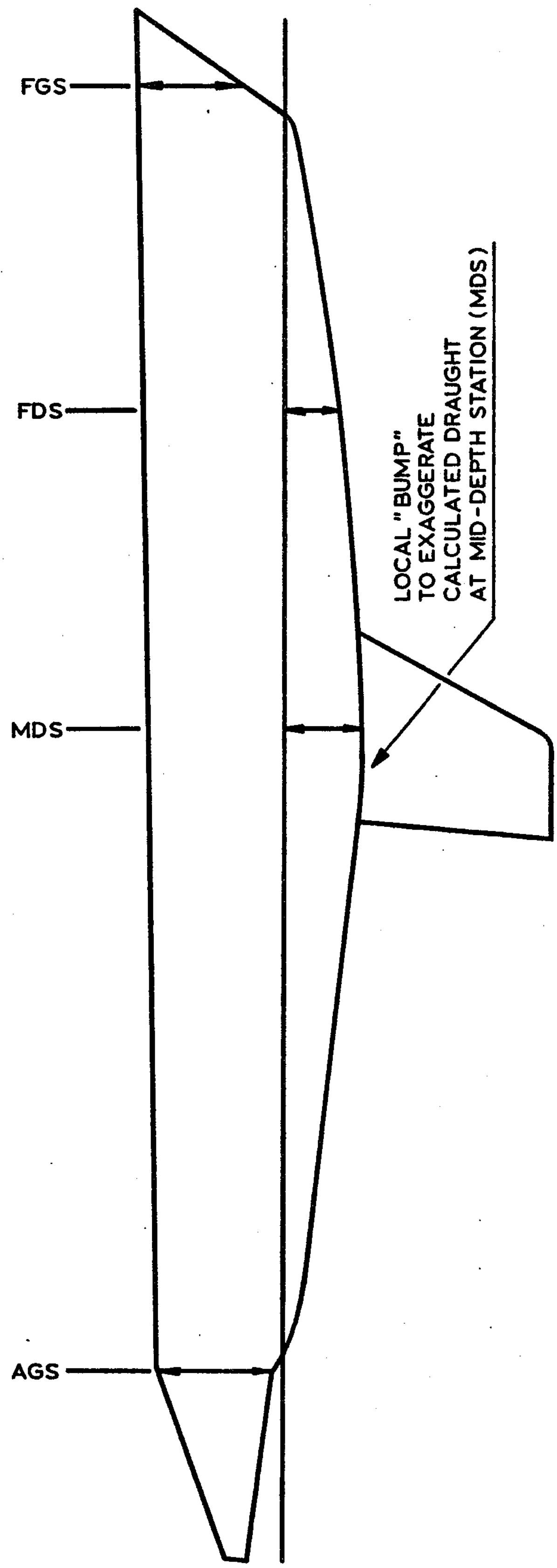
- TO MAKE THE YACHT LONGER THEN THE RULE FIGURES, PLACE FORE AND AFT GIRTH STATIONS CLOSE TOGETHER → SMALL LBG FOR LARGER LOA.

- DSPL (DISPLACEMENT) = L.B.MDIA.32
  MDIA: MID-DEPTH IMMERSED ADJUSTED BASED ON 4 SEPARATE DEPTH MEASUREMENTS → FIG 49
  THIS LEADS TO FLAT BOTTOM SHAPES.



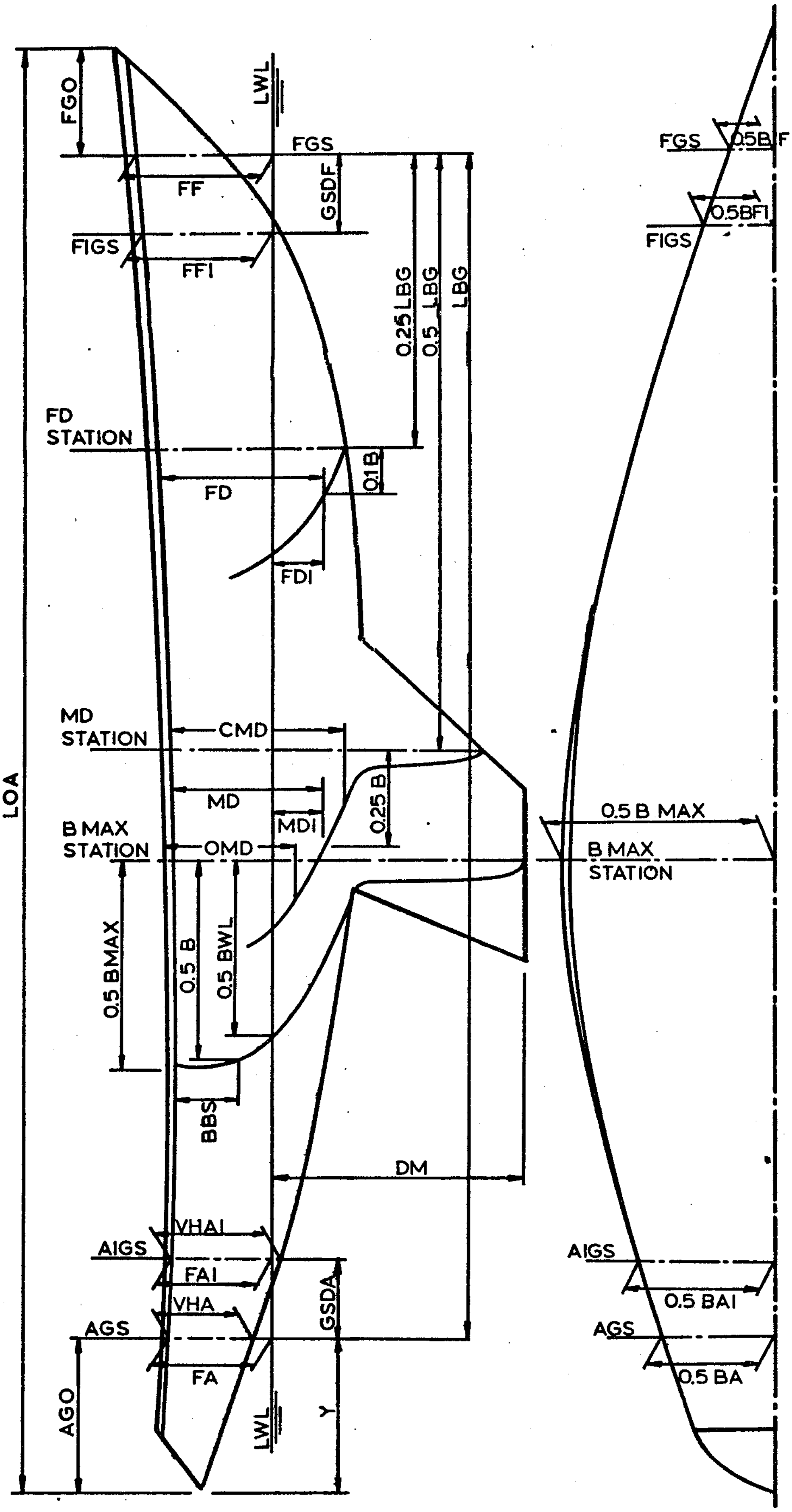
SECTION AT MID-DEPTH STATION

7.



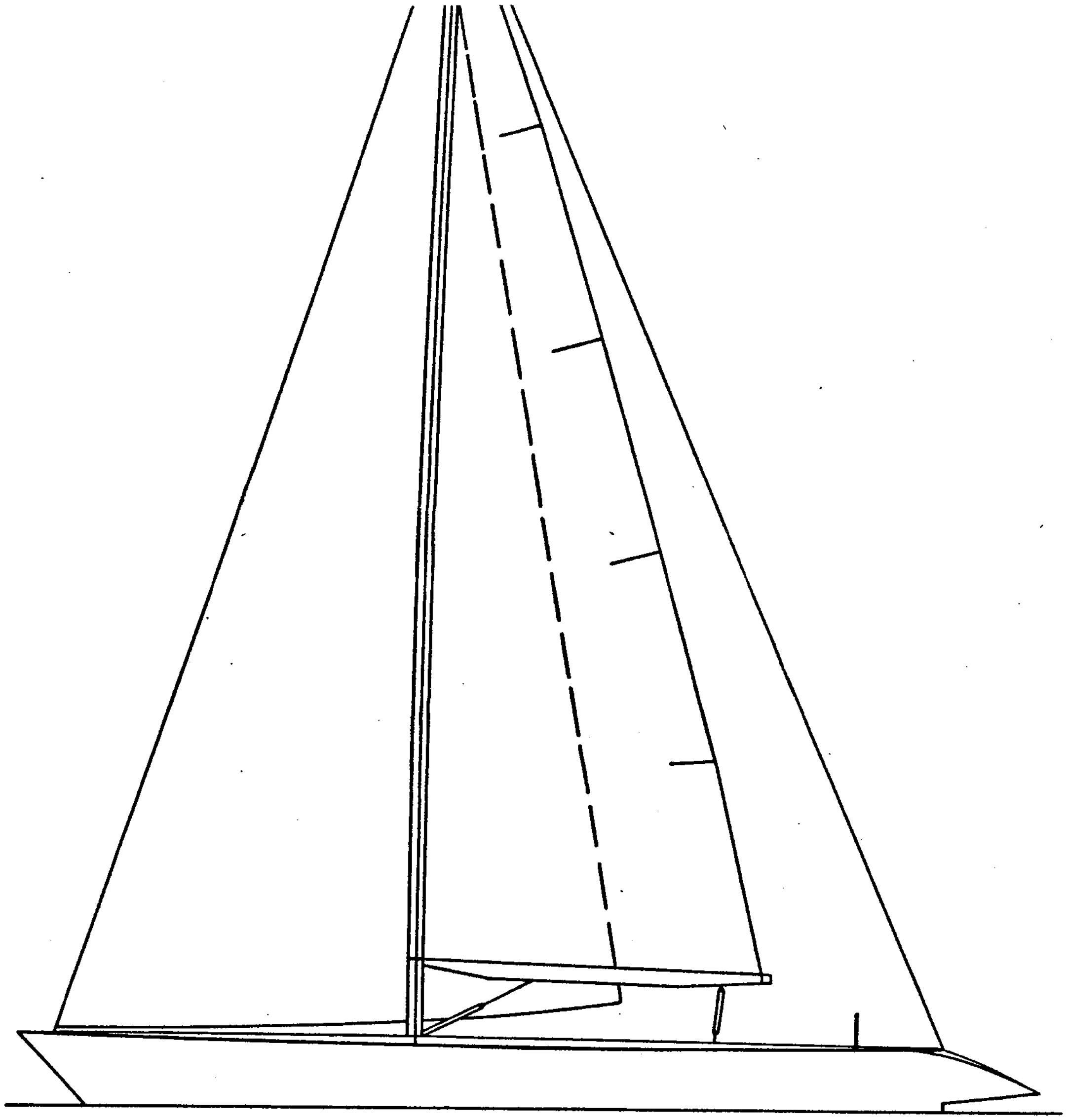
8.

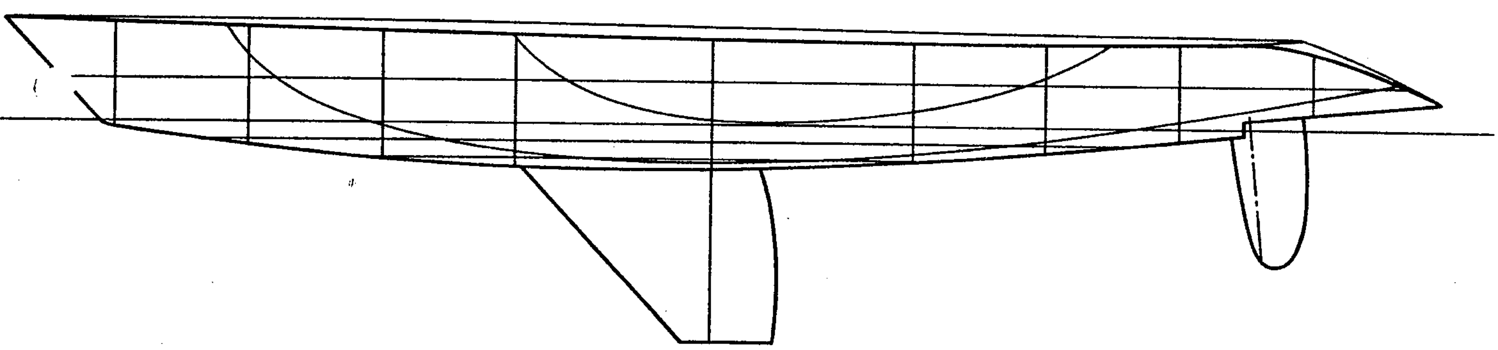
IOR PARAMETERS

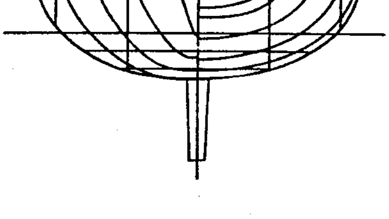


6.

IOR PARAMETERS







IOR MAXI "COTE D'OR"

7. 5.5 metre rule

the only rating rule to be introduced after 1945, still widely used is the 5.5 metre rule. Introduced by the IYRU in 1950 to replace the 6 metres as an olympic class.

The use of 6-metre and later also the 5.5 metre rule in the olympics was considered too flexible – not leading to fair competition. Later the Etchell and Soling were introduced to replace the 5.5.

$$\text{Rating (5.5m)} = 0.9 \frac{L \times \sqrt{S}}{12\ \Delta^{1/3}} + 0.25L + 0.25\sqrt{S}$$

8 The new International Americas Cup Class

This is the first rule based solely on the results of VPP analysis

$$\frac{L + 1.25\sqrt{S} - 9.8\,\nabla^{1/3}}{0.388} = 42\text{ m}$$

$$L = LM + \left(1 + 0.01 * (LM - 21.8)^{8}\right) + FP + DP + WP + BP$$

9 $LM = LBG + G$

NEW AMERICA'S CUP YACHT

NEW RULE :

$$\frac{L + 1.25\sqrt{S} - 9.8\,DSP^{1/3}}{0.388} = 42$$

$$L = LM + \left(1 + 0.01(LM - 21.8)^{8}\right) + FP + DP + WP + BP$$

$LM = LBG + G$, where

LBG = waterline length at a height 200 mm above measurement waterline (MWL)

MWL = plane of flotation in sea water without crew, sails.

G = girth component

$G = FGC + AGC$

FGC = forward girth correction

AGC = aft girth correction

FGC = the greater of

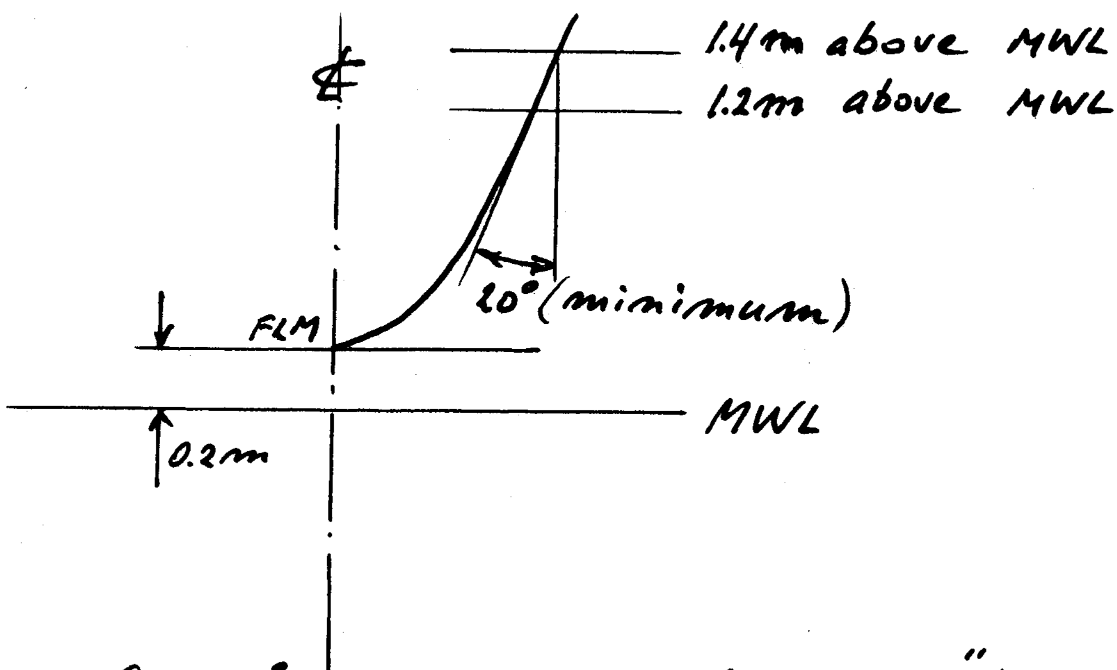
$$0.3 \quad \text{or} \quad 1.25(FG - 2.4 + FBC)$$

where FG = girth around hull through forward LBG-ending (the forward length mark FLM) in a transverse plane 1.2 m above FLM

FBC = the greater of

$$-0.116 \quad \text{or} \quad -1.8\left(\frac{1}{\cos\theta} - 1\right)$$

θ = slope of hull in transverse plane at FLM between 1 and 1.2 m above FLM, relative to the vertical



$\theta = 20°$ minimum for zero "penalty"

AGC = the greater of

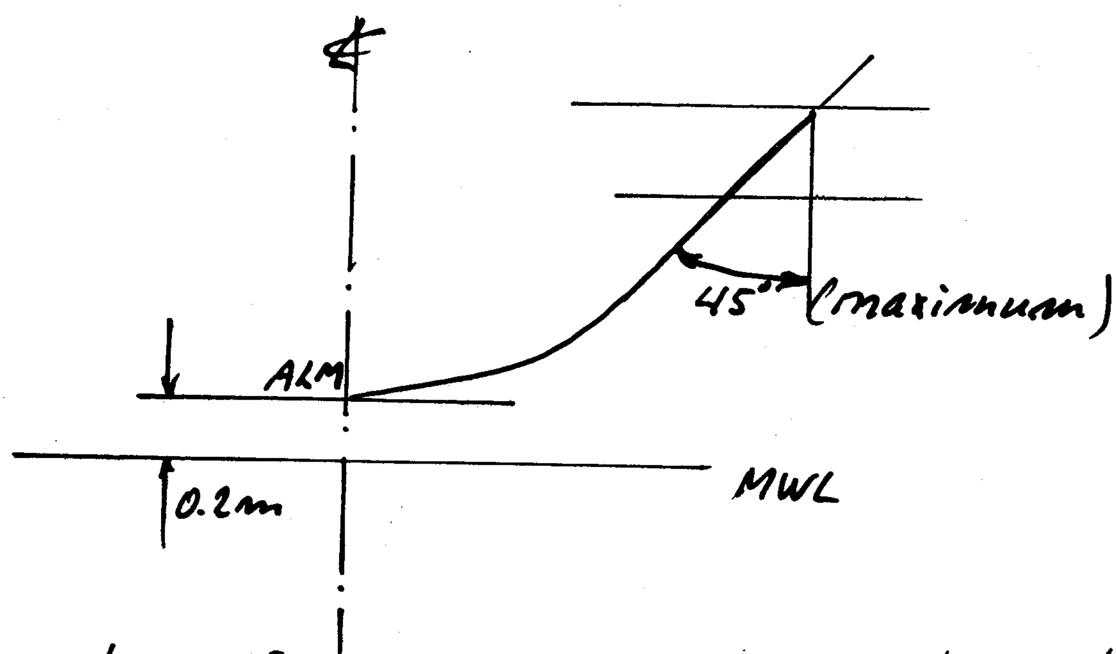
$1.6\,m$ or $0.75(AG - 1.8 + ABC)$

AG = girth around hull through aft LBG-ending (the aft length mark ALM) in a transverse plane 0.9m above ALM

ABC = the greater of

0 or $1.414 - \frac{1}{\cos\phi}$

ϕ = slope of hull in transverse plane at ALM between 0.7m and 1.0m above ALM (relative to the vertical)



$\phi = 45°$ maximum for zero "penalty"

max draught to bottom of keel = 4m (from MWL) without penalty

max. mass = 25000 kg (without penalty)

min. mass = 16000 kg (" ")

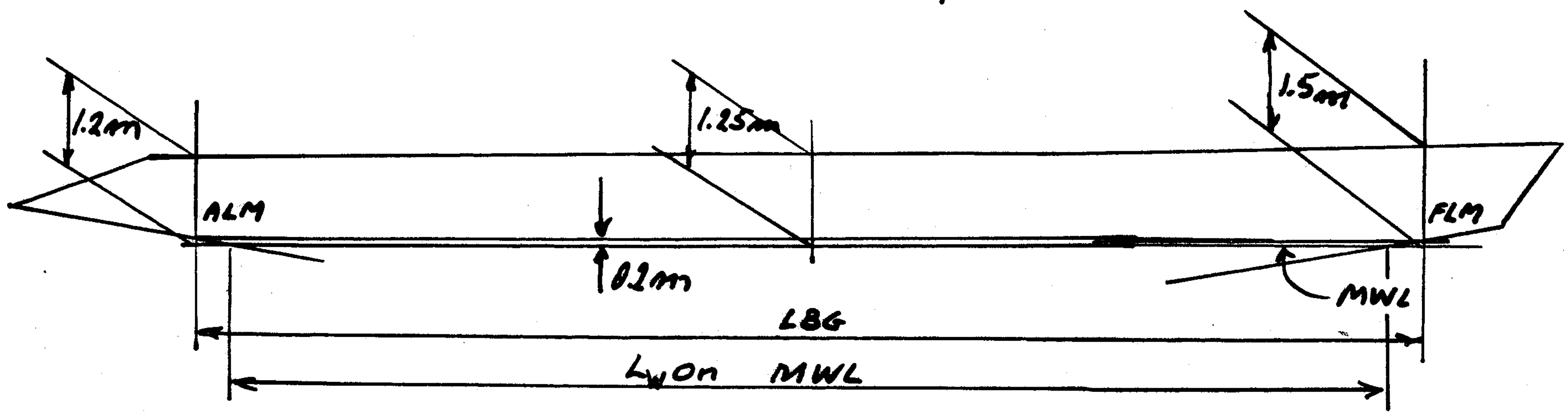
max beam = 5.5 m (" ")

$DP = 4(D-4)$ for $D > 4$

$WP = 4(25.198 - W^{1/3})$ for $W < 16000$ kg

$WP = 4(W^{1/3} - 29.240)$ for $W > 25000$ kg

$BP = 4(B - 5.5)$ for $B > 5.5$



SAIL AREA S:

$$S = SM\left(1 + 0.001\,(SM^{1/2} - 16.9)^8\right)$$

$$SM = MSA + IJ/2$$

MSA = mainsail area
($P_{max} = 30.5\,m$)

$$I_{max} = 0.8\,(P + 1.5)$$

E and J such that S complies with class formula

typical design data:

	12M	IAC	IOR maxi
LOA	19.8 (65)	23.2 (76)	24.4 (80)
LWL	14.0 (46)	17.4 (57)	20.1 (66)
BEAM	3.6 (12)	5.5 (18)	6.1 (20)
DRAUGHT	2.7 (9)	4.0 (13)	4.3 (14)
SAIL AREA	174 (1870)	270 (2900)	288 (3100)
DISPL	28000 (58000)	17000 (35000)	39000 (80000)

↑ m, m^2, kg ↑ ft, ft^2, lbs

$SP = 1.25\,I$ (hoist)

$SPA_{max} = 1.5\,SM$

THE DESIGN PROCESS

1. ASCERTAIN WIND CONDITIONS

- STATISTICAL DATA ON WIND SPEED RANGE
- DETERMINE AVERAGE WIND SPEED FOR WHICH THE YACHT IS TO BE DESIGNED
- DETERMINE WIND DIRECTION — IMPORTANT FOR DETERMINING POINT OF SAIL
- FOR A SPECIFIC RACE OR RACING EVENT DETERMINE PERCENTAGES FOR SAILING ON-THE WIND, REACHING, RUNNING, ETC.

2. ASCERTAIN SEA CONDITIONS

- COLLECT STATISTICAL DATA ON WAVE HEIGHT AND WAVE PERIOD
- ASCERTAIN SEA CONDITIONS CORRESPONDING WITH AVERAGE WIND CONDITIONS
- IS ADDED RESISTANCE IN WAVES AN IMPORTANT FACTOR?

3. DESIGN OF CANOE BODY

- MANY PATHS CAN BE TAKEN TO FIND THE OPTIMUM SET OF MAIN CANOE BODY PARAMETERS FOR PREVAILING WIND AND SEA CONDITIONS.

- THE MAIN DESIGN PARAMETERS ARE:

- L_{WL}, ∇_c } $L/\nabla^{1/3}$

- B_{WL}, T_c } B_{WL}/T_c

- C_B, C_P } C_M

- LCB

- A POSSIBLE DESIGN PATH IS AS FOLLOWS:

1 - MAKE FIRST ESTIMATE OF L_{WL}
2 - ASSUME 3 VALUES FOR ∇_c
3 - FOR EACH $L/\nabla^{1/3}$ VALUE, ASSUME 3 B_{WL}/T_c VALUES
4 - DEDUCE B_{WL} AND T_c FOR EACH B_{WL}/T_c

5- FOR EACH $L/\nabla^{1/3}$ AND B_{WL}/T_C COMBINATION DEDUCE APPROXIMATE STABILITY CHARACTERISTICS (KM, KG AND GM)

6- FOR EACH $L/\nabla^{1/3}$ AND B_{WL}/T_C COMBINATION DETERMINE POSSIBLE SAIL AREAS AND RIG HEIGHT

7- FOR EACH $L/\nabla^{1/3}$ AND B_{WL}/T_C NOW CHECK RIG WEIGHT, BALLAST DETAILS (TO SUIT ∇_C) AND RE-CALCULATE KG AND GM. ASSUME C_P, C_M, ETC.

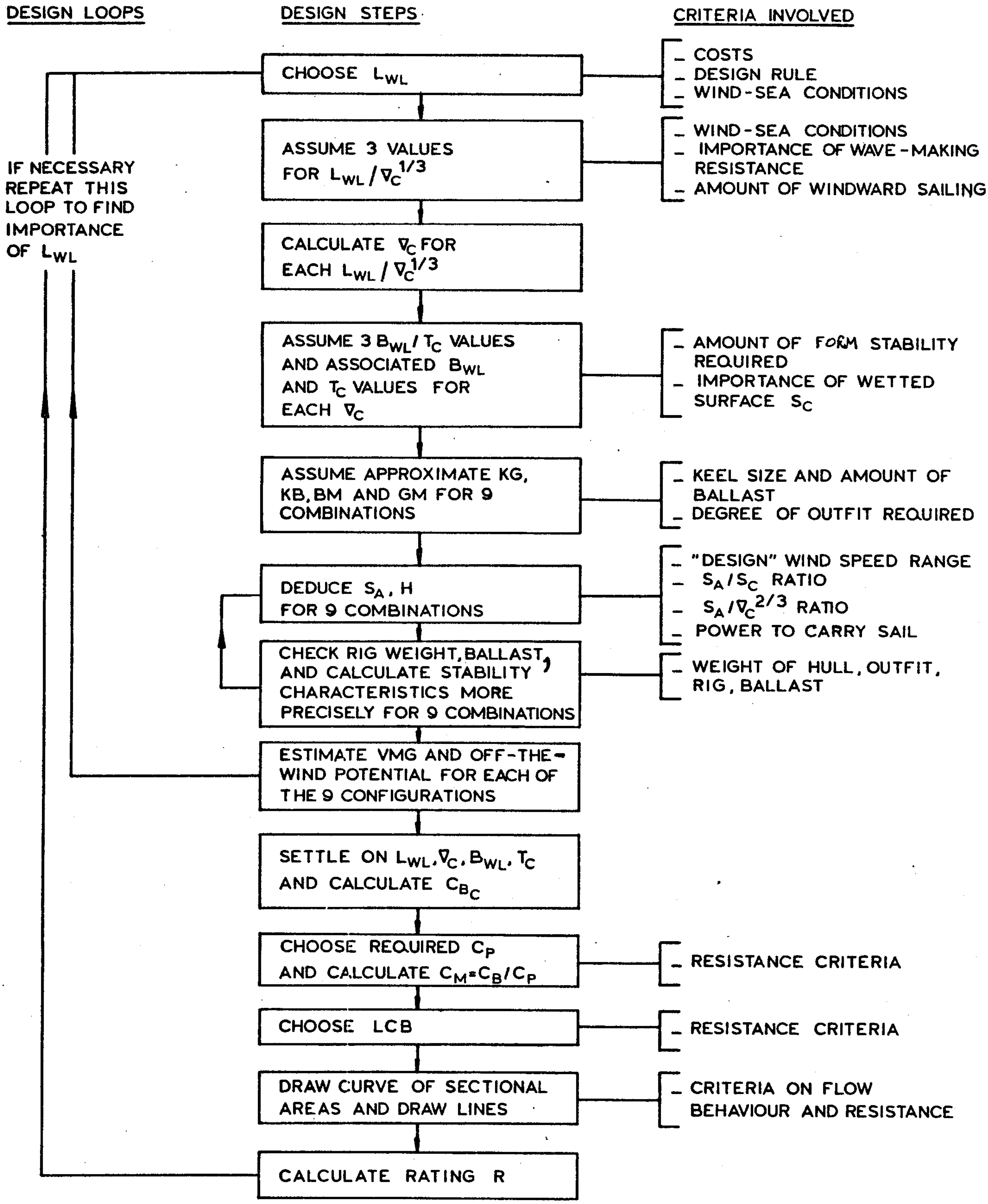
8- DO VPP CALCULATIONS FOR EACH OF THE 9 COMBINATIONS AND CALCULATE TIME OVER THE COURSE TO BE SAILED. DETERMINE OPTIMUM VALUES

9- FOR THE CALCULATED AVERAGE SPEED, CHECK C_P, C_M, LCB

10- CHECK TO SEE IF RATING OR DESIGN RULE VALUES CORRESPOND TO THOSE ASSUMED

11- REPEAT FOR 2 OTHER L_{WL} VALUES

(SEE FIGURE)



10.

DESIGN PROCEDURE

3.1 CHOICE OF LWL

- FOR CRUISING YACHTS, LWL MAINLY DETERMINES COST. THUS THE POTENTIAL OWNER MUST SPECIFY COST OR EQUIVALENT

IN 1985 THE FOLLOWING FORMULA WAS VALID FOR "AVERAGE" CRUISING YACHT

$$US\$ = 62.5 (L_{OA})^3 \quad (L_{OA} \text{ in metres})$$

WITH $L_{OA} \approx 1.15\, L_{WL}$

- FOR RACING YACHTS LWL IS MAINLY DETERMINED BY SPEED POTENTIAL, IN COMBINATION WITH RATING RULE OR DESIGN FORMULA CONSIDERATIONS

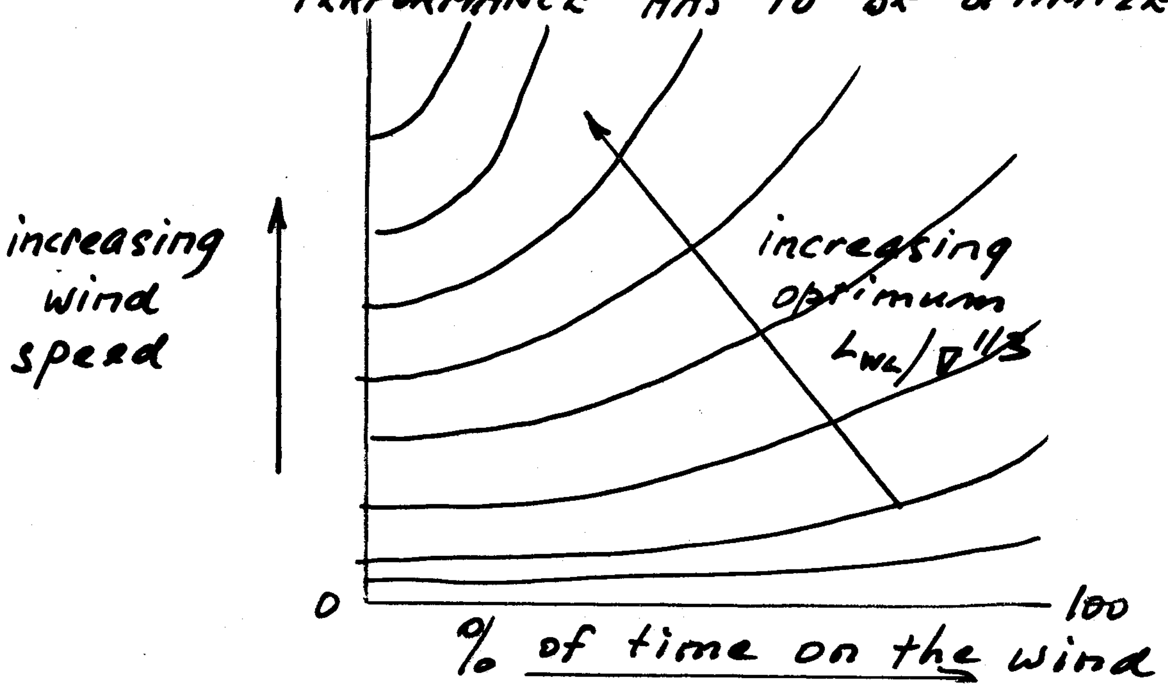
L_{WL} (metres)	$2.75\sqrt{L_{WL}}$ "hull" speed (knots)
2	3.9
4	5.5
8	7.8
16	11.0
32	15.6
64	22.0
128	31.1
256	44.0

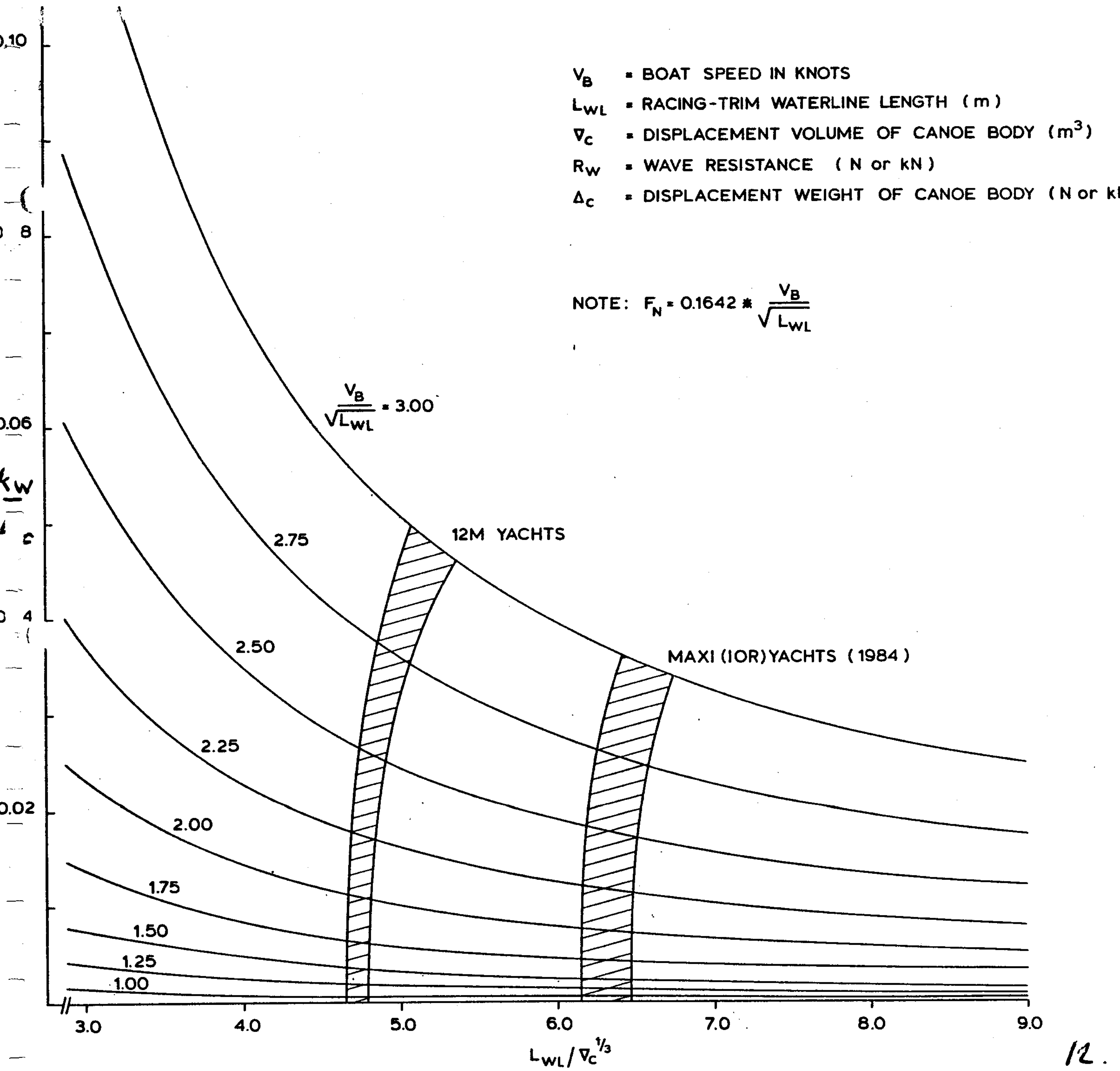
- WHEN DESIGNING ACCORDING TO A COMPLEX RATING RULE (IOR) OR DESIGN FORMULA IT IS BEST TO CHOOSE 3 L_{WL} VALUES, A LOW, MEDIUM, AND HIGH VALUE

3.2 INITIAL ESTIMATE OF DISPLACEMENT

- DISPLACEMENT VOLUME ∇_c DETERMINES LENGTH-DISPLACEMENT RATIO $L_{WL}/\nabla^{1/3}$ ONCE L_{WL} IS KNOWN

- $L_{WL}/\nabla^{1/3}$ HAS IMPORTANT INFLUENCE ON WAVE RESISTANCE. IF AVERAGE WIND SPEED IS LOW, BOAT SPEEDS ARE LOW AND RESISTANCE WILL BE PREDOMINANTLY VISCOUS. IN THAT CASE L/∇ CAN BE LOW (SEE FIGURE)
- ∇_c DETERMINES THE AMOUNT OF BALLAST THAT CAN BE CARRIED. THIS IS IMPORTANT WHEN ON-THE-WIND PERFORMANCE HAS TO BE OPTIMIZED.





12.

EFFECT OF $L_{WL}/\nabla_c^{1/3}$ ON R_W/Δ_c

- AT HIGH WIND SPEED AND FOR OFF-THE WIND PERFORMANCE $L_{WL}/\nabla^{1/3}$ HAS TO BE AS HIGH AS POSSIBLE

- AT HIGH WIND SPEED AND ON-THE-WIND PERFORMANCE STABILITY IS PARAMOUNT. THIS IS OFTEN THE MOST DIFFICULT DESIGN CASE. TO STILL OBTAIN ACCEPTABLE $L_{WL}/\nabla^{1/3}$ VALUES THE MAXIMUM POSSIBLE L_{WL} NEEDS TO BE CHOSEN.

- FOR OPTIMIZING ON-THE-WIND PERFORMANCE THE "POWER-TO-CARRY-SAIL" NEEDS TO BE STUDIED (SEE SECTION 3.5)

- FOR THE PRELIMINARY ASSESSMENT OF THE EFFECT OF $L_{WL}/\nabla^{1/3}$, ASSUME 3 VALUES OF ∇_c, A LOW, MEDIUM AND HIGH VALUE.

3.3 INITIAL ESTIMATION OF BEAM AND DRAUGHT

- BEFORE SELECTING SPECIFIC VALUES OF B_{WL} AND T_c, FIRST CONSIDER THE EFFECT OF B_{WL}/T_c ON PERFORMANCE

- THE METACENTRIC RADIUS BM APPROXIMATELY INCREASES WITH $(B_{WL}/T_c)^2$

- IN FACT, AN APPROXIMATE FORMULA FOR BM (FOR YACHT-TYPE HULLS) IS:

$$BM = 0.036\left(\frac{B_{WL}}{T_c}\right)^2 \cdot \frac{T_c}{C_B}$$

- ON USING THE APPROXIMATE FORMULA (FOR YACHT TYPE HULLS) FOR KB AS:

$$KB = T_c\left(0.833 - \frac{C_B}{3C_{WP}}\right)$$

- WE THEN OBTAIN:

$$KM = KB + BM$$

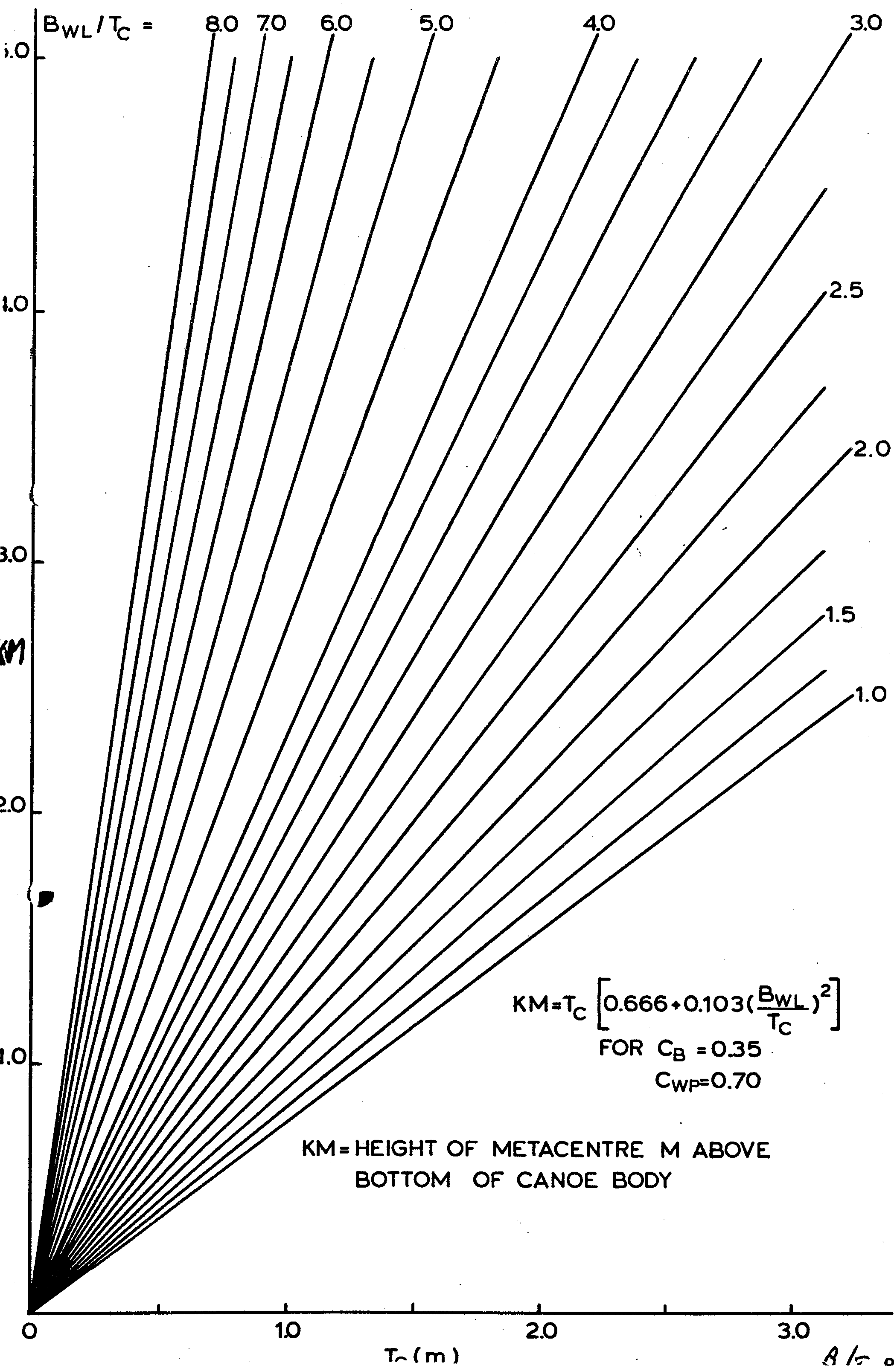
$$KM = T_c\left[0.666 + 0.103\left(\frac{B_{WL}}{T_c}\right)^2\right]$$

FOR $C_B = 0.35$ and $C_{WP} = 0.7$ (average values)

- ON REALIZING THAT STABILITY (AT SMALL ANGLES OF HEEL) IS DETERMINED BY

$$GM = KM - KG$$

IT FOLLOWS THAT A LARGE VALUE OF B_{WL}/T_c IS VERY ADVANTAGEOUS (AT SMALL TO MODERATE HEEL ANGLES) — SEE FIGURE



BWL / TC =
8.0 7.0 6.0 5.0 4.0 3.0
2.5
2.0
1.5
1.0
KM
0
1.0
2.0
3.0
TC (m)
KM=TC [0.666+0.103(BWL/TC)2]
FOR CB =0.35
CWP=0.70
KM=HEIGHT OF METACENTRE M ABOVE BOTTOM OF CANOE BODY

14.

B/T ON KM

- B_{WL}/T_C ALSO GREATLY INFLUENCES WETTED AREA

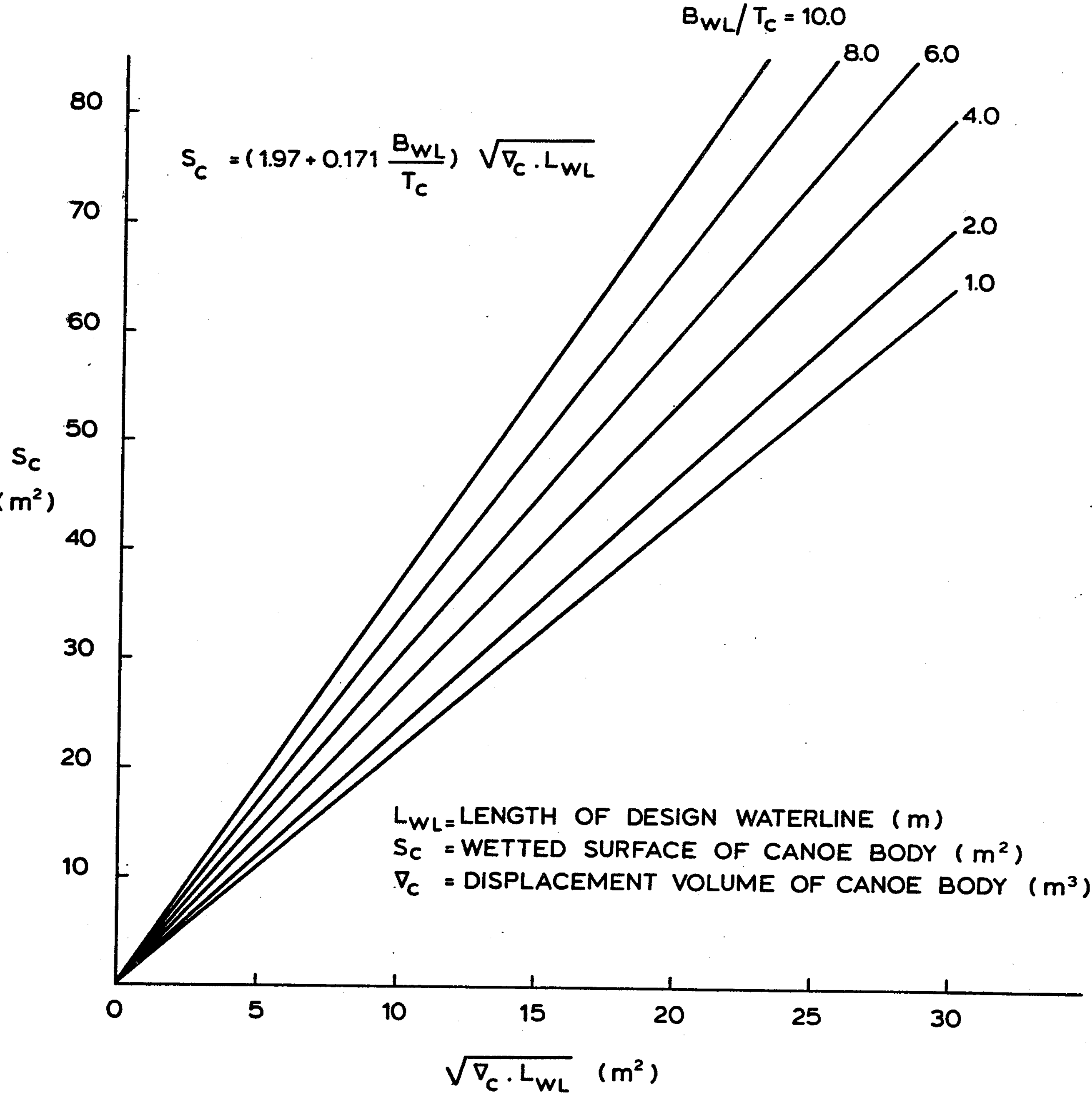
$$S_C = \left(1.97 + 0.171 \frac{B_{WL}}{T_C}\right)\sqrt{\nabla_C \cdot L_{WL}}$$

(Gerritsma et al)

(see figure)

- B_{WL}/T_C ALSO HAS "MINOR" INFLUENCE ON WAVE RESISTANCE. UP TO A FROUDE NUMBER OF 0.4 AN INCREASE IN B_{WL}/T_C RESULTS IN AN INCREASE IN WAVE RESISTANCE. IN EXCESS OF THIS FROUDE NUMBER THE REVERSE IS TRUE.

- THE CHOICE WITH RESPECT TO B_{WL}/T_C IS DIFFICULT TO MAKE BECAUSE OF THE EFFECTS INDICATED. IN AN EARLY DESIGN STAGE IT IS BEST TO ADOPT 3 VALUES — AS WAS DONE FOR $L_{WL}/\nabla^{1/3}$


B_{WL}/T_c = 10.0
8.0
6.0
4.0
2.0
1.0
$S_c = (1.97 + 0.171 \frac{B_{WL}}{T_c}) \sqrt{\nabla_c . L_{WL}}$
S_c (m^2)
80
70
60
50
40
30
20
10
0
5
10
15
20
25
30
$\sqrt{\nabla_c . L_{WL}}$ (m^2)
L_{WL} = LENGTH OF DESIGN WATERLINE (m)
S_c = WETTED SURFACE OF CANOE BODY (m^2)
∇_c = DISPLACEMENT VOLUME OF CANOE BODY (m^3)

13.

B/T ON S_c

3.4 INITIAL ESTIMATION OF STABILITY CHARACTERISTICS

- THIS IS REQUIRED IN AN EARLY DESIGN STAGE SO AS TO DETERMINE BALLAST REQUIREMENTS, SAILPLAN, ETC

- THE FOLLOWING FORMULAS CAN NOW BE USED

$$KM = KB + BM$$

$$KB = T_c \left(\frac{5}{6} - \frac{C_B}{3 C_{WP}} \right)$$

$$BM = 0.036 \left(\frac{B_{WL}}{T_c} \right)^2 \cdot \frac{T_c}{C_B}$$

- WITH INITIAL ASSUMPTIONS RELATIVE TO L_{WL}, ∇_c, B_{WL}, T_c (and hence C_B), these formulas can now be used. For C_{WP} an average value of about 0.7 CAN BE USED.

$$C_B = \frac{\nabla_c}{L_{WL} \cdot B_{WL} \cdot T_c} \qquad C_{WP} = \frac{A_{WP}}{L_{WL} \cdot B_{WL}}$$

$C_{WP} \approx 0.65$ to 0.68 (IOR)

≈ 0.70 to 0.73 (others)

- AS AN INITIAL ASSUMPTION RELATIVE TO $GM = KM - KG$, $KG \approx T_c$

3.5 INITIAL ESTIMATE OF REQUIRED SAIL AREA

- BEFORE CALCULATING THE "POWER-TO-CARRY-SAIL" PARAMETER, FIRST ASSESS 2 NON-DIMENSIONAL RATIOS:

  - SAIL AREA – WETTED SURFACE ARE RATIO

    S_A / S_c

    IS A MEASURE OF PERFORMANCE IN LIGHT WINDS

  - SAIL AREA – $\nabla_c^{2/3}$ RATIO

    IS A MEASURE OF PERFORMANCE IN MODERATE WINDS AND HEAVY CONDITIONS

  - NOTE THAT THESE RATIOS DO NOT TAKE STABILITY INTO ACCOUNT. ACCORDINGLY THESE RATIOS CAN ONLY GIVE GUIDANCE

- A STUDY OF EXISTING RACING YACHTS REVEAL THAT

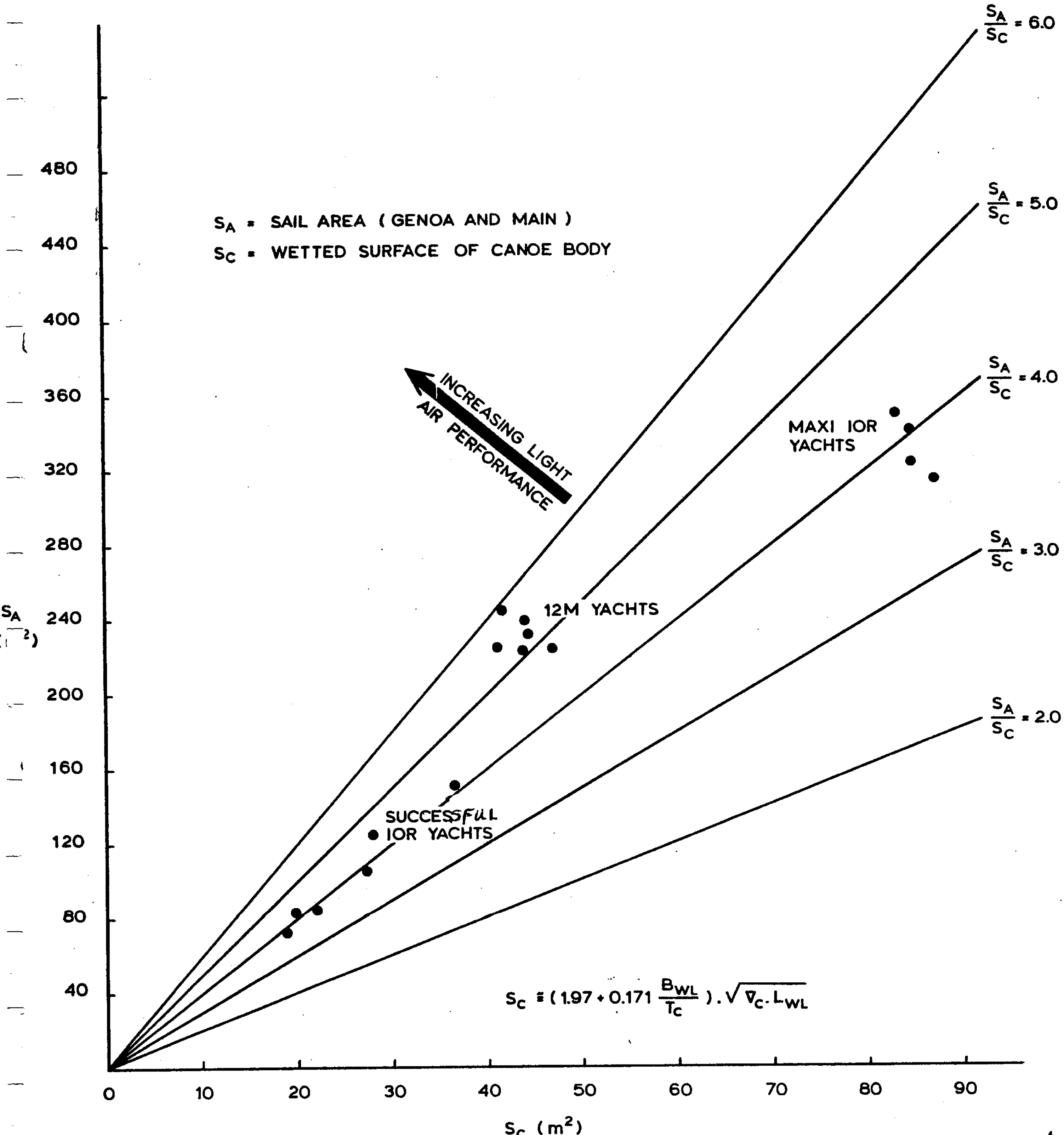
$$S_A / S_c \approx 4 \quad (IOR)$$

$$S_A / S_c \approx 5 \quad (12M)$$

$$S_A / \nabla_c^{2/3} \approx 5 \, L_{WL} / \nabla_c^{1/3} \quad (IOR)$$

OR $S_A \approx 5 \, L_{WL} \cdot \nabla^{1/3}$ (IOR)

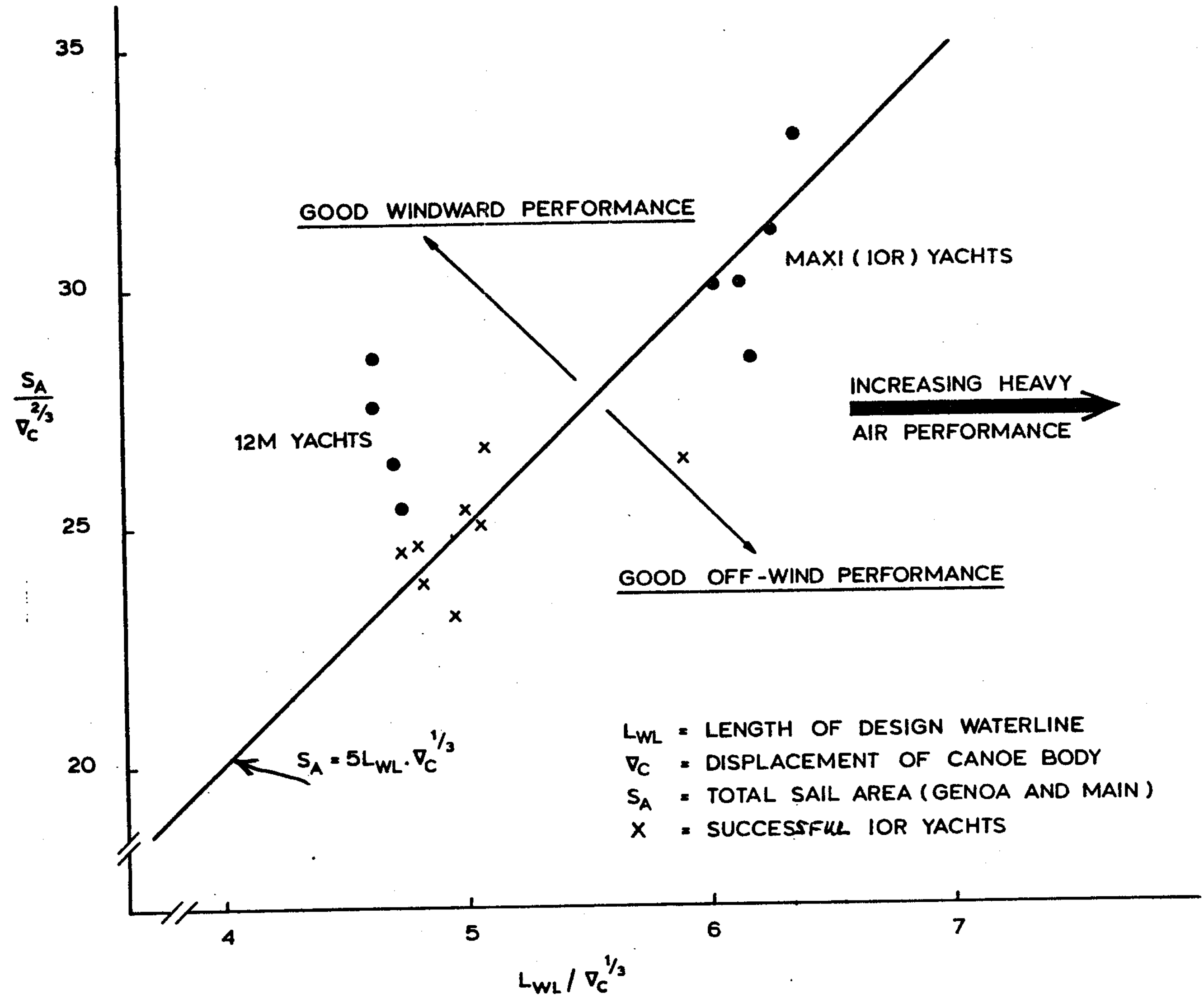
(see figures)



SA = SAIL AREA (GENOA AND MAIN)
SC = WETTED SURFACE OF CANOE BODY
INCREASING LIGHT AIR PERFORMANCE
SA/SC = 6.0
SA/SC = 5.0
SA/SC = 4.0
SA/SC = 3.0
SA/SC = 2.0
MAXI IOR YACHTS
12M YACHTS
SUCCESSFUL IOR YACHTS
SC = (1.97 + 0.171 BWL/TC) . √(∇C . LWL)
SA
480
440
400
360
320
280
240
200
160
120
80
40
0
10
20
30
40
50
60
70
80
90
SC (m2)

16

$S_A = f(S_c)$



17.

$S_A = f(L_{WL}, \nabla_c)$

- FOR DESIGN PURPOSES IT IS MORE SUITABLE TO ACTUALLY CALCULATE THE REQUIRED OR PERMISSABLE (MAXIMUM) SAIL AREA.

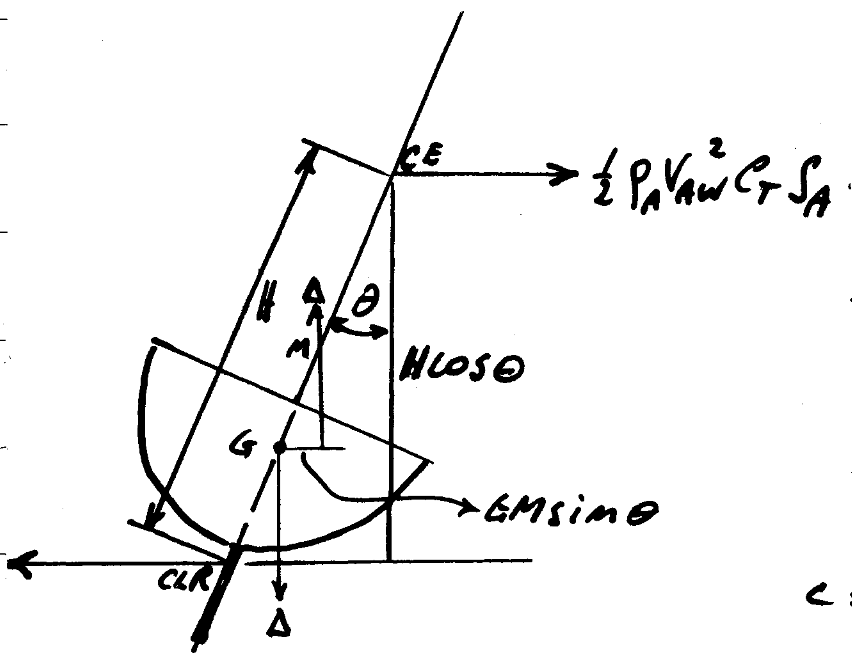
- THE RIGHTING MOMENT IS:

$$\Delta . GM \sin\theta$$

- THE OVERTURNING MOMENT IS:

$$\frac{1}{2}\rho_A V_{AW}^2 C_T S_A . H \cos\theta$$

with $C_T = \left(C_{L_S}^2 + C_{D_S}^2\right)^{1/2} \cos\left(\beta_{AW} - \varepsilon_S\right)$



and $\Delta = \rho_W g \nabla$

it follows:

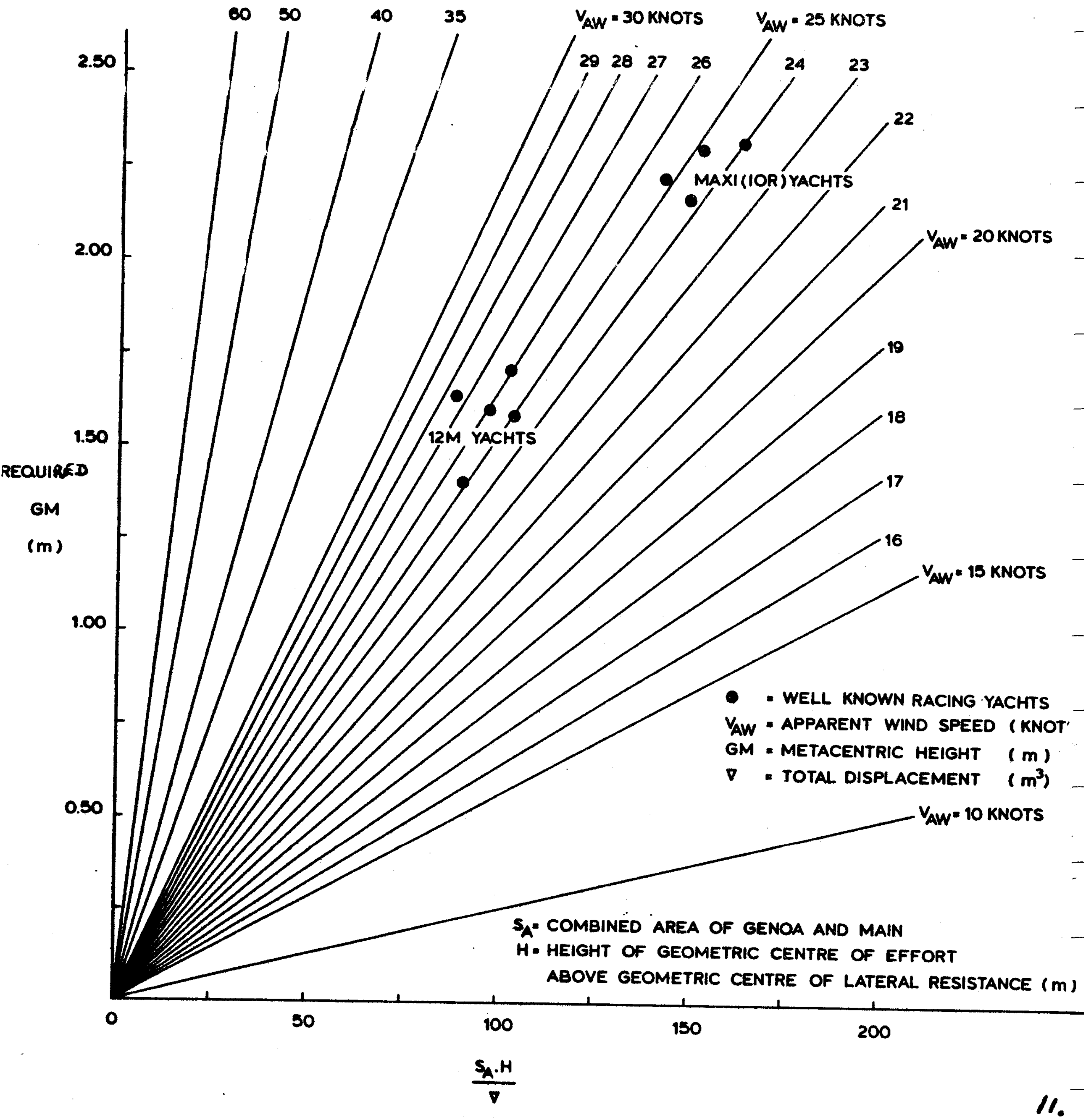
$$\frac{S_A H}{\nabla} = c \frac{GM}{V_{AW}^2}$$

$$c = \frac{g \rho_W \sin\theta}{\frac{1}{2}\rho_A C_T \cos\theta}$$

FOR $g = 9.81$; $\rho_W = 1025.9$, $\rho_A = 1.33$ kg/m³

$C_{L_S} = 0.95$ $C_{D_S} = 0.15$ $\beta_{AW} = 22°$ $\varepsilon_S = 8.97°$

$\theta = 30° \longrightarrow c = 10758$ (FIGURE)



11.

REQUIRED GM

3.6 REQUIRED VALUE OF PRISMATIC COEFFICIENT

- C_p depends on the speed of the yacht

- at high speeds ($F_n = 0.4$ or higher), a relatively large C_p leads to a greater separation of the bow and stern waves – ie a greater wave length because the yacht has greater effective wave-making length in that case

- a relatively large C_p leads to a greater wetted area (more volume in the ends). Hence when wave resistance is less important choose a lower C_p-value.

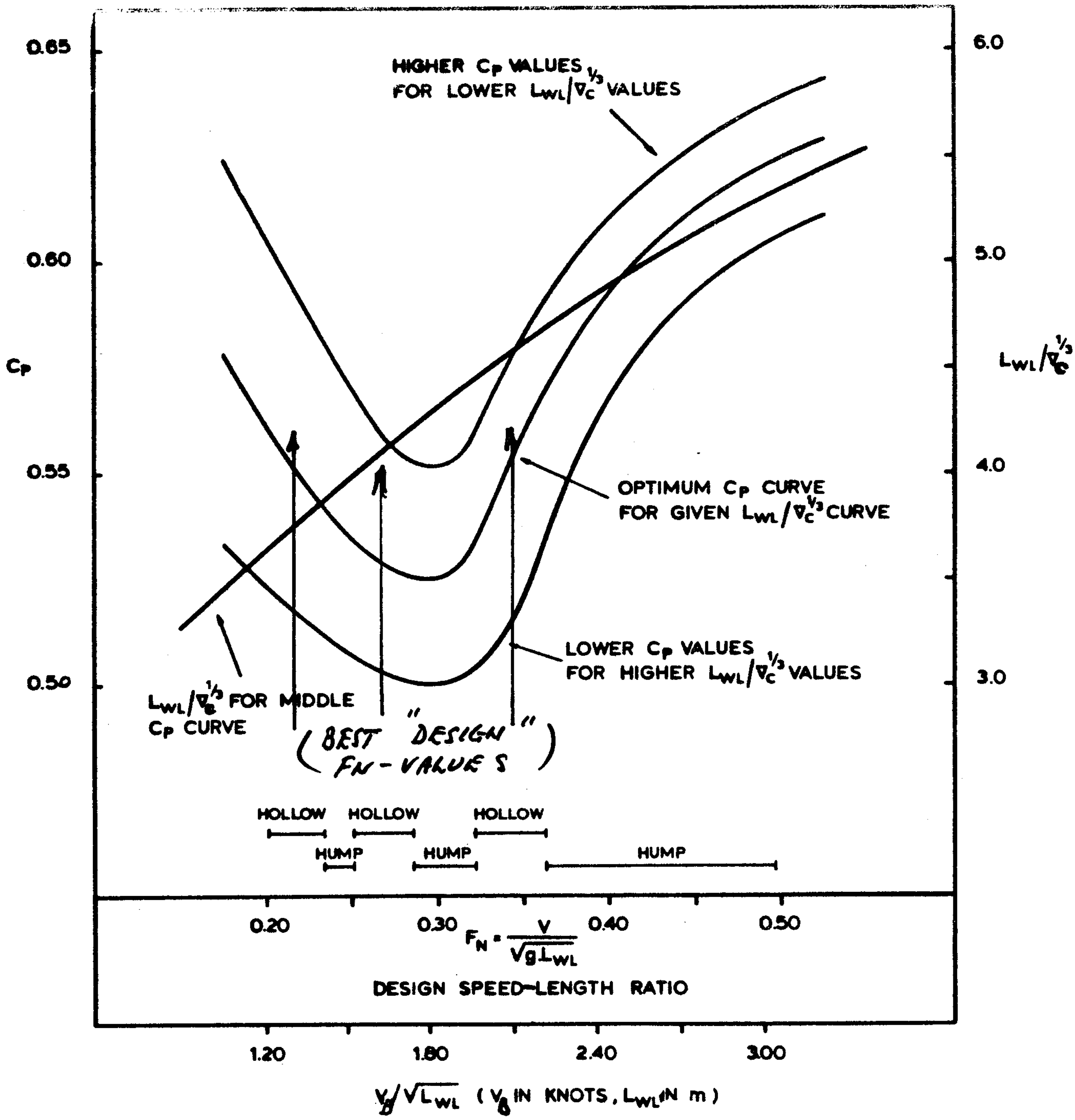
- C_p depends on $L/\nabla^{1/3}$ also. This is due to limitations in acceptable midship section coefficient values

  large $L/\nabla^{1/3}$ requires lower C_p values (i.e. larger C_M values)

  low $L/\nabla^{1/3}$ requires higher C_p values (i.e smaller C_M values)

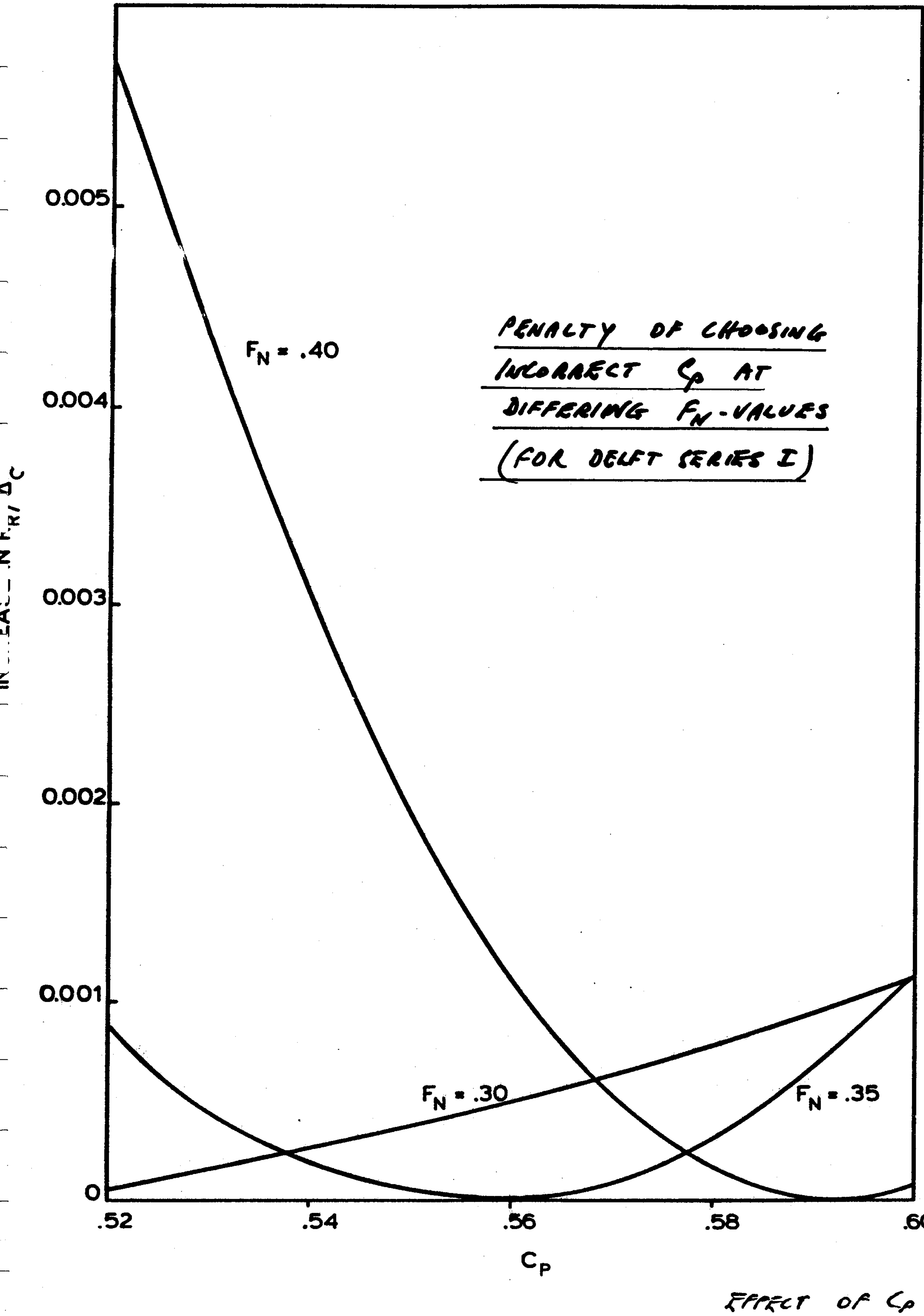
$$\left(\frac{L}{\nabla^{1/3}} = \frac{L^3}{\nabla_c} = \frac{L^3}{C_B\, L.B.T} = \frac{L^3}{C_p . C_M B T . L} \right)$$

(see figure)



19.

OPTIMUM C_p

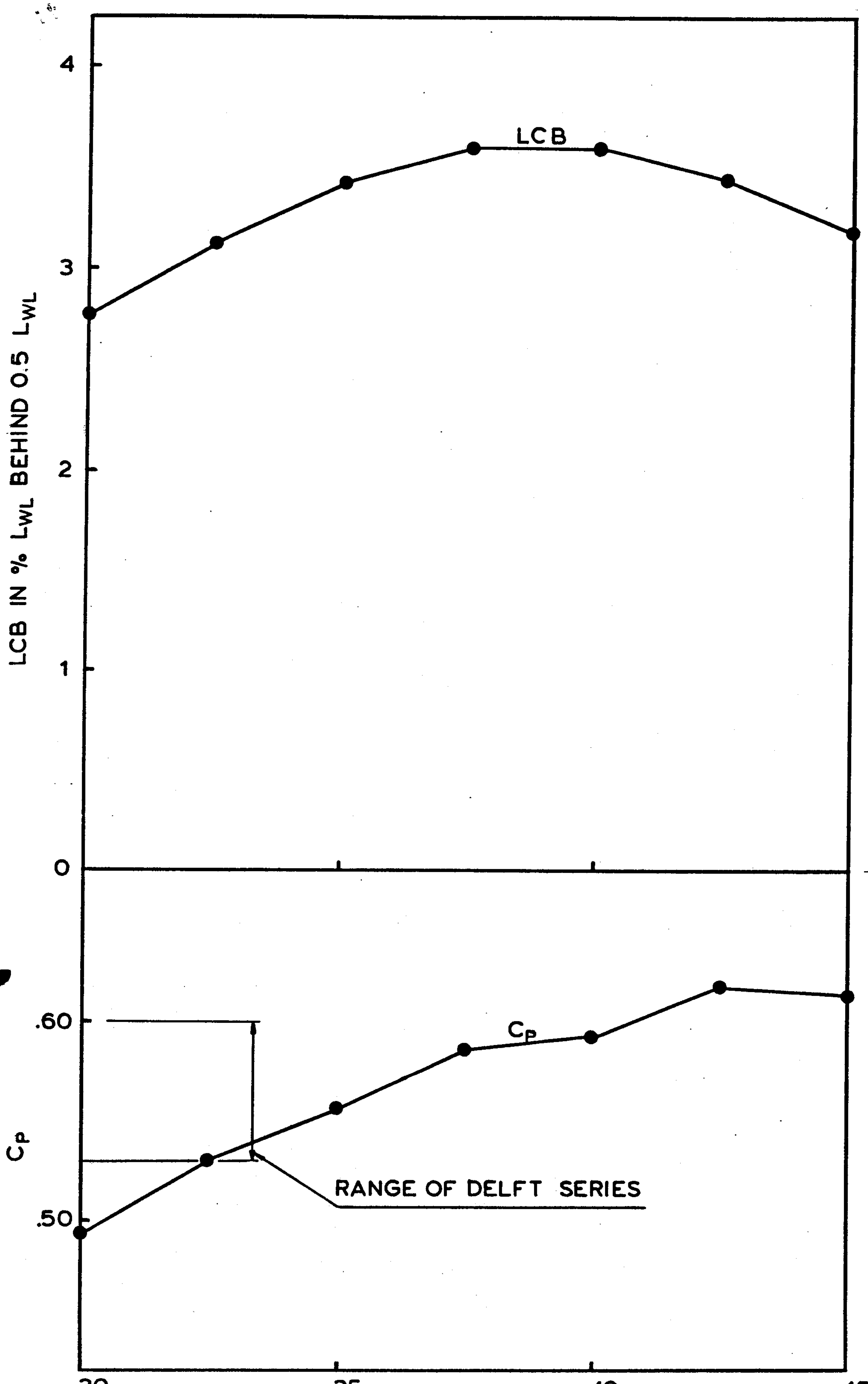


22

EFFECT OF C_p ON R_R/Δ

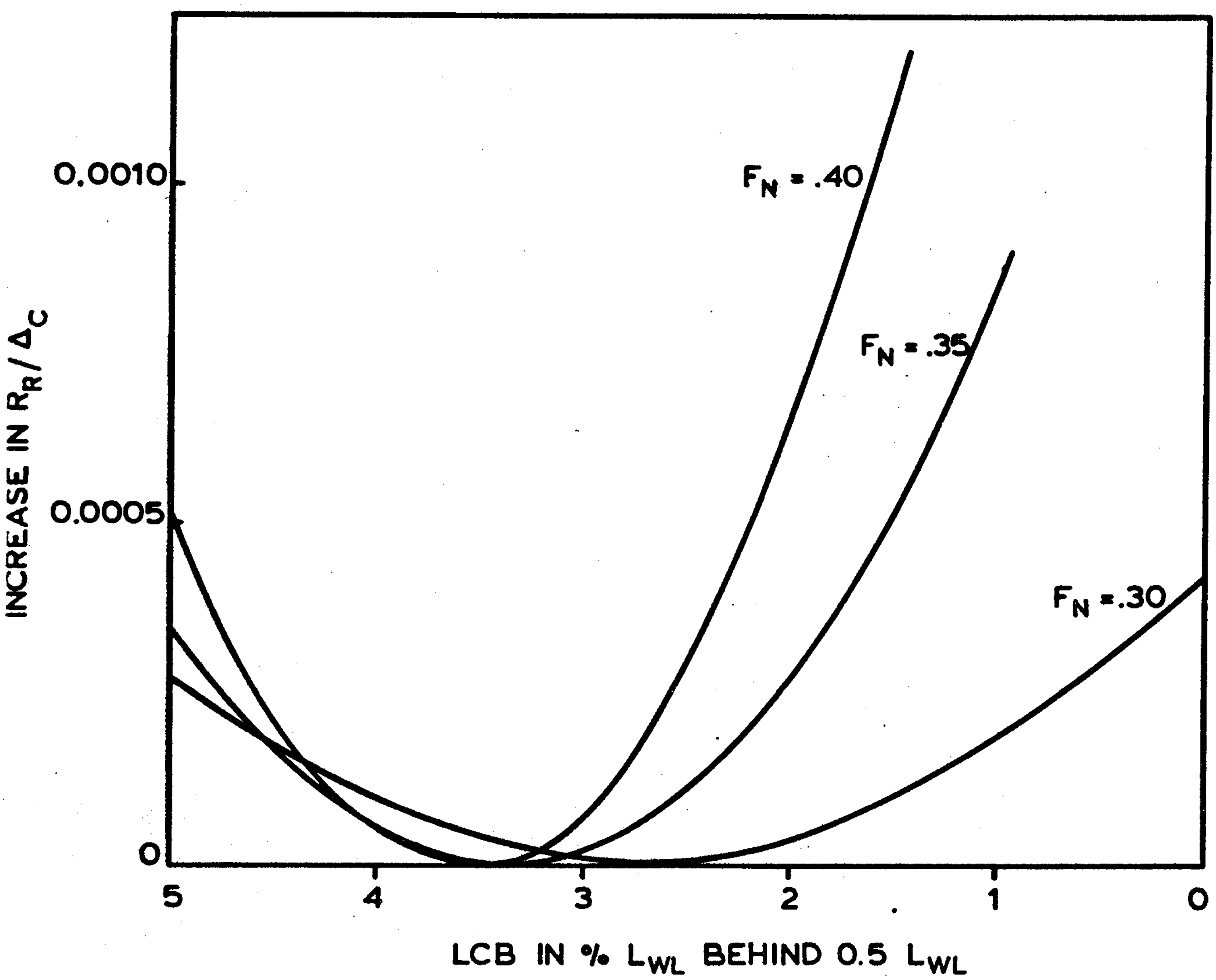
2.7 REQUIRED LONGITUDINAL CENTRE OF BUOYANCY POSITION

- THE LCB POSITION, WITH RESPECT TO THE EFFECT ON RESISTANCE IS AS IMPORTANT AS C_p
- AS SPEED INCREASES AND WAVE RESISTANCE INCREASES, THE FORWARD PART OF THE HULL NEEDS TO BECOME FINER AND LCB MUST BE POSITIONED FURTHER AFT.
- FOR YACHTS, LCB IS USUALLY BETWEEN 2 AND 5% OF LWL BEHIND 0.5 LWL.
- ON POSITIONING LCB TO FAR AFT THE AFT-BODY BECOMES TOO FULL RESULTING IN FLOW SEPARATION
- OPTIMUM C_p AND LCB VALUES FOR THE DELFT SERIES (22 MODELS) ARE GIVEN IN NEXT FIGURE (80)
- THE EFFECT OF NON-OPTIMUM LCB POSITION ON THE INCREASE IN RESIDUAL RESISTANCE ACCORDING TO THE DELFT SERIES, FOR DIFFERENT SPEEDS IS SHOWN IN FIGURE (SHEET 81)



20.

OPTIMUM C_p & LCB

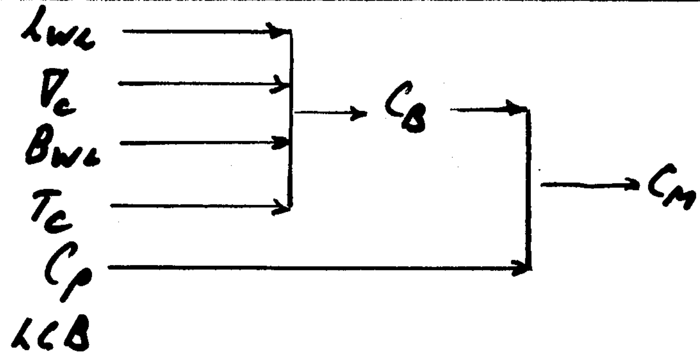


21.

EFFECT OF LCB ON R_R / Δ_C

2.8 DESIGN OF LINESPLAN

- AT THIS STAGE THE FOLLOWING INFORMATION MUST BE AVAILABLE



- SET-UP CURVE OF SECTIONAL AREAS BY POSITIONING SECTION OF GREATEST AREA BEHIND $0.5 L_{WL}$ (USUALLY BETWEEN $0.55 L_{WL}$ AND $0.6 L_{WL}$ FROM THE FORWARD L_{WL}-ENDING)

- A SLIGHTLY HOLLOW SECTIONAL AREA CURVE (IN THE BOW REGION) IS ACCEPTABLE! TOO HOLLOW MEANS LOSS OF EFFECTIVE WAVE-MAKING LENGTH HOWEVER

- HALF-ANGLE OF THE DESIGN WATERLINE IN THE BOW TO BE LESS THAN ABOUT 20° IF POSSIBLE

- ADOPT STRAIGHT WATERLINES IN THE BOW REGION

- IN THE AFT-BODY EITHER THE WATERLINES OR THE BUTTOCKS SHOULD RUN RATHER STRAIGHT :- FOR LARGE B_{WL}/T_C VALUES THE BUTTOCKS

- AVOID A CONVEX CURVE OF SECTIONAL AREAS IN THE AFT-BODY (RUN) - TO AVOID FLOW SEPARATION.

- DIAGONALS ALONG HEELED WATERLINES SHOULD BE AS STRAIGHT AS POSSIBLE.

- AFTER SETTING-UP ROUGH LINESPLAN CHECK GIRTHS, PENALTIES, ETC WHEN DESIGNING TO A RATING RULE OR DESIGN FORMULA.

- POSITION TRANSOM SUCH THAT FLOW WILL SEPARATE "CLEANLY"

- PAY ATTENTION TO RUNNING TRIM EFFECTS AT HIGHER SPEEDS

DESIGN OF KEEL AND RUDDER

- THE FUNCTION OF BOTH KEEL AND RUDDER IS TO GENERATE A HIGH SIDE FORCE FOR A LOW INCREASE IN RESISTANCE
- BOTH THE ATTAINABLE LIFT AND THE LIFT-DRAG RATIO ARE IMPORTANT
- DRAG INCREASE DUE TO LIFT SHOULD BE ASSESSED FOR THE HULL AS A WHOLE. THIS USUALLY LEADS TO THE REQUIREMENT THAT THE REQUIRED SIDE FORCE MUST BE GENERATED AT SMALL ANGLES OF LEEWAY.

- NOWADAYS THE RUDDER IS NO LONGER CONSIDERED ONLY FOR DIRECTIONAL CONTROL BUT ALSO FOR GENERATING SIDE FORCE ON-THE-WIND. WEATHER HELM IS DESIGNED "INTO" THE CONFIGURATION

- TO ENSURE THAT LEEWAY ANGLES REMAIN SMALL "SIZING" OF BOTH KEEL AND RUDDER IS IMPORTANT.

- IN ORDER OF IMPORTANCE, WE WILL LOOK AT
  - LATERAL AREA REQUIREMENTS
  - EFFECTS OF ASPECT RATIO
  - TAPER RATIO AND SWEEP ANGLES
  - EFFECTS OF WINGLETS
  - EFFECT OF SECTION SHAPES

1. LATERAL AREA REQUIREMENTS

- IF KEEL AND/OR RUDDER LATERAL AREA IS TOO LARGE THE YACHT WILL PRODUCE THE REQUIRED SIDE FORCE AT SMALL LEEWAY ANGLES. WETTED AREA WILL BE EXCESSIVE HOWEVER. THE YACHT WILL SAIL "HIGH AND SLOW"

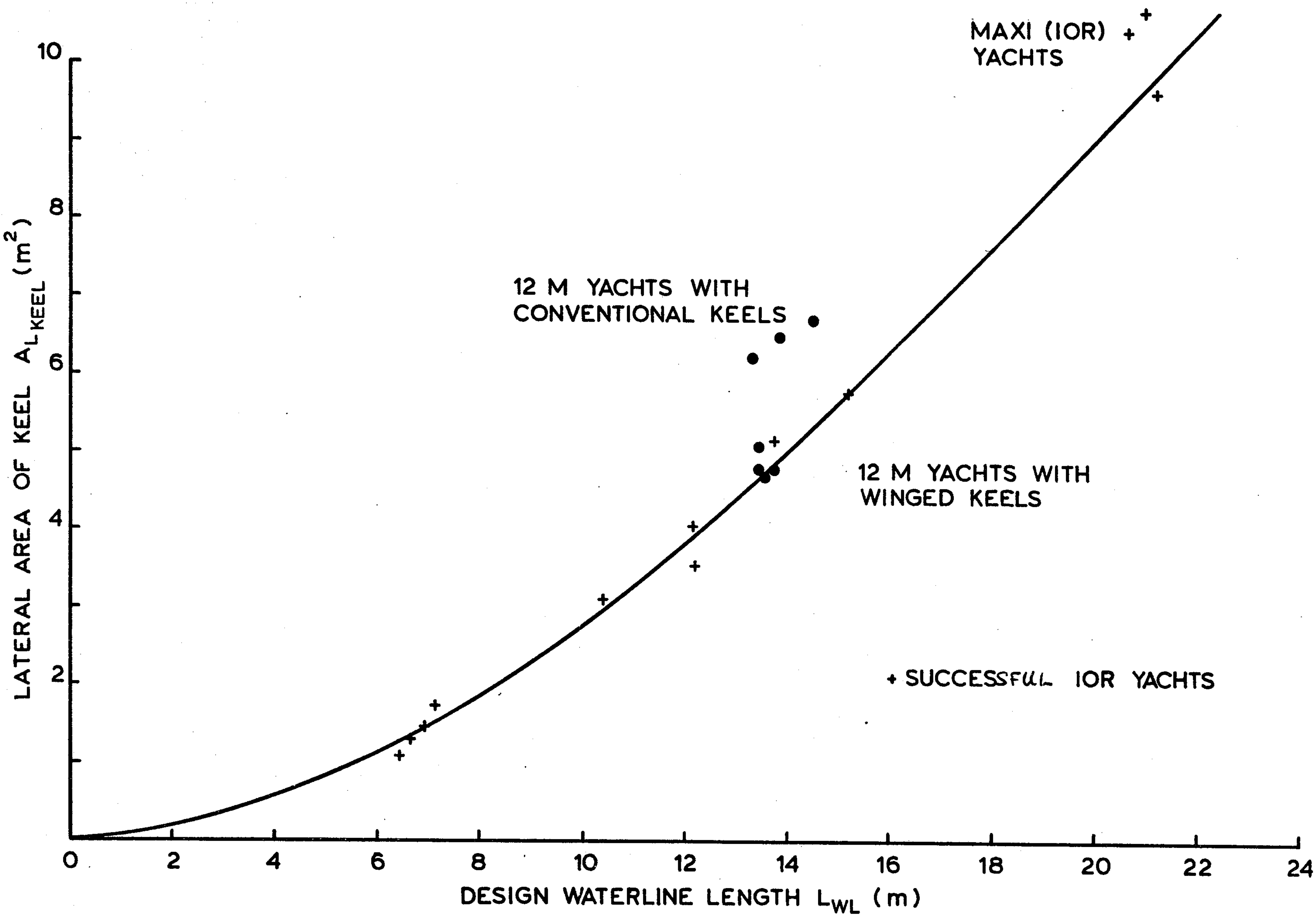
- IF KEEL AND/OR RUDDER LATERAL AREA IS TOO SMALL, THE REQUIRED SIDE FORCE CAN ONLY BE DEVELOPED AT RELATIVELY LARGE LEEWAY ANGLES. WETTED SURFACE WILL BE SMALL HOWEVER: THE YACHT WILL SAIL "LOW AND FAST"

- THE CORRECT LATERAL AREA MUST BE DETERMINED TO OBTAIN OPTIMUM SPEEDS

- AMOUNT OF "ON-THE-WIND" AND "OFF-THE WIND" SAILING MUST BE CONSIDERED IN THIS CONNECTION AS WELL. IF PREDOMINANT "OFF-THE-WIND" (WHITBREAD) SMALLER KEEL SIZES CAN BE ADOPTED.

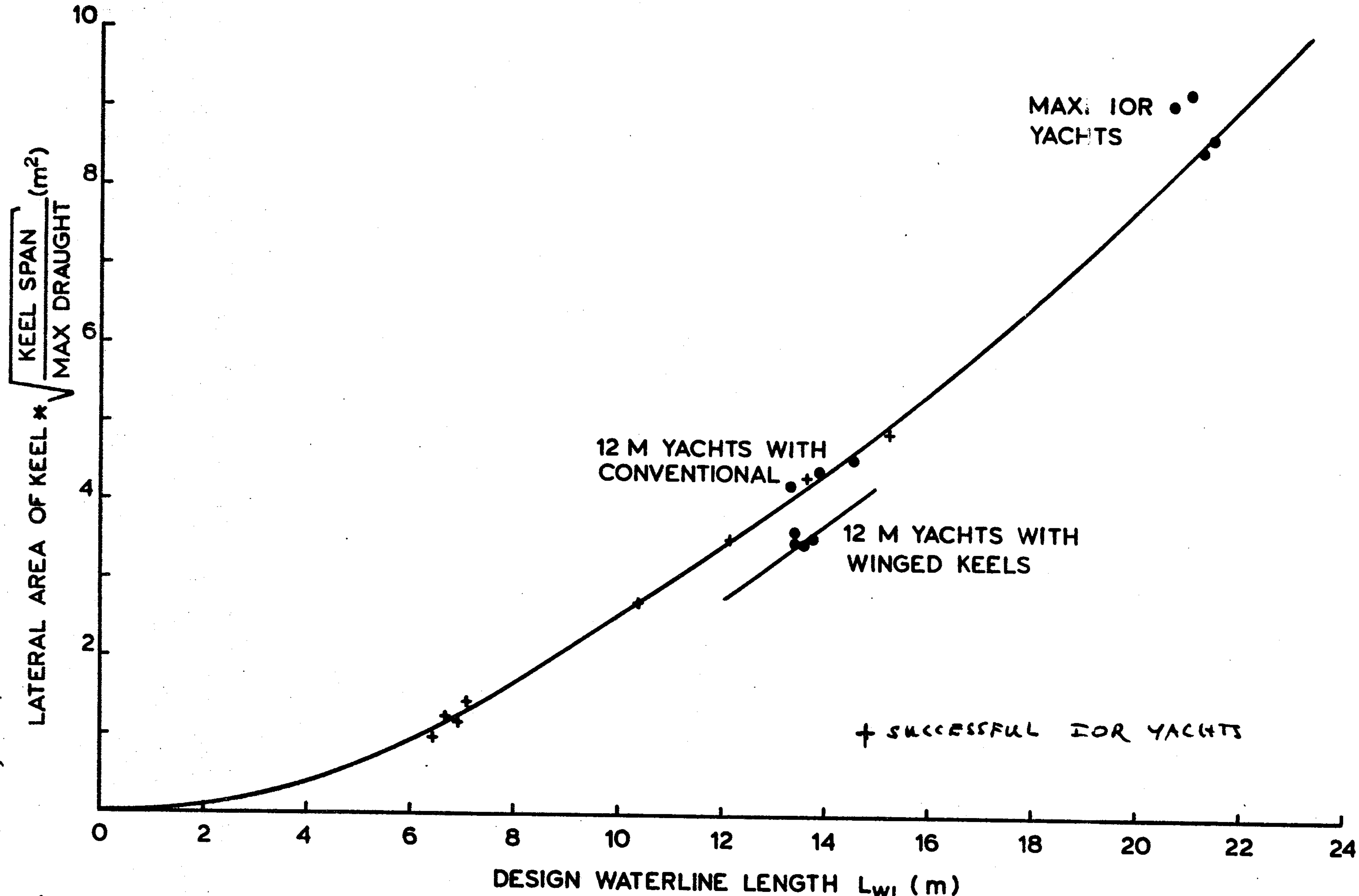
- PAY ATTENTION, ALSO, TO KEEL SIZE FROM THE POINT-OF-VIEW OF HAVING TO CARRY BALLAST (LEAD)

- FIGURE (86) FOR GENERAL GUIDANCE
- FIGURE (87) TO REDUCE EFFECT OF SPAN



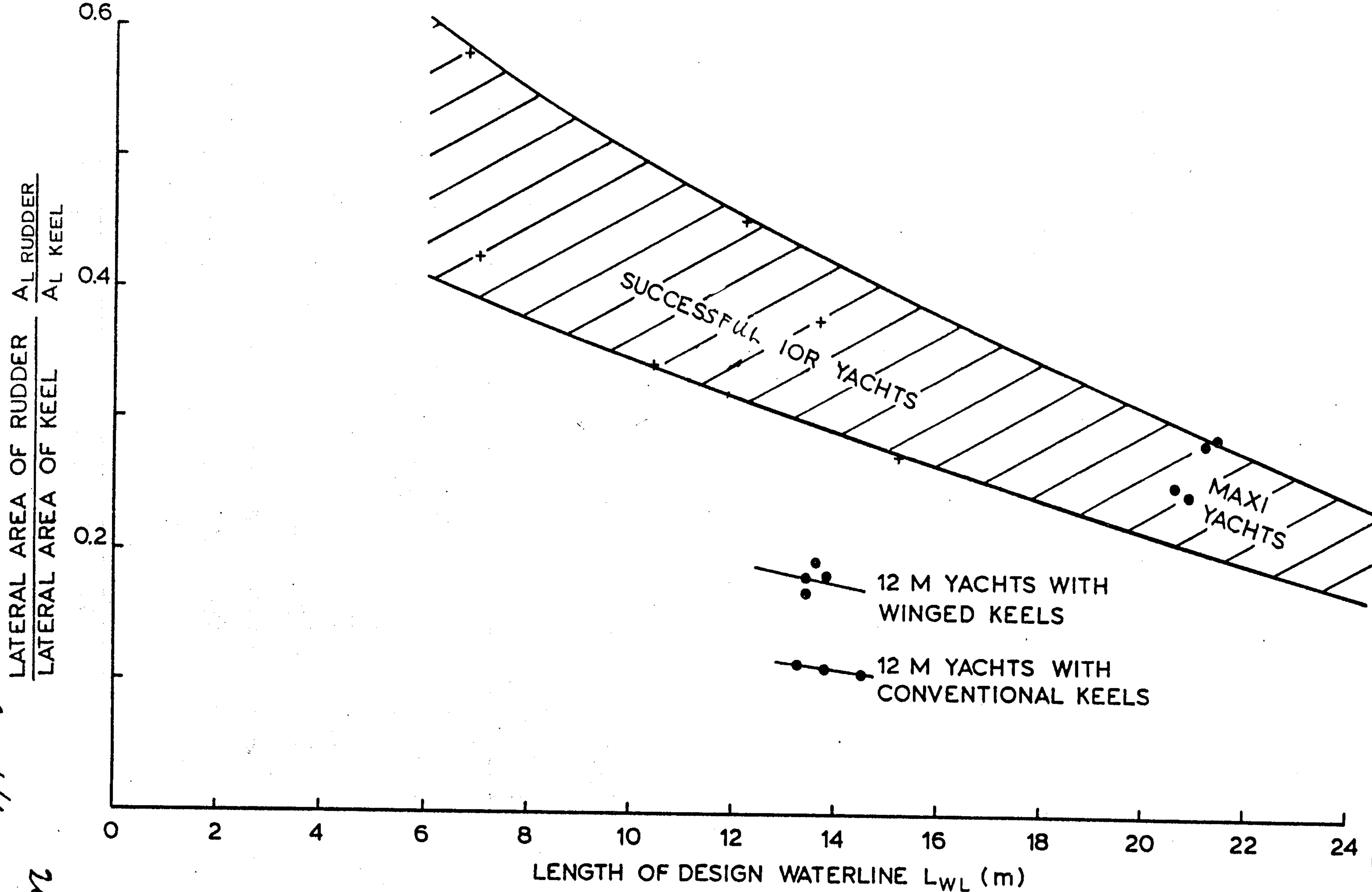
24.

KEEL AREA = f(LWL)



25.

KEEL AREA = $f(T_c, T_{max}, L_{WL})$



26.

$A_{LR} = f(A_{LK}, L_{WL})$

2. SIGNIFICANCE OF ASPECT RATIO

- PARTICULARLY WHEN THE MAXIMUM DRAUGHT CAN BE CHOSEN FROM AN OPTIMUM PERFORMANCE POINT OF VIEW, CAREFUL CONSIDERATION MUST BE GIVEN TO THE VALUE OF KEEL (OR RUDDER) HEIGHT DIVIDED BY AVERAGE CHORD LENGTH

$$AR = \frac{span}{\bar{c}_{k,R}} \quad \text{OR} \quad \frac{T_{k,R} - T_c}{\bar{c}_{k,R}}$$

WHEN SUFFICIENTLY SUBMERGED

$$\text{effective } AR_e = \frac{2\,span}{\bar{c}_{k,R}} \quad \text{OR} \quad \frac{2(T_{k,R} - T_c)}{\bar{c}_{k,R}}$$

- REMEMBER THAT UP TO $AR \approx 2.0$

$$L = \tfrac{1}{2}\rho V_B^2 \; \tfrac{1}{2}\pi \, AR \, \beta . A_L$$

V_B = boat speed (m/sec)

β = leeway angle (radians)

A_L = lateral area (m^2)

(without any keel - canoe body interaction)

- doubling AR at constant A_L doubles the lift in this (low) AR region common to most yacht keels

- Since wetted surface remains the same in that case, L/D approximately doubles

- In fact L/D more than doubles because D_I decreases with AR, viz:

$$D_I = \frac{\frac{1}{2}\rho V_B^2 A_L C_L^2}{\pi AR}$$

- suppose $\beta = 0.05$, $C_L = 0.2$, $R_n = 1\times10^6$

$$\frac{L}{D} \cong \frac{\frac{1}{2}\rho V_B^2 \frac{1}{2}\pi AR A_L \beta}{\frac{1}{2}\rho V_B^2 C_F 2A_L + \frac{1}{2}\rho V_B^2 A_L \frac{C_L^2}{\pi AR}}$$

on doubling AR:

$$\frac{\left(\frac{L}{D}\right)_{AR=2}}{\left(\frac{L}{D}\right)_{AR=1}} = \frac{\dfrac{\frac{1}{2}\cdot\pi\cdot2\cdot(0.05)}{0.0047\cdot2 + (0.2)^2/\pi/2}}{\dfrac{\frac{1}{2}\pi\cdot1\cdot(0.05)}{0.0047\cdot2 + (0.2)^2/\pi}}$$

$$= \frac{\dfrac{0.157}{0.0094 + 0.0064}}{\dfrac{0.0785}{0.0094 + 0.0127}}$$

$$= \frac{9.94}{3.55} = 2.80$$

NOTE: $C_F \approx 0.075/(\log_{10} R_N - 2)^2$

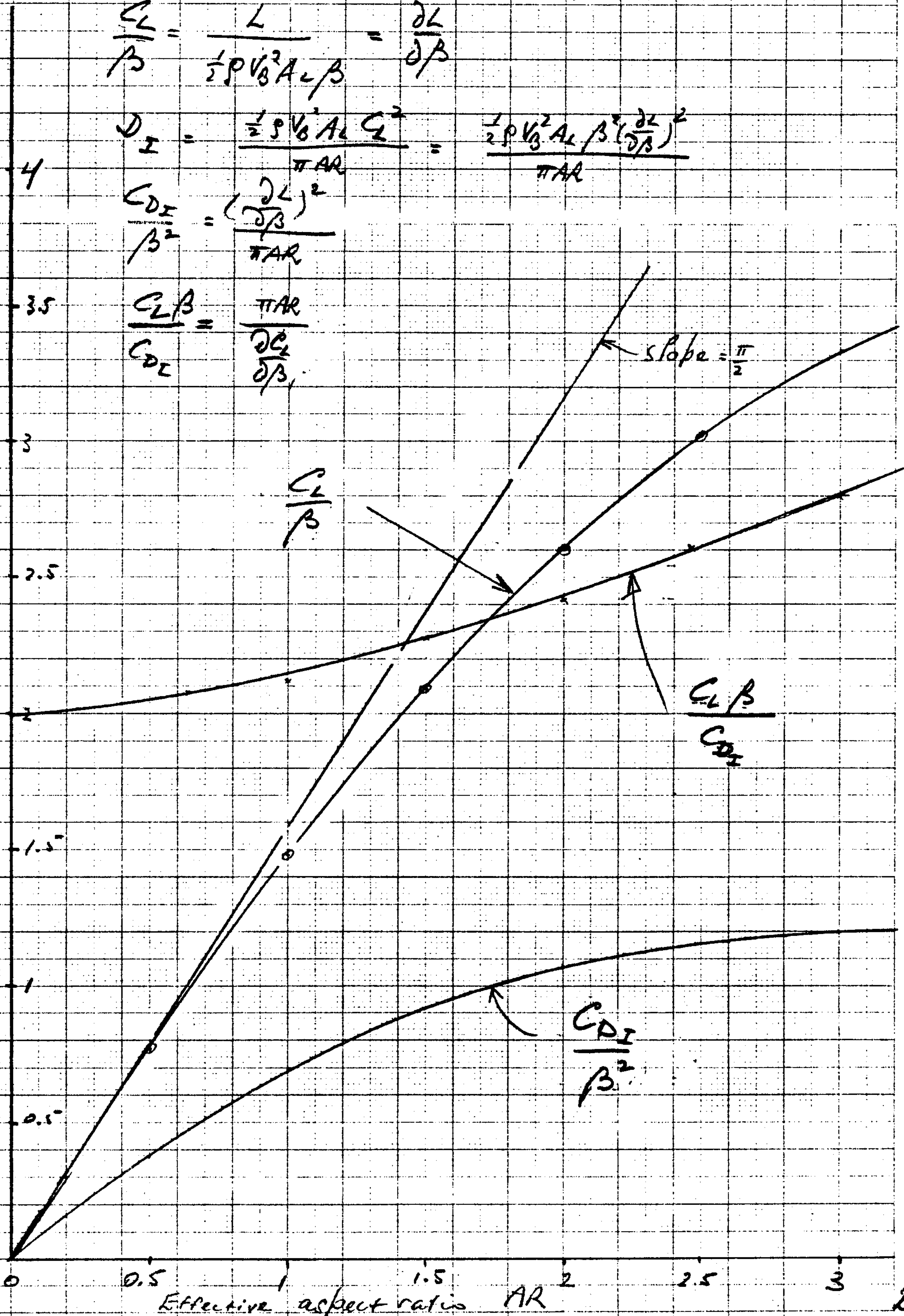
$$L = \tfrac{1}{2}\rho V_B^2 A_L \beta \cdot \frac{\partial L}{\partial \beta}, \text{ with } \frac{\partial L}{\partial \beta} \cong \frac{2\pi AR}{2+\sqrt{AR+4}}$$

$$\frac{C_L}{\beta} = \frac{L}{\tfrac{1}{2}\rho V_B^2 A_L \beta} = \frac{\partial L}{\partial \beta}$$

$$D_I = \frac{\tfrac{1}{2}\rho V_B^2 A_L C_L^2}{\pi AR} = \frac{\tfrac{1}{2}\rho V_B^2 A_L \beta^2 \left(\frac{\partial L}{\partial \beta}\right)^2}{\pi AR}$$

$$\frac{C_{D_I}}{\beta^2} = \frac{\left(\frac{\partial L}{\partial \beta}\right)^2}{\pi AR}$$

$$\frac{C_L \beta}{C_{D_I}} = \frac{\pi AR}{\frac{\partial C_L}{\partial \beta}}$$



C_L/C_{D_I} on AR

- WHEN A DRAUGHT RESTRICTION APPLIES (AS IN MOST CASES IN RACING YACHT DESIGN), IT PAYS TO TRY AND MINIMIZE DRAUGHT OF CANOE BODY TO OBTAIN GREATEST POSSIBLE KEEL HEIGHT

- IN A 12 METRE MAXIMUM DRAUGHT IS:

$$\boxed{0.16\, L_{WL} + 0.5 \ (m)}$$

for $L_{WL} \approx 14$ m and $T_c \approx 1.4$ m

$$T_k - T_c = 0.16(14) + 0.5 - 1.4$$
$$= 1.34\, m$$

- IN IOR DESIGN, THE SO-CALLED BASE DRAUGHT IS:

$$\boxed{0.146\, L + 0.6096 \ (m)}$$

for $L = 14$, $T_c = 0.667$ $\left(\frac{L}{B} = 3.5, \ \frac{B}{T} = 6\right)$

$$T_k - T_c = 0.146(14) + 0.6096 - 0.667$$
$$= 1.99\, m$$

- NO DRAUGHT RESTRICTION IS PLACED ON RUDDER SPAN (YET?)

3. REQUIRED PLANFORM: TAPER RATIO AND SWEEP ANGLE

- $\text{TAPER RATIO} = \dfrac{c_{tip}}{c_{root}}$

- SWEEP ANGLE DEFINED BY ANGLE OF QUARTER-CHORD LINE RELATIVE TO VERTICAL

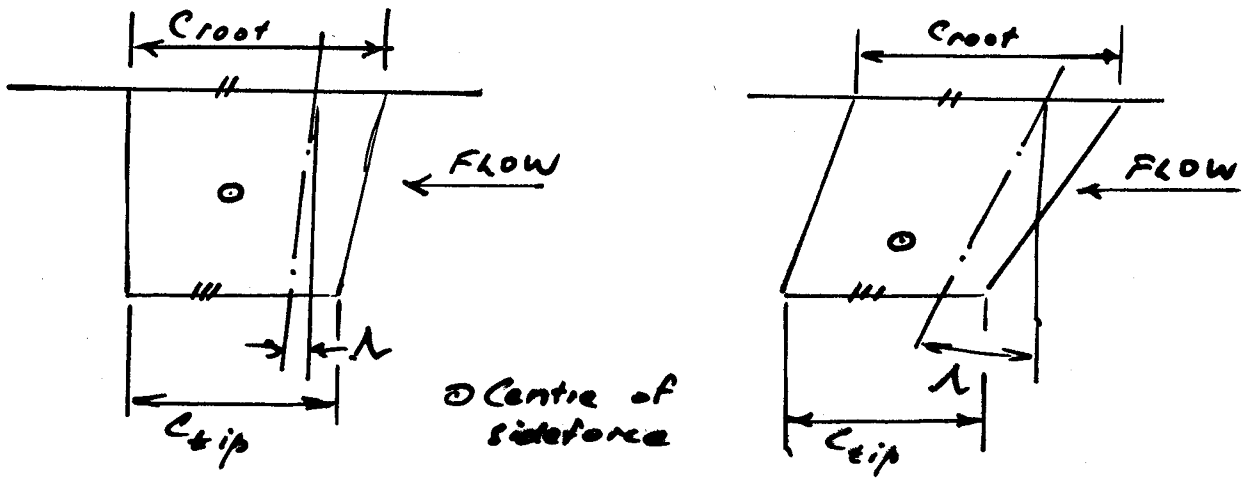
- UP TO 1983 IT WAS COMMON TO INCREASE SWEEP ANGLE WITH DECREASING ASPECT RATIO

  - DUE TO TOWING TANK RESULTS REVEALING THAT FOR TRAPEZIUM-SHAPED KEELS WITH TAPER RATIO < 1 THAT RESISTANCE AT VARIOUS COMBINATIONS OF HEEL & LEEWAY DECREASED WITH INCREASING SWEEP ANGLE.

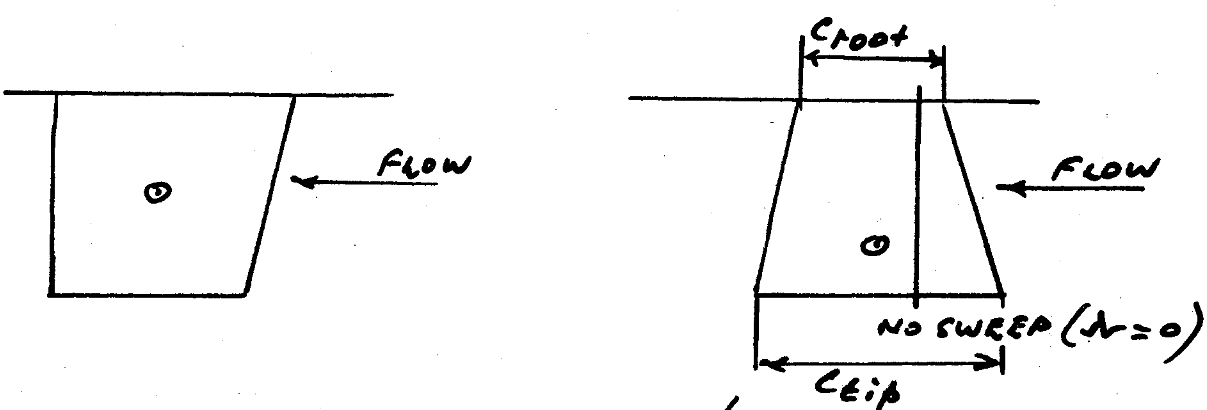
- IN 1981/1982 THE "AUSTRALIA II" RESEARCH REVEALED THIS TO BE DUE TO THE IMPORTANCE OF PLACING THE LIFT ON THE KEEL FURTHER DOWN FROM THE FREE SURFACE – THEREBY REDUCING WAVE RESISTANCE

  - AS A CONSEQUENCE IT WAS FOUND TO BE ADVANTAGEOUS TO ADOPT INVERSE TAPER RATHER THAN SWEEP.

- SINCE LIFT REDUCES WITH COS(Λ) IT IS BETTER TO MAXIMIZE SIDEFORCE BY ADOPTING LITTLE OR NO SWEEP AND TO UTILIZE INVERSE TAPER FOR ENSURING THAT THE CENTRE OF SIDEFORCE IS AS FAR AWAY FROM THE WATER SURFACE AS POSSIBLE



effect of sweep on centre of sideforce



effect of inverse taper on centre of sideforce

- taper ratio values > 1 adopted for "Australia II" based on this finding

- NOTE: Sweep on aircraft wings only to delay onset of shock wave

- This result, leading to non-elliptic loading distributions is only valid in the presence of the free water surface, particularly when at moderate to high heel angles, the pressure difference between the 2 sides of the keel leads to an unfavourable secondary wave system and an increase in wave resistance

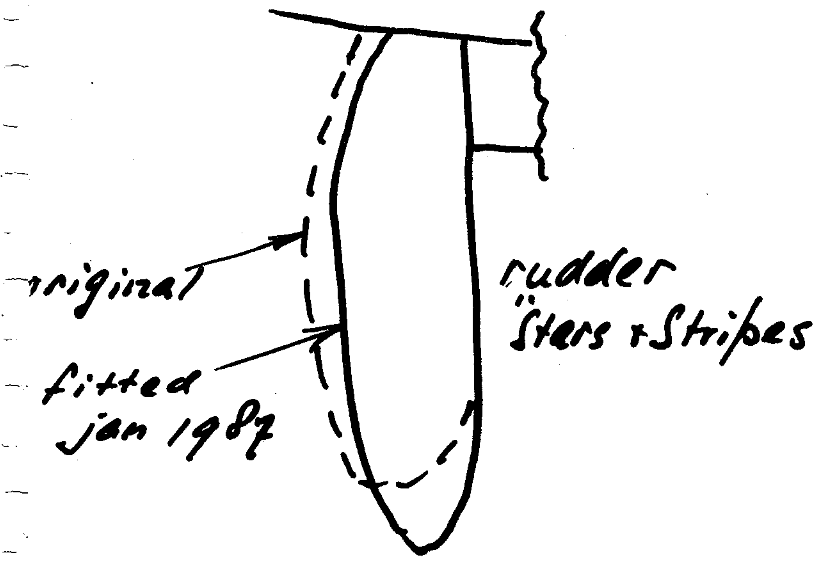
- for relatively large aspect ratio values the influence of keel loading on wave resistance is less because of shorter chord lengths at the canoe body (less lift-carry-over from keel to canoe body and vice-versa)

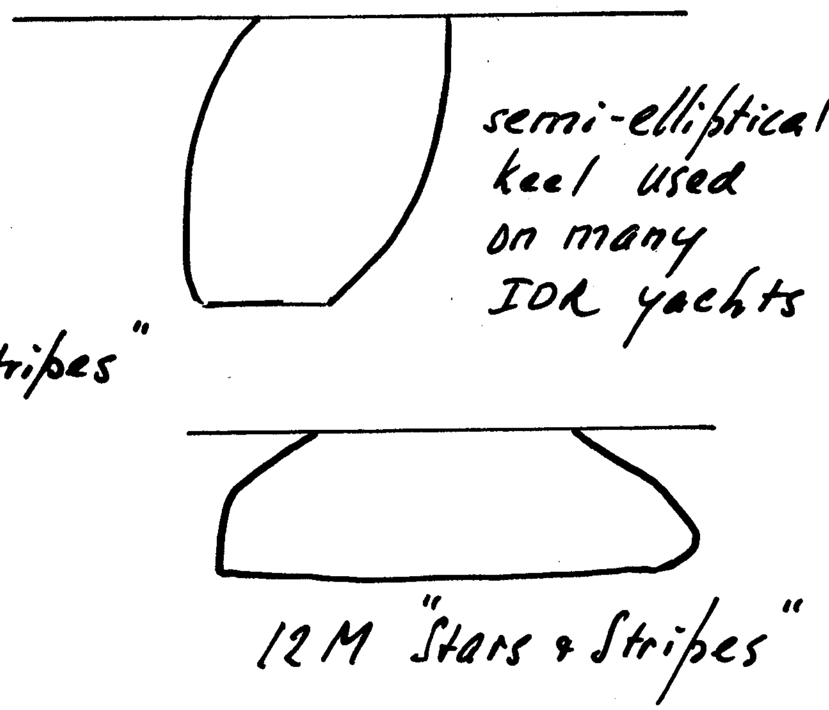
- note that in a trapezium planform a taper ratio of 0.333 is equivalent to the case of elliptical loading.

- A sweep angle of about 30° requires a taper ratio of 0.15 to maintain near-elliptical loading

- The 12 Metre "Advance" (1983), in having a large keel (and skeg), both with small taper ratio, had a high wave resistance

• Publication of the "Australia II" research in 1984 lead to the <u>semi-elliptical</u> and near-elliptical keel and rudder shapes we now see regularly





• <u>Rudders in particular need to carry little load at the root chord.</u> Besides greater proximity to the water surface, the deflected rudder also destroys local boundary layer flow.

• <u>leading edge sweep</u> also triggers onsett of turbulent flow. This is another reason for avoiding large sweep angles.

4. Application of Winglets

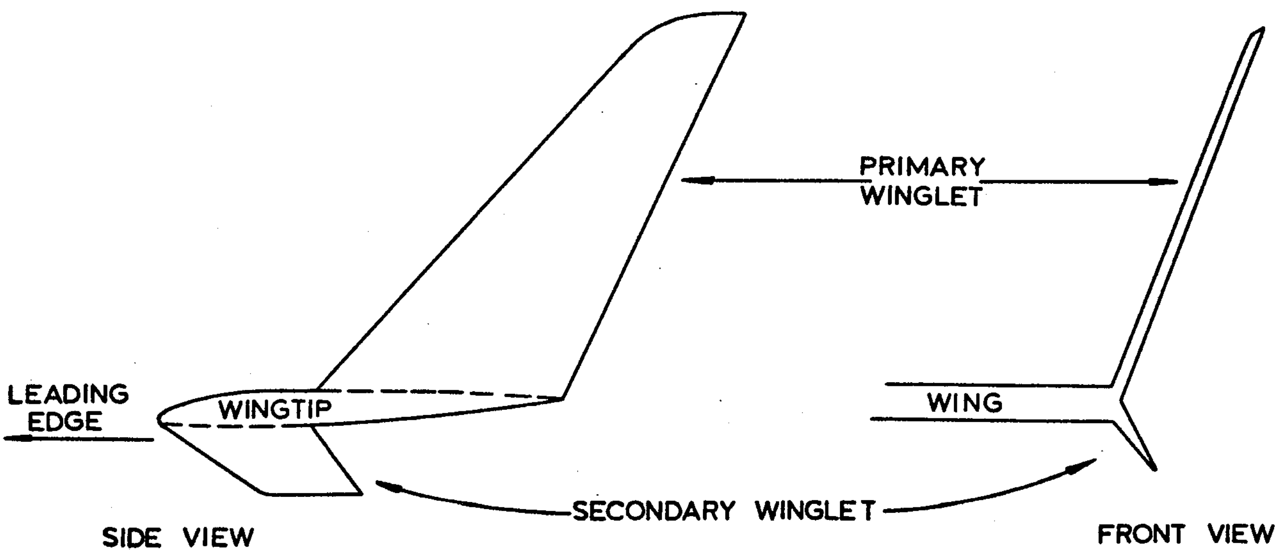
- On having found that an "upside-down" keel was favourable for a 12 Metre, it was only a small step to look for means to reduce tip losses on a planform with a large load near the tip

- The only disadvantage of large taper ratio keels is the relatively large tip vortex and the associated loss in lift and greater induced resistance.

- Winglets, as patented by R.T. Whitcomb (NASA) in 1976, were almost the first means studied in 1981/1982 to reduce the loss in lift, etc for "Australia II"

- winglets are lifting surfaces that generate lift such that the associated induced velocities cancel the induced flow field caused by the wing-tip itself.

- Only a small tip vortex from the winglet tip remains (in a good design)

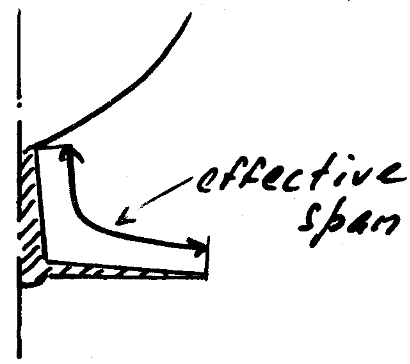

PRIMARY WINGLET
LEADING EDGE
WINGTIP
WING
SECONDARY WINGLET
SIDE VIEW
FRONT VIEW

33.

WINGLETS

- The lift-effectiveness of a keel-winglet configuration can be assessed by taking an effective span equal to the girth of the keel-winglet in cross-section

- For "Australia II" the lift was increased by 25% (leading to a reduction in lateral area) by adding winglets.



- Winglets allow more ballast to be carried

- Winglets are difficult to design. Design variables are
  - span and chord
  - taper ratio
  - dihedral
  - anhedral
  - camber and twist

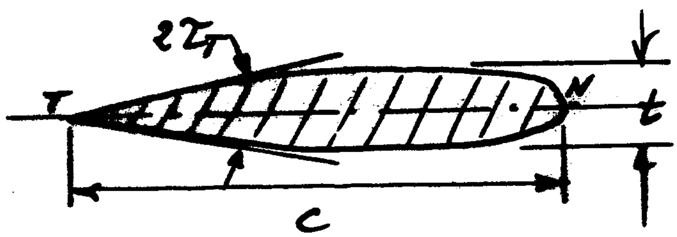
- The large span winglets seen in Fremantle in 1987 was a move to let the winglets produce more than 50% of the keel side force. Is only successful when wetted area is less important.

5 Section shapes of keels, rudders and winglets

- Requirements:
  - sufficient volume for ballast
  - " strength
  - avoid boundary layer separation
  - obtain a high- lift-drag ratio

- Boundary layer behaviour and a high lift-curve slope are mainly controlled by

  - the thickness-chordlength ratio $\frac{t}{c}$
  - half trailing edge angle τ_T



$$C_L = 2\pi\left[1 + 0.82\frac{t}{c} - \tan\tau_T\left(\frac{0.117}{t/c} + 3.2\frac{t}{c} + 3.9\left(\frac{t}{c}\right)^2\right)\right]\alpha$$

this is an accurate formula of a 2-dimensional section (wing with infinite aspect ratio) for C_L

α = angle-of-attack (radians)

τ_T = half trailing-edge angle (radians)

• for symmetrical section-shapes

$$C_D = C_F\left[1 + 1.2\frac{t}{c} + 70\left(\frac{t}{c}\right)^4\right]\left(1 + C_L^2\right)$$

$$C_F = \frac{0.075}{\left[LOG(R_n) - 2\right]^2} - \frac{1800}{R_n}$$

$$R_n = \frac{V_B \cdot \bar{c}}{\nu}$$

V_B = boat speed (m/sec)

$\bar{c}$ = average chord length (m)

ν = kinematic viscosity
(1.191×10^{-6} for seawater at 15°C)

• for $R_n < 5 \times 10^5$ laminar flow prevails for which

$$C_F = \frac{1.328}{\sqrt{R_n}}$$

• for NACA 4-digit type section shapes with location of maximum thickness less than 30% of chord to the leading edge

$$C_D = C_F\left[1 + 2\frac{t}{c} + 60\left(\frac{t}{c}\right)^4\right]\left(1 + C_L^2\right)$$

- surfaces must be kept very smooth to maintain laminar flow near leading edge (even at high R_n)

- Very rough surfaces:

  C_f has a terminal value approx. equal to

$$C_{f_{ROUGH}} = 0.032 (k/\ell)^{0.2}$$

where k = height of surface roughness (average value)
ℓ = average chord length

- A completely laminar boundary layer leads to about 25% of the drag associated with a completely turbulent boundary layer (at equal R_n.

- At equal thickness, different sections can accommodate varying amounts of ballast

type of section	area coefficient
63-009	0.630
64-009	0.632
65-009	0.651
66-009	0.681
0012	0.715

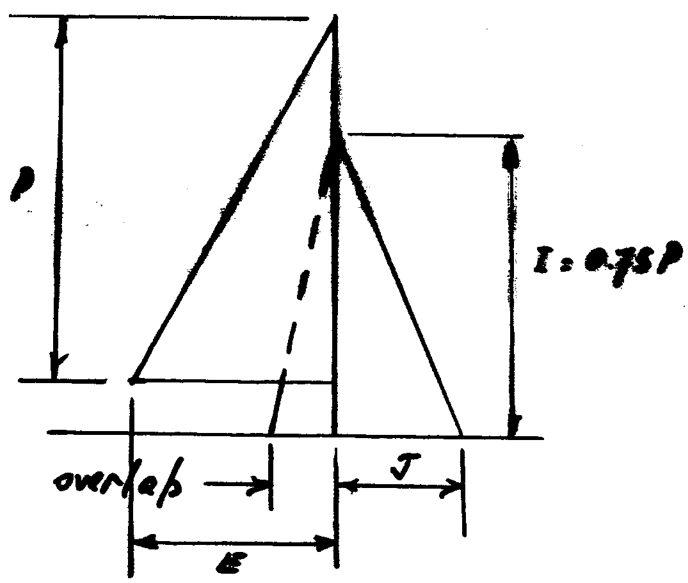
DESIGN EXAMPLE

CONSIDER THE DESIGN ACCORDING TO A "MODERN" 12 METRE RULE

$$\frac{L + \sqrt{S} - F}{2.37} = 12$$

where L = length of hull 0.18 m above measurement WL (for which no crew, sails and miscellaneous gear are not on board)

S = measured sail area
$= P.E/2 + I.J/2$



with $P_{max} = 25\,m$

$I_{max} = 0.75 * 25 = 18.75\,m$

overlap = 3.8 m (max) - not rated

and F = average freeboard
= 1.25 m maximum for use in formula

In addition:

$$\nabla_c = (0.15 L_{WL} + 0.1)^3 \quad (m^3)$$

$$T_{max} = 0.16 L_{WL} + 0.5 \quad (m)$$

$$B_{max} = 3.6 m$$

$$L_{WL\,min} = 13.5 m$$

In the above formulas the existing 12 Metre rule requirements have been retained except for leaving G (girth) requirement out of main formula and for the ∇_c requirement. The above ∇_c formula allows smaller displacement volumes to be adopted.

Volume of keel, rudder/skeg (max) = 40% ∇_c
Here stability is not rated

For the sake of this example, let's approximate L_{rule} as follows:

$$L_{Rule} = L_{WL} + \frac{0.18}{\tan(\text{aft slope counter})} + \frac{0.18}{\tan(\text{fwd slope overhang})}$$

or

$$L = L_{WL} + \frac{0.36}{\tan(12.5°)} = L_{WL} + 1.624$$

REQUIREMENT: FIND OPTIMUM L_{WL}, B_{WL}, T_c, I, J, P, E, ∇_c, KEEL SPAN/CHORD, ETC, FOR

THE OPTIMUM ON-THE-WIND (HIGHEST VMG) POINT OF SAIL AND A TRUE WIND SPEED OF 14 KNOTS

STEP 1 : CALCULATION OF CANOE BODY CHARACTERISTICS

- MASS OF CREW + PERSONAL GEAR
  11 * 85 = 935 kg
- MASS OF SAILS ≈ 300 kg
- MISCELLANEOUS ITEMS ≈ 100 KG
  TOTAL = 1335 kg

- $\nabla_{crew/sails/misc.} = 1335/1025.9 = 1.301\ m^3$

- CHOOSE 3 (MEASUREMENT) L_{WL} VALUES:

  e.g. 13.50 (min), 14.50 AND 15.50m

L_{WL} (meas.)	∇_c (rule)	∇_c (racing trim)
13.50	9.596	10.897
14.50	11.775	13.076
15.50	14.261	15.562

- FOR EACH L_{WL} CHOOSE 3 (racing trim) B_{WL} VALUES, e.g. 3.00, 3.25 and 3.50 m

- now calculate sinkage and L_{WL} to racing trim WL, ASSUMING $C_{wp} \approx 0.70$

∇_c (racing trim)	B_{WL}	A_{wp}	$\Delta T \left(\frac{1.301}{A_{wp}}\right)$	L_{WL} $\left(+\frac{2\Delta T}{\tan(12.5)}\right)$
10.897	3.00	28.35	0.046	13.915
"	3.25	30.71	0.042	13.879

∇_c (Racing trim)	B_{WL}	A_{wp}	$\Delta T\left(\frac{1.301}{A_{wp}}\right)$	$L_{WL}\left(+\frac{2\Delta T}{\tan(12.5)}\right)$
13.076	3.00	30.45	0.043	14.888
"	3.25	32.99	0.039	14.852
	3.50	35.53	0.037	14.834
15.562	3.00	32.55	0.040	15.861
	3.25	35.26	0.037	15.834
	3.50	37.98	0.034	15.807

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- PRELIMINARY CHOICE OF C_p AND C_M
  - FROM C_p CHART, FOR HIGH $L/\nabla^{1/3}$ CURVE AND AT $F_N \approx 0.37$ $C_p \approx 0.54$
  - TAKE $C_M \approx 0.68$

- NOTE THAT $L_{WL}/\nabla_c^{1/3}$ FOR RACING TRIM WL

L_{WL} (RACING TRIM)	∇_c (r.t.)	$L_{WL}/\nabla_c^{1/3}$
13.915	10.897	6.276
13.879	10.897	6.260
13.852	10.897	6.248
14.888	13.076	6.319
14.852	13.076	6.304
14.834	13.076	6.297
15.861	15.562	6.353
15.834	15.562	6.342
15.807	15.562	6.331

- FROM $C_{B_c} = \dfrac{\nabla_c}{L_{WL} \cdot B_{WL} \cdot T_c}$ AND $C_p = \dfrac{C_B}{C_M}$

$$C_p \cdot C_M = \frac{\nabla_c}{L_{WL} \cdot B_{WL} \cdot T_c}$$

OR

$$T_c = \frac{\nabla_c}{C_p \cdot C_M \, L_{WL} \cdot B_{WL}}$$

L_{WL} (R.T.)	B_{WL}	V_C (R.T.)	T_C (m)	B_{WL}/T_C
13.915	3.000	10.897	0.711	4.219
13.879	3.250	10.897	0.658	4.939
13.852	3.500	10.897	0.612	5.719
14.888	3.000	13.076	0.797	3.764
14.852	3.250	13.076	0.738	4.404
14.834	3.500	13.076	0.686	5.102
15.861	3.000	15.562	0.891	3.367
15.834	3.250	15.562	0.824	3.944
15.807	3.500	15.562	0.766	4.569

- CALCULATION OF STABILITY CHARACTERISTICS.

• USING :

$$KB = T_C\left(\frac{5}{6} - \frac{C_B}{3C_{WP}}\right)$$

$$BM = \frac{0.036\, B_{WL}^2}{C_B . T_C}$$

Where $C_B = C_P . C_M = 0.54 * 0.68 = 0.367$

$C_{WP} = 0.70$

L_{WL} (R.T. value)	B_{WL}	T_C	KB	BM	KM	$KM+(T_M-T$
13.915	3.000	0.711	0.460	1.242	1.710	3.705
13.879	3.250	0.658	0.434	1.575	2.009	4.053
13.852	3.500	0.612	0.403	1.963	2.366	4.453
14.888	3.000	0.797	0.525	1.108	1.633	3.699
14.852	3.250	0.738	0.486	1.404	1.890	4.011
14.834	3.500	0.686	0.452	1.752	2.204	4.375
15.861	3.000	0.891	0.587	0.991	1.578	3.707
15.834	3.250	0.824	0.543	1.257	1.800	3.993
15.807	3.500	0.766	0.504	1.569	2.073	4.321

- IT IS BEST TO CALCULATE KM RELATIVE TO A UNIFORM BASE LINE AND NOT RELATIVE TO THE BOTTOM OF THE CANOE BODY ; e.g. RELATIVE TO LINE THROUGH BOTTOM OF KEEL.

$$T_{MAX} = 0.16 L_{WL} + 0.5$$ (relative to measurement waterline)

where L_{WL} = waterline length on measurement draught

L_{WL} (racing trim)	L_{WL} (meas. trim)	T_{max} (meas trim)	T_{max} (racing trim)	$T_{max} - T_c$
13.9[illegible]	13.500	2.660	2.706	1.995
13.879	13.500	2.660	2.702	2.044
13.852	13.500	2.660	2.699	2.087
14.888	14.500	2.820	2.863	2.066
14.852	14.500	2.820	2.859	2.121
14.834	14.500	2.820	2.857	2.171
15.861	15.500	2.980	3.020	2.129
15.834	15.500	2.980	3.017	2.193
15.807	15.500	2.980	3.014	2.248

NOTE THAT $(T_{MAX} - T_C)$ = maximum height of keel!

- STEP 2 : CALCULATION OF SAIL AREAS
  - ON ASSUMING THAT THE FREEBOARD AS DEFINED BY THE RULE = 1.25 m, S FOLLOWS FROM :

$$S = (12 * 2.37 + 1.25 - L_{RULE})^2$$

  - ON ADOPTING MAXIMUM I AND P

$$18.75 J + 25 E = 25$$

• ON USING $E/J = 1.3$

$$18.75\frac{E}{1.3} + 25E = 25$$

$$\text{OR } E = \frac{25}{39.423} \longrightarrow S = 0.05075$$

$$\& \quad J = 0.039025$$

with $L_{RULE} = L_{WL}\left(\begin{smallmatrix}\text{meas.}\\ \text{trim}\end{smallmatrix}\right) + 1.624$

L_{WL}	L_{RULE}	S	E	J
13.50	15.124	212.17	10.757	8.279
14.50	16.124	184.04	9.331	7.181
15.50	17.124	157.90	8.006	6.161

• TO FIND HOW THESE VALUES RELATE TO EXISTING DESIGNS, CALCULATE

S_A/S_C AND $S_A/(L_{WL} \cdot \nabla_C^{1/3})$

where $S_C = \left(1.97 + 0.171\frac{B_{WL}}{T_C}\right)\sqrt{\nabla_C \cdot L_{WL}}$

L_{WL} (racing trim)	∇_C (racing trim)	$\frac{B_{WL}}{T_C}$	S_C	S_A/S_C	$S_A/(L_{WL} \cdot \nabla_C^{1/3})$
13.915	10.897	4.219	33.142	6.402	6.878 "LIGHT AI
13.879	10.897	4.939	34.613	6.130	6.895
13.852	10.897	5.719	36.218	5.858	6.909
14.888	13.076	3.764	36.467	5.047	5.247
14.852	13.076	4.404	37.948	4.850	5.260
14.834	13.076	5.102	39.588	4.649	5.266
15.861	15.562	3.367	39.996	3.948	3.987
15.834	15.562	3.944	41.511	3.804	3.994
15.807	15.562	4.569	43.151	3.659	4.001 "HEAVY AIR"

FROM STATISTICS, FOR $\frac{S_A}{S_C} > 4.0$ GOOD LIGHT AI PERFORMANCE

FOR $\frac{S_A}{L_{WL} \cdot \nabla_C^{1/3}} < 5$ GOOD HEAVY AIR PERFORMANCE

STEP 3: APPROXIMATE VCG (KG) CALCULATION
(FOR USE IN EXAMPLE ONLY)

- STATISTICAL INFORMATION SUGGESTS THAT FOR GRP RACING YACHTS:

- MASS OF HULL AND DECK STRUCTURE

$$\Delta_{HULL/DECK} = 20\left(L_{OA} \cdot B_{OA} \cdot (T_c + F)\right)^{0.9} \quad (kg)$$

- MASS OF DECK EQUIPMENT

$$\Delta_{DECK\ GEAR} = 5\left(L_{OA} \cdot B_{OA}\right) \quad (kg)$$

- MAST, RIGGING, BOOM & POLE

$$\Delta_{MAST\ ETC} = 3.2\, S_A \quad (kg)$$

- VCG OF $\Delta_{HULL/DECK} \approx (T_c + F)/2 + (T_{MAX})$
- VCG OF $\Delta_{DECK\ GEAR} \approx F + T_c + (T_{MAX})$
- VCG OF $\Delta_{MAST\ ETC} \approx 0.45(F + P) + T_c + (T_{MAX})$
- VCG OF CREW/SAILS $\approx T_c + F + 1 + (T_{MAX})$

- ASSUME

$L_{OA} = L_{WL} + 5$

$B_{OA} = 3.6$

$F = 1.25$

L_{WL} (meas trim)	L_{OA}	B_{OA}	T_c	F	Δ Hull Deck	Δ Deck Gear	Δ Mast Etc
13.50	18.50	3.6	0.711	1.25	1605	333	679
13.50	18.50	"	0.658	"	1565	333	679
13.50	18.50	"	0.612	"	1530	333	679
14.50	19.50	"	0.797	"	1750	351	589
14.50	19.50	"	0.738	"	1705	351	589
14.50	19.50	"	0.686	"	1665	351	589
15.50	20.50	"	0.891	"	1905	369	505
15.50	20.50	"	0.824	"	1850	369	505
15.50	20.50	"	0.766	"	1805	369	505

L_{WL} (meas trim)	T_c	F	T_{MAX}	Δ Hull Deck	KG	Moment
13.50	0.711	1.25	2.706	1605	3.687	5918
13.50	0.658	"	2.702	1565	3.656	5722
13.50	0.612	"	2.699	1530	3.630	5554
14.50	0.797	"	2.863	1750	3.887	6802
14.50	0.738	"	2.859	1705	3.853	6569
14.50	0.686	"	2.857	1665	3.825	6369
15.50	0.891	"	3.020	1905	4.091	7793
15.50	0.824	"	3.017	1850	4.054	7499
15.50	0.766	"	3.014	1805	4.020	7260

L_{WL} meas trim	Δ Deck Gear	Moment	Δ Mast Etc	Moment	Δ Crew Etc	Moment
13.50	333	1554	679	10341	1335	7565
13.50	333	1535	679	10302	1335	7489
13.50	333	1519	679	10269	1335	7424
14.50	351	1723	589	9113	1335	7890
14.50	351	1701	589	9076	1335	7806
14.50	351	1662	589	9044	1335	7734
15.50	369	1904	505	7940	1335	8225
15.50	369	1879	505	7905	1335	8131
15.50	369	1856	505	7874	1335	8050

STEP 4: BEFORE FINALIZING VCG CALCULATION WE MUST SIZE KEEL AND RUDDER AND CALCULATE BALLAST DETAILS

- DEDUCE REQUIRED LATERAL AREA OF KEEL FROM FIGURE OR OTHERWISE (TO BE VERIFIED LATER);
- DO THE SAME FOR RUDDER:

L_{WR} (RACING TRIM)	$\sqrt{\frac{\text{KEEL SPAN}}{T_{MAX}}}$	A_{LK}	$\frac{A_{LK}}{T_{MAX}-T_c}$	$\frac{\bar{t}}{c}$	∇_K
13.915	0.859	4.89*	2.45	0.10	0.778
13.879	0.870	4.83*	2.36	"	0.740
13.852	0.879	4.78+	2.29	"	0.711
14.885	0.849	5.24+	2.54	"	0.866
14.852	0.861	5.17+	2.44	"	0.821
14.834	0.872	5.10+	2.35	"	0.779
15.861	0.840	5.54Δ	2.60	"	0.935
15.834	0.853	5.45Δ	2.49	"	0.884
15.807	0.864	5.38Δ	2.39	"	0.835

* $A_{LK} * \sqrt{\frac{\text{KEEL SPAN}}{T_{MAX}}} = 4.2$

\+ " $= 4.45$

Δ " $= 4.65$

NOTES: - AVERAGE KEEL CHORD $= \frac{A_{LK}}{T_{MAX} - T_c}$

- $\nabla_k \approx \bar{C}_k^2 \cdot \frac{\bar{t}}{c} \cdot 0.65 \cdot \text{span}$

- NOTE THAT ∇_K could be increased until $\nabla_K + \nabla_R = 20\% \; \nabla_c$. This large keel volume could not be utilized, however (for ballast).

L_{WL} (RACING TRIM)	∇_c	∇_k	∇_R	∇_{tot}	Δ_{tot}	$\Delta_{ballast}$	$\nabla_{ballast}$
13.915	10.897	0.778	0.091	11.766	12071	8119	0.718
13.879	10.897	0.740	0.091	11.728	12032	8120	0.719
13.852	10.897	0.711	0.091	11.699	12002	8125	0.719
14.888	13.076	0.866	0.103	14.045	14409	10384	0.919
14.852	13.076	0.821	0.103	14.000	14363	10383	0.919
14.834	13.076	0.779	0.099	13.954	14315	10375	0.918
15.861	15.562	0.935	0.115	16.612	17042	12928	1.144
15.834	15.562	0.884	0.111	16.557	16986	12927	1.144
15.807	15.562	0.835	0.107	16.504	16931	12917	1.143

NOTE

$$\Delta_{ballast} = \Delta_{tot} - \Delta_{\substack{HULL\\DECK}} - \Delta_{\substack{DECK\\GEAR}} - \Delta_{\substack{MAST\\ETC}} - \Delta_{\substack{CREW\\ETC}}$$

$$\nabla_{ballast} = \frac{\Delta_{ballast}}{11300}$$

density of lead = 11300 kg/m^3

- the thickness of some of the keels needs to be increased to accommodate the amount of ballast

L_{WL}	$\nabla_{ballast}$	∇_{keel}	$\frac{t}{c}$ keel	∇_{tot}	Δ_{total}
13.915	0.718	0.778	0.0920	11.706	12009
13.879	0.719	0.740	0.0972	11.707	12010
13.852	0.719	0.711	0.1011	11.707	12010
14.888	0.919	0.866	0.1061	14.098	14463
14.852	0.919	0.821	0.1119	14.098	14463
14.834	0.918	0.779	0.1178	14.093	14458
15.861	1.144	0.935	0.1224	16.821	17257
15.834	1.144	0.884	0.1294	16.817	17253
15.807	1.143	0.835	0.1369	16.812	17247

this new value for Δ_{tot} leads to a 2nd iteration for $\Delta_{ballast}$.

LWL	Δ ballast	KG ballast	MOMENT
13.915	8057	0.9975	8037
13.879	8098	1.022	8276
13.852	8133	1.044	8487
14.888	10438	1.033	10782
14.852	10483	1.061	11117
14.834	10518	1.086	11417
15.861	13143	1.065	13991
15.834	13194	1.097	14467
15.807	13233	1.124	14874

NOTE: KG ballast is assumed to be equal to $(T_{max} - T_c)/2$. THIS WILL DEPEND ON TAPER RATIO, THICKNESS DISTRIBUTION, ETC.

- KG OF TOTAL CONFIGURATION CAN NOW BE CALCULATED FROM:

$$KG = \frac{\sum \text{MOMENTS}}{\Delta_{total}}$$

LWL	Σ MOMENTS	Δ total	KG (m)	KM−KG
13.915	33415	12009	2.782	0.923
13.879	33324	12010	2.775	1.278
13.852	33253	12010	2.769	1.684
14.888	36310	14463	2.511	1.188
14.852	36269	14463	2.508	1.503
14.834	36246	14458	2.507	1.868
15.861	39853	17257	2.309	1.398
15.834	39881	17253	2.312	1.681
15.807	39914	17247	2.314	2.007

NOTE: $GM = KM - KG$

RIGHTING MOMENT $\approx \Delta_{tot} \cdot GM \cdot \sin\theta$

STEP 5 SAIL FORCE, RESISTANCE AND SIDE FORCE CALCULATIONS

- FOR ON-THE-WIND CONDITIONS MOST DISPLACEMENT-TYPE YACHTS WILL REACH SPEEDS BETWEEN $F_N = 0.325$ AND $F_N = 0.375$.

- THEREFORE DO CALCULATIONS FOR 3 SPEEDS, VIZ:

  $F_N = 0.325$, 0.35 AND 0.375

- THEN $V_B = F_N * \sqrt{9.81 * L_{WLRT}}$ m/sec

- WETTED AREA OF CANOE BODY IS:

$$S_{CB} = \left(1.97 + 0.171 \frac{B_{WL}}{T_c}\right) * \sqrt{\nabla_{CB} * L_{WLRT}}$$

(all entities relative to racing trim WL)

- WETTED AREA OF KEEL AND RUDDER:

$$S_K \cong 2 * (T_K - T_c) * \bar{c}_K$$

T_K = DRAUGHT TO BOTTOM OF KEEL
T_c = " " " " CANOE BODY
$\bar{c}_K$ = average chord length keel

$$S_R \cong 2 * (T_R - T_{CR}) * \bar{c}_R$$

T_R = DRAUGHT TO BOTTOM OF RUDDER
T_{CR} = " " " " CANOE BODY AT LOCATION OF RUDDER
$\bar{c}_R$ = AVERAGE CHORD LENGTH OF RUDDER

• REYNOLDS NUMBERS:

$$R_{NCB} = V_B * L_{WLRT} / 0.000001191 \quad (15°C)$$

$$R_{NK} = VB * \bar{c}_K / 0.00000 1191$$

$$R_{NR} = VB * \bar{c}_R / 0.00000 1191$$

• FRICTION COEFFICIENTS:

$$C_{FCB} = \frac{0.075}{[0.43429 \, LN(R_{NCB}) - 2]^2} - \frac{1800}{R_{NCB}}$$

$$C_{FK} = \frac{0.075}{[0.43429 \, LN(R_{NK}) - 2]^2} - \frac{1800}{R_{NK}}$$

$$C_{FR} = \frac{0.075}{[0.43429 \, LN(R_{NR}) - 2]^2} - \frac{1800}{R_{NR}}$$

• FRICTION RESISTANCE:

$$R_{FCB} = \frac{1025.9}{2} \cdot V_B^2 \cdot S_{CB} \cdot C_{FCB}$$

$$R_{FK} = \frac{1025.9}{2} \cdot V_B^2 \cdot S_K \cdot C_{FK}$$

$$R_{FR} = \frac{1025.9}{2} V_B^2 \cdot S_R \cdot C_{FR}$$

• VISCOUS RESISTANCE:

$$R_{VCB} \cong 1.1 \, R_{FCB}$$

$$R_{VK} = \left(1 + 1.2\left(\frac{\bar{t}}{\bar{c}}\right)_K + 70\left(\frac{\bar{t}}{\bar{c}}\right)_K^4\right) \cdot R_{FK}$$

$$R_{V_R} = \left(1 + 1.2\left(\frac{\bar{t}}{c}\right)_R + 70\left(\frac{\bar{t}}{c}\right)_R^4\right) \cdot R_{F_R}$$

where $\left(\frac{\bar{t}}{c}\right)_K, \left(\frac{\bar{t}}{c}\right)_R$ = thickness-chord length ratio of average section of keel, rudder

• NOW CALCULATE RESIDUAL RESISTANCE FOR UPRIGHT CONDITION FOR EACH SPEED USING DELFT POLYNOMIALS

$$RR325 = 85.16 - 315.C_p + 296.8\,C_p^2 + 0.5725.LCB + 0.093\,LCB^2 + 0.4526\,\frac{B_{WL}}{T_c} + 0.4662\,L_{WLRT}/\nabla_c^{1/3}$$

$$RR350 = 195.6 - 687.8\,C_p + 617\,C_p^2 + 1.009\,LCB + 0.1476\,LCB^2 + 0.464\,\frac{B_{WL}}{T_c} + 0.6776\,L_{WLRT}/\nabla_c^{1/3}$$

$$RR375 = 272.8 - 901.2\,C_p + 777.1\,C_p^2 + 1.54\,LCB + 0.2142\,LCB^2 + 0.3431\,\frac{B_{WL}}{T_c} + 0.3463\,L_{WLRT}/\nabla_c^{1/3}$$

where $C_p = 0.54$

$LCB = -3.5$

• NOW CALCULATE 2D LIFT SLOPE:

$$\frac{\partial L}{\partial \beta}_{2D\,K} = 2\pi\left(1 + 0.82\left(\frac{\bar{t}}{c}\right)_K - \tan(\tau_{T_K})\left(\frac{0.117}{\left(\frac{\bar{t}}{c}\right)_K} + 3.2\left(\frac{\bar{t}}{c}\right)_K + 3.9\left(\frac{\bar{t}}{c}\right)_K^2\right)\right)$$

likewise for rudder

$$\tau_{T_E} \approx 1.05\left(\frac{t}{c}\right) \quad (\text{NACA63A})$$

- CALCULATE ASPECT RATIO'S (EFFECTIVE)

$$AR_K = 2(T_K - T_C)/\bar{C}_K$$

$$AR_R = (T_R - T_{C_R})/\bar{C}_R$$

$$AR_{CB} \cong T_C/(0.75\, L_{WL_{RT}}) \quad \left(= \frac{T_C^2}{0.75\, T_C\, L_{WL_{RT}}}\right)$$

- CALCULATE 3D LIFT CURVE SLOPES:

$$\left(\frac{\partial L}{\partial \beta}\right)_{3D_K} = \left(\frac{\partial L}{\partial \beta}\right)_{2D_K} \cdot AR_K / \left(\frac{\partial L}{\partial \beta}_{2D_K} / \pi + \cos\Lambda_K \sqrt{AR_K^2 / \cos\Lambda_K^4 + 4} \right)$$

i.e

$$\left(\frac{\partial L}{\partial \beta}\right)_{3D_K} = \frac{\left(\frac{\partial L}{\partial \beta}\right)_{2D_K} \cdot AR_K}{\left(\frac{\partial L}{\partial \beta}\right)_{2D_K} / \pi + \cos\Lambda_K \sqrt{\frac{AR_K^2}{\cos\Lambda_K^4} + 4}}$$

LIKEWISE FOR RUDDER

- THE FOLLOWING CALCULATIONS NOW FORM PART OF AN ITERATION PROCEDURE TO DETERMINE HEEL AND LEEWAY ANGLES
- PUT HEEL1 = 0 AND LEEWAY1 = 0

REPEAT:

- RESIDUAL RESISTANCE

RRC = RR325, RR350 OR RR375

$$RR = RRC/1000 * \nabla_{CB} * 9.81 * 1025.9/\cos^2(heel1)$$

• CALCULATE SIDE FORCE "COEFFICIENTS"
(i.e. coefficient of leeway angle)

$$SIDEFORCECOEF_{CB} = \frac{1025.9}{2} V_B^2 \left[\frac{\pi}{2} T_c^2 + 1.8\, T_c\, LWL_{RT} * 0.75\, LEEWAY1 \right]$$

$$SIDEFORCECOEF_{K} = \frac{1025.9}{2} V_B^2 \left(\frac{\partial L}{\partial \beta}\right)_{3D_K} \cdot (T_K - T_c)\, \bar{c}_K \cdot \cos(HEEL1) * \left(1 + T_c / T_K\right)^2$$

$$SIDEFORCECOEF_{R} = \frac{1025.9}{2} V_B^2 \left(\frac{\partial L}{\partial \beta}\right)_{3D_R} \left(T_R - T_{c_R}\right) \bar{c}_R \cos(HEEL1)$$

• CALCULATE INDUCED RESISTANCE:

$$RI_K = \frac{2 * 1025.9\, V_B^2 (T_K - T_c)\, \bar{c}_K\, \pi\, AR_K \cdot LEEWAY1^2}{\left(2 + \sqrt{AR_K + 4}\right)^2}$$

$$R_{IR} = \frac{2 * 1025.9\, V_B^2 (T_R - T_{c_R})\, \bar{c}_R\, \pi\, AR_R\, LEEWAY1^2}{2 + \sqrt{AR_R + 4}}$$

$$R_{ICB} = 2 * 1025.9\, V_B^2 * 0.75\, T_c\, LWL_{RT}\, \pi\, T_c^2 / (0.75\, T_c\, LWL_{RT}) * LEEWAY1^2 / \left(2 + \sqrt{T_c^2 / (0.75\, T_c\, LWL_{RT}) + 4}\right)^2$$

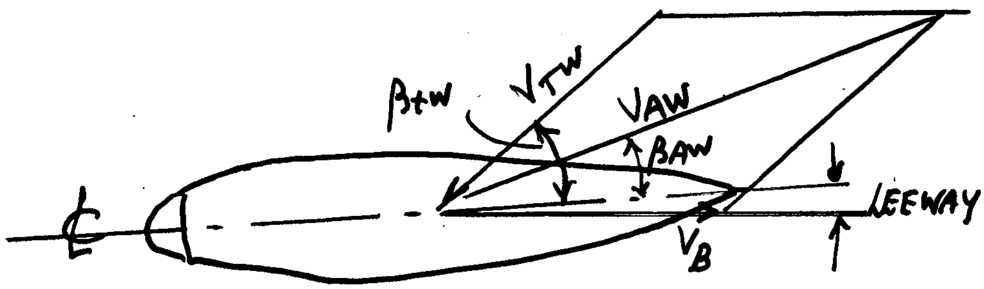
CALCULATE TOTAL RESISTANCE

$$R_{TOT} = R_{VCB} + R_{VK} + R_{VR} + R_{IK} + R_{IR} + R_{ICB} + RR$$

• NOW CALCULATE WIND SPEEDS AND ANGLES

$$V_{AW} = \sqrt{\left[V_{TW} \sin(\beta_{tw} + \text{leeway}_1)\right]^2 + \left[V_{TW} \cos(\beta_{tw} + \text{leeway}_1) + V_B\right]^2}$$

$$\beta'_{AW} = \tan^{-1}\left[V_{TW} \sin(\beta_{tw} + \text{leeway}_1) / V_{AW}\right]$$



V_{TW} = true wind speed (14 knots)
V_{AW} = apparent wind speed
β_{tw} = true wind angle (to be input)
β_{AW} = apparent wind angle
V_B = boat speed (equal to $F_N = 0.325$, 0.350 or 0.375)

NOTE: $\beta_{AW}' = \beta_{AW} + \text{LEEWAY}$

• CALCULATE SAIL AREAS

$$GEA = I * (J + 3.8)/2$$
$$MSA = P * E/2 * 1.15$$

where $I = 18.75$ m
$P = 25.00$ m

note: – overlap = 3.8
– effect of roach ≈ $1.15 * P.E/2$

• CALCULATE SAIL COEFFICIENTS:

AGAIN:

FLAT = 1

FLATITER:

$$CL = FLAT\left[\left(0.041\beta'_{AW}*\beta'_{AW} + 0.344\beta'_{AW} + 1.04\right)GEA + \left(2.357\beta'_{AW} - 1.073\beta'_{AW}*\beta'_{AW}\right)MSA\right]$$

$$CD = 0.044 + \frac{0.089\,CL^2}{(GEA+MSA)(1-HEEL1)}$$

• CALCULATE SAIL DRIVING FORCE AND SAIL HEELING FORCE:

$$RSD = \frac{1.226}{2}V_{AW}^2\left[CL\sin\beta'_{AW} - CD\cos\beta'_{AW}\right]$$

$$RSH = \frac{1.226}{2}V_{AW}^2\left[CL\cos\beta'_{AW} + CD\sin\beta'_{AW}\right]\cos(HEEL1)$$

• CALCULATE HEEL ANGLE

$$HEEL2 = \tan^{-1}\left[\frac{RSH*H}{\nabla_{tot}*9.81*1025.9\,GM}\right]$$

where H = distance between C.E. of sailplan and C.L.R. of hull

GM = metacentric height

• IF HEEL2 > 30° then

FLAT = 0.99 * FLAT

GOTO "FLATITER"

• IF |HEEL2 - HEEL1| > 0.00175 then

HEEL1 = HEEL2

GOTO "AGAIN"

- ON HAVING OBTAINED CONVERGENCE FOR FLAT AND HEEL, PROCEED TO CALCULATE LEEWAY:

$$LEEWAY2 = \frac{RSH - SIDEFORCECOEFR * \delta_R * \cos(HEEL)}{[SIDEFORCECOEFCB + SIDEFORCECOEFK + SIDEFORCECOEFR)} * \frac{1}{\cos(HEEL)}$$

IF $|LEEWAY2 - LEEWAY1| > 0.00017$ then

LEEWAY1 = LEEWAY2
GOTO "REPEAT"

INPUTS USED

PARAMETER	1	2	3	4	5	6	7	8	9
I	18.75	18.75	18.75	18.75	18.75	18.75	18.75	18.75	18.75
J	8.279	8.279	8.279	7.181	7.181	7.181	6.161	6.161	6.161
P	25	25	25	25	25	25	25	25	25
E	10.757	10.757	10.757	9.331	9.331	9.331	8.006	8.006	8.006
H	12.08	12.08	12.08	12.16	12.16	12.16	12.24	12.24	12.24
L_{WLRT}	13.915	13.874	13.852	14.888	14.852	14.834	15.861	15.834	15.807
B_{WL}	3.00	3.25	3.50	3.00	3.25	3.50	3.00	3.25	3.50
T_c	0.711	0.658	0.612	0.797	0.738	0.686	0.891	0.824	0.766
C_p	0.54	0.54	0.54	0.54	0.54	0.54	0.54	0.54	0.54
LCB	-3.5	-3.5	-3.5	-3.5	-3.5	-3.5	-3.5	-3.5	-3.5
∇_{CB}	10.897	10.897	10.897	13.076	13.076	13.076	15.562	15.562	15.562
∇_{tot}	11.706	11.707	11.707	14.098	14.098	14.093	16.821	16.817	16.812
$T_k - T_c$	1.995	2.044	2.087	2.066	2.121	2.171	2.129	2.193	2.248
$\bar{c}_k$	2.45	2.36	2.29	2.54	2.44	2.35	2.60	2.49	2.39
$T_R - T_{cR}$	2.19	2.19	2.19	2.28	2.28	2.25	2.37	2.34	2.31
$\bar{c}_R$	0.73	0.73	0.73	0.76	0.76	0.76	0.79	0.78	0.77
$(\bar{t}/c)_k$	0.0928	0.0972	0.1011	0.1061	0.1119	0.1178	0.1224	0.1294	0.136
$(t/c)_R$	0.12	0.12	0.12	0.12	0.12	0.12	0.12	0.12	0.12
τ_{TK}	5.57	5.83	6.06	6.36	6.70	7.05	7.32	7.74	8.18
τ_{TR}	7.18	7.18	7.18	7.18	7.18	7.18	7.18	7.18	7.18

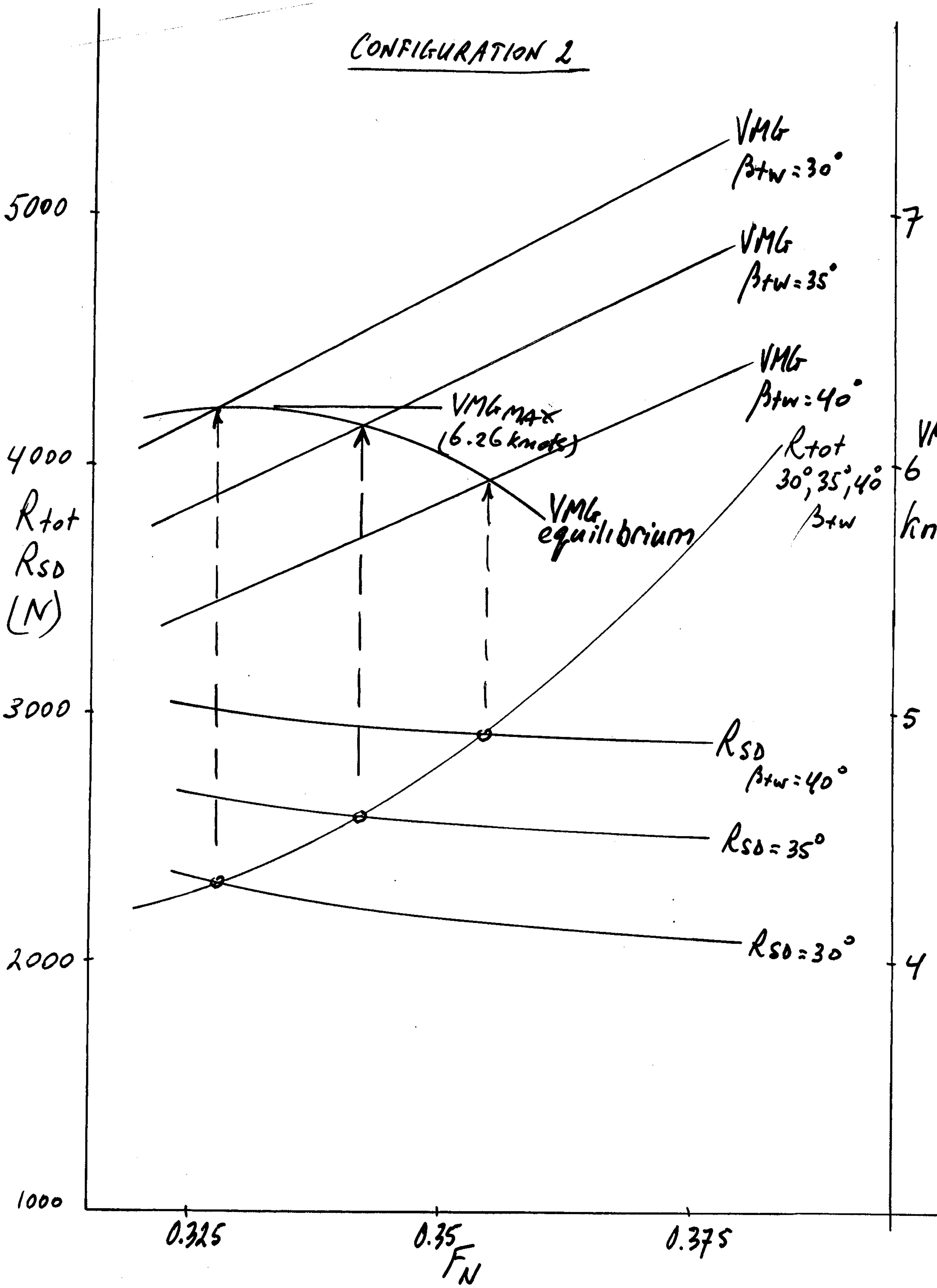
INPUTS USED (CONT)

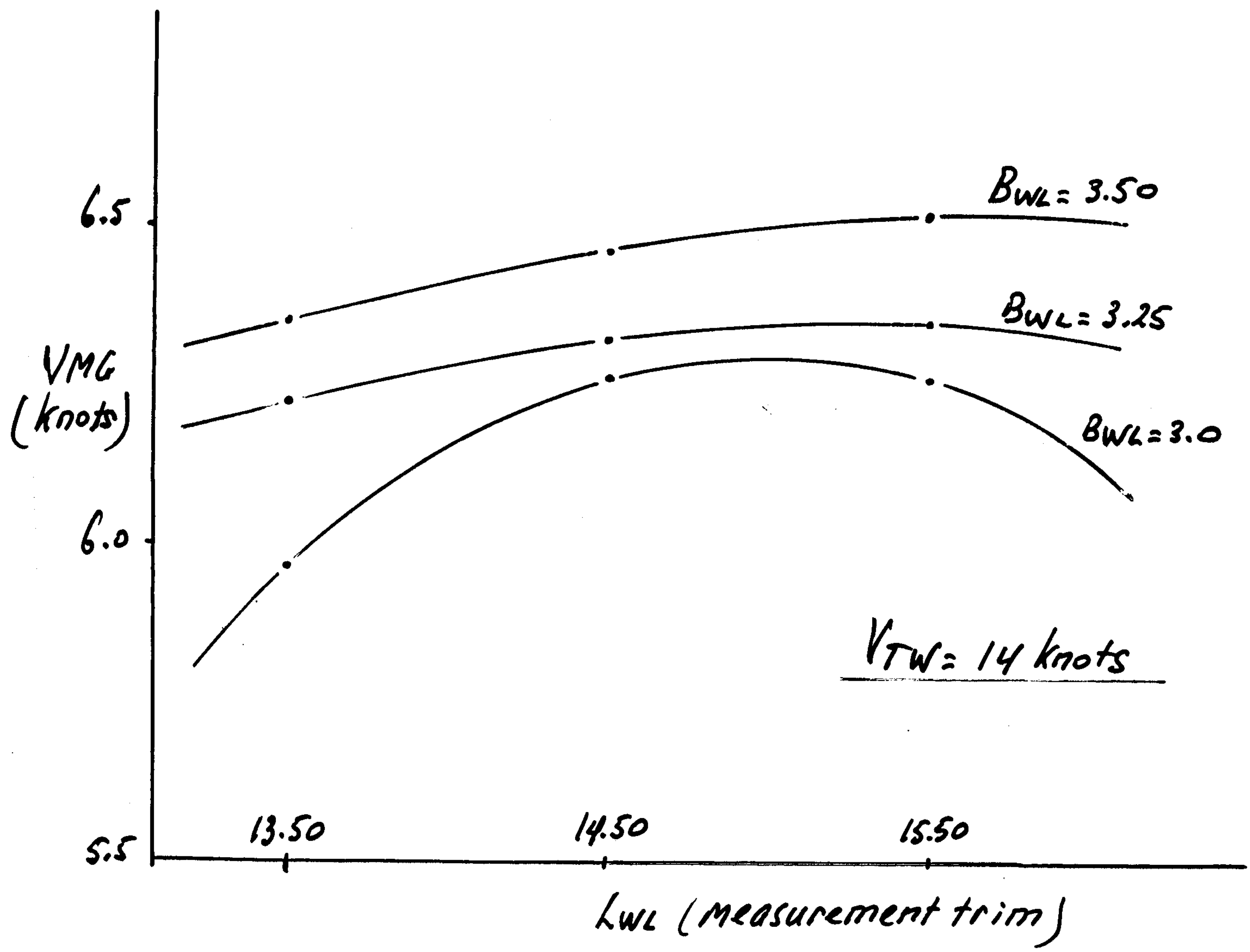
	1	2	3	4	5	6	7	8	9
Ω_K	0	0	0	0	0	0	0	0	0
Ω_R	0	0	0	0	0	0	0	0	0
δ_R	4	4	4	4	4	4	4	4	4
GM	0.923	1.278	1.684	1.188	1.503	1.868	1.398	1.681	2.007

RESULTS FOR $F_N = 0.35$; $\beta_{tw} = 35°$

	1	2	3	4	5	6	7	8	9
R_{TOT}	2579	2795	3086	3129	3379	3717	3749	3906	3859
C_L	0.328	0.457	0.600	0.552	0.694	0.866	0.840	0.942	0.942
C_D	0.0201	0.0389	0.0668	0.0568	0.0898	0.139	0.131	0.154	0.141
R_{SD}	1947	2581	3199	2618	3074	3515	2956	3200	3440
R_{SH}	5154	7187	9445	7953	10033	12567	11083	12573	12877
HEEL	30	30	30	30	30	30	30	28.7	25.08
LEEWAY	1.413	2.415	3.511	2.185	3.035	4.086	2.788	3.254	3.168
OPT. VMG	5.96	6.26	6.26	6.22	6.32	6.35	6.35	6.46	6.52

- RESULTS FOR VMG_{MAX} WERE OBTAINED BY SETTING OUT R_{tot}, R_{SD} AGAINST F_N AND GRAPHICALLY DETERMINING FOR WHICH F_N $R_{tot} = R_{SD}$ AT β_{tw} = 30, 35 AND 40 DEGREES. FOR THESE F_N VALUES THE VMG VALUES WERE READ FROM THE VMG-β_{tw} CURVES, AND THE MAXIMUM VALUE FOUND; SEE SLIDE 125


CONFIGURATION 2
VMG β_{tw} = 30°
VMG β_{tw} = 35°
VMG β_{tw} = 40°
VMG MAX (6.26 knots)
VMG equilibrium
R_{tot} 30°, 35°, 40° β_{tw}
R_{SD} β_{tw} = 40°
R_{SD} = 35°
R_{SD} = 30°
R_{tot}
R_{SD}
(N)
5000
4000
3000
2000
1000
0.325
0.35
0.375
F_N
7
6
5
4
kn



CONCLUSION: MAXIMUM B_{WL} to be adopted for an L_{WL} (measurement trim) equal to ≈ 15.70m

4th YEAR STUDENT PROJECT/EXERCIZE

- DETERMINE OPTIMUM CHARACTERISTICS OF A YACHT FOR MAXIMUM SPEED-MADE-GOOD TO WINDWARD (VMG) WHEN $V_{TW} = 8$ KNOTS, ACCORDING TO THE FOLLOWING DESIGN RULE:

$$L + 1.25\sqrt{S} - 10\,DSP^{1/3} = 7.0$$

where L = length of canoe body at a waterline 0.085 m above the "measurement" FLOTATION WATERLINE (THE CASE WHEN NO CREW, SAILS AND SOME MISCELLANEOUS GEAR ITEMS ARE ON BOARD)*,

$$S = I.J/2 + P.E/2$$

in which I = 10.40 m or less
P = 13.00 m or less
J = FREE TO CHOOSE
E = " " "

(GENOA OVERLAP NEED NOT BE CONSIDERED)

$$DSP = \frac{W}{1025.9}$$

where: W = mass of total yacht in measurement trim condition (no crew, etc on board)

with: $W_{max} = 2000$ kg
$W_{min} = 1250$ kg

* The overhang slopes are to be no greater then $12\frac{1}{2}°$

- maximum draught to bottom of keel = 2.50m

- maximum beam anywhere on the hull = 2.50m

- minimum freeboard = 0.50m

- maximum crew weight = 320kg
- maximum weight of sails and miscellaneous GEAR ITEMS = 100 kg

- crew weight can be used to reduce heel. The moment arm of crew C.G from ℄ CAN BE NO MORE THAN 2m (equivalent to using a trapeze) however.

- Write a 2 to 3 page account of how you selected L, S AND DSP for optimum windward performance at $V_{TW} = 8$ knots. Include a table giving:
  I, J, P, E, L_{WL} (racing trim), B_{WL}, T_c, C_p, LCB, ∇_c, ∇_{total}, span of keel and rudder, average chord length of keel and rudder, thickness-chord length ratios of keel and rudder and GM (metacentric height). Performance values (speed, heel, leeway, etc) need not be given.
  NOTE: All canoe body parameters above refer to racing trim condition