RATIONAL MAST DESIGN.

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Abstract

The problem with the majority of mast design procedures is that they are based almost entirely on empiricism and observation. Such methods do not provide a clear path for progress because one can only produce slight improvements on the same theme. They are therefore not suited to yachts of a different nature - for example, it is not usually possible to design a safe multihull rig using keel yacht methods.

This paper outlines the way in which the authors have attempted to produce a more objective method that is not influenced by fashion or precedent in its evaluation of structural integrity. It is based on a computer model of the yacht which uses well established engineering techniques. The results have been very good and the programs have been successfully used in several practical applications.

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1. Introduction

In general the design of yacht masts has proceeded by a combination of simple analysis and experience. There are very few books or papers on the topic, but those that there are often refer to mast design as an art, or even as requiring a little black magic. The authors have developed an alternative approach which uses standard engineering design methods as much as possible - this paper outlines this procedure. The original work was carried out by the first author as a Masters thesis (Mitchell 1986). Since then it has been applied to many different yachts, most notably to those of two America's Cup syndicates.

Mast design falls naturally into a certain sequence. First the overall configuration is chosen, then the loads in the mast and rigging sections are found, leading in turn to the specification of the overall dimensions of each section. Finally the detailed design of the fittings and connections is attended to. This last step is relatively easy, as the required strength of each component is known at this point, so here we shall concentrate on determining the loads and primary dimensions of the mast and rigging.

2. Existing methods of mast design

The basic configuration and overall length of the rig is often known before their detailed design commences, as other factors like waterline length, sail plan, hull stability and class rules determine the rig plan required. The usual sequence is to arrive at the cross-section dimensions of the mast by considering its stability in compression, and then to calculate rigging strengths.

In the existing methods of mast design a basic estimate of mast compression is obtained using a moment balance of the form:

mast comp. x 1/2 chainplate beam = righting mom. at 30 deg of heel (1)

This estimate is then increased by various factors in order to compensate for further compression induced by halyards, fore and aft rigging, running rigging, pretensions in standing rigging and increases in angle of heel.

The final value of mast compression is then entered into a formula that estimates the second moments of area for the mast required to prevent buckling. The formula is easily recognised as Euler's equation for long slender columns; it is normally used for predicting elastic instability (so-called Euler buckling). Yacht designers usually use it in the following form:

sec. mom. area = mast compression x $(\text{panel length})^2$ x factor (2)

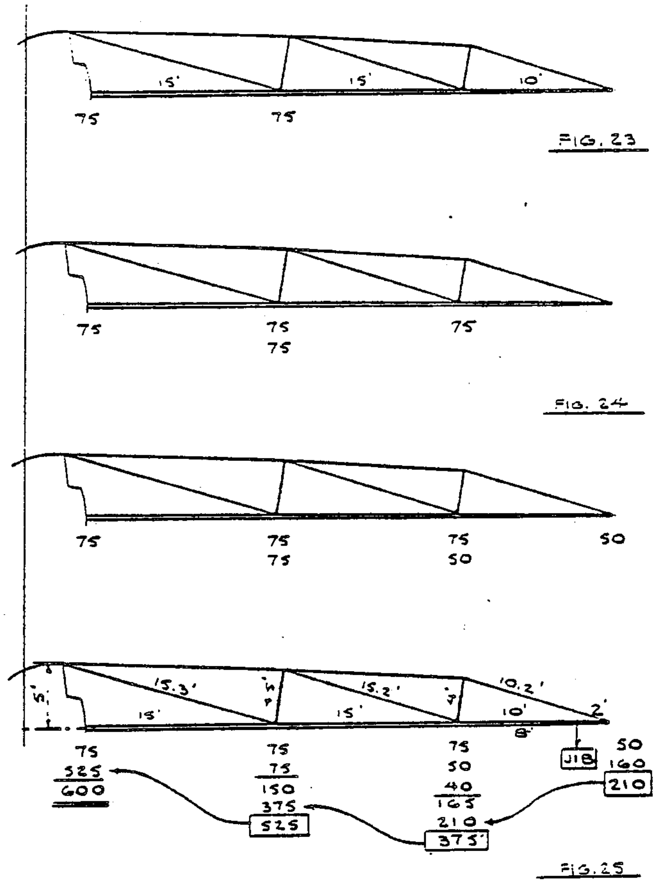
Here the panel length is the length of unsupported mast between any two supports. The value of this factor varies depending on the type of panel - that is, on its position up the mast, on the number of spreaders and whether it is a fore-aft panel or an athwartship panel. The factor also incorporates the elastic modulus of the material to be used.

Although this procedure is based on fundamental equations, it is only the addition of the empirical factors which makes it work. This is because Euler's equation was developed for concentrically loaded compression struts, while masts are eccentrically loaded, tapered, subject to bending and torsion, and have varying fixivity. The factors inserted into Euler's equation have been historically proven to provide sufficient safety margins. The method is therefore risky for less common yachts such as catamarans, and 12 metre yachts for which these factors have not been established.

The final step is to calculate rigging strengths. This is often done by calculating the rigging needed to support a lateral load distributed along the mast. The load distribution is often quite unrealistic, as discussed later. To simplify the analysis most texts (Kinney 1973) assume that the leeward rigging is slack, that the mast remains straight under load, and that the fore-aft and athwartships planes are considered separately. To make the analysis statically determinate it is also assumed that all connections are pin-jointed. The authors question whether any of the above simplifications are acceptable for application by professional naval architects. Figure 1 shows an example of this treatment.

A second and probably more common method of design is to specify the strength of rigging members as a percentage of the mast compression found in equation (1). For a variety of common rig configurations these factors have been found through full-scale measurements such as those by S.N.A.M.E. (1973) and also by using the first method to produce these percentages for common rig configurations. This second method is still very limited for development purposes since it relies on the new design being similar to the original.

Figures 23, 24, 25.
Shroud loads



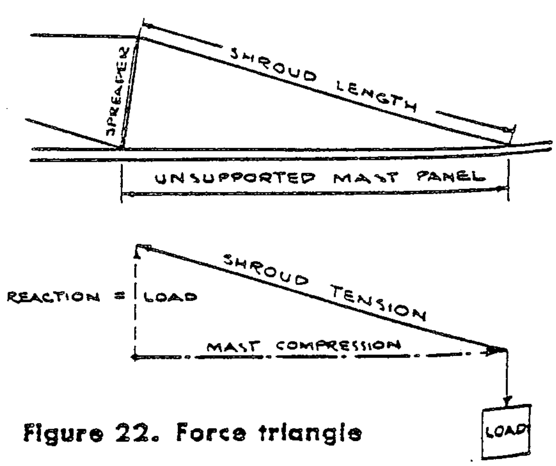


Figure 22. Force triangle

Figure 1 Traditional Shroud loading procedure (Kinney 1973)

3. Rational design of a mast

Masts should be designed using imagination, lateral thinking and reliable calculations. Collectively called 'engineering' the process involves mixing qualitative aspects such as looks, complexity and crew ability with quantitative aspects such as structural integrity, to meet the client's requirements and budget.

The computer aided rig analysis developed by the authors provides structural information for a given yacht sailing under certain load conditions. Rather than designing the optimal rig, it analyses a particular mast with speed and accuracy. This approach has been adopted because of the non-structural constraints mentioned above. The programs have therefore been written for ease of use as a design tool. In the process the calculations involved have been made more accurate.

It is suggested that designers use their imagination and experience to produce prototypes that are analysed by the programs. The results can then be used to see if improvements could be made to structural aspects such as size, weight and bend. In practice a designer can analyse several designs and compare the results.

The method followed by the authors is represented by Figure 2. Much more emphasis is given here on estimating the loads on the rig, as discussed below. The design loads are then applied to a computer model of the rig structure which outputs the rig deflections, and then calculates the load in each member. The deformed rig shape is then plotted, and member loads printed out. The data input is organised so that changes to the rig are easily made, so that the package performs well as a design tool. It has been used by two America's Cup syndicates, and is commercially available on a consultation basis (Advanced Engineering Services 1987).

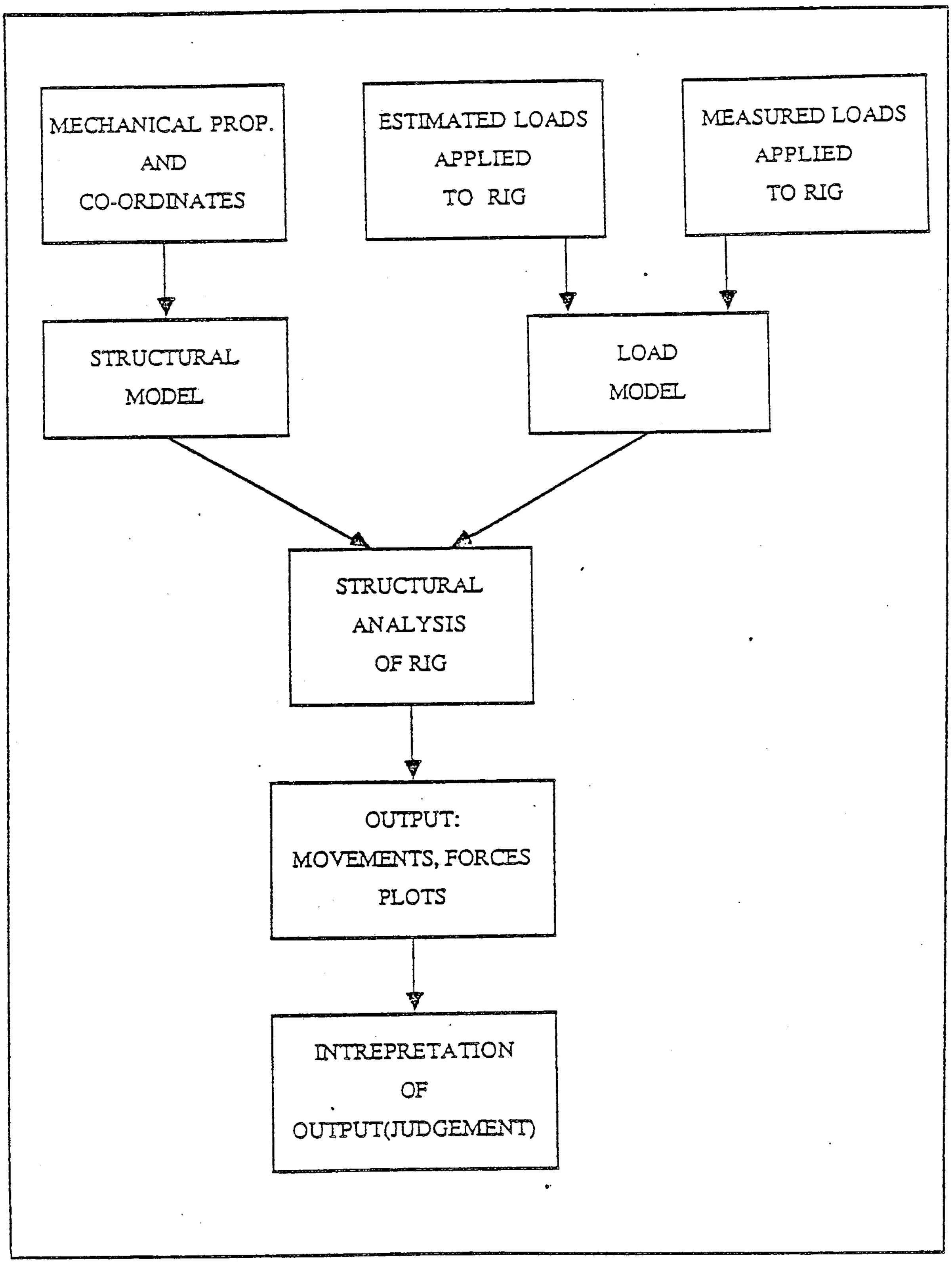


Figure 2 The analysis procedure.

3.1 Structural Analysis

The programs are new in that the incorporation of all their facilities have not been regularly applied to yacht rigs. However, the individual techniques are quite well established and comparatively old for the engineering community.

The analysis uses a three dimensional space frame modelling the rig by a series of one dimensional 'beam' elements. The elements are such that while deflections due to bending and elongation are calculated those due to shear are neglected. A modified Newton-Raphson approach is taken to solve the problem repeatedly, making use of the direct stiffness method of structural analysis. This allows two types of nonlinearity to be accounted for - large displacements or (geometric nonlinearity) and compressive elastic instability. Large displacements cause stay angles to alter to such an extent that the subsequent change in rig stiffness must be accounted for. In addition, large axial compression can erode flexural stiffness to the point of compressive elastic instability (Euler buckling). The bending stiffness of each element is therefore modified according to its axial load. Any stay that is found to be in compression is removed from the structural model. This can cause convergence difficulties in accounting for pretensions and nonlinearities. Fortunately these been overcome.

Elastic stability of the entire rig could be measured by the eigen-values of the stiffness matrix, but in practice it is found that the convergence behaviour of the modified Newton-Raphson method is a good indication of marginal elastic stability.

3.2 Load Model

Procuring accurate information on loads is often the most difficult part of the analysis. As yachts are sailed in many different directions and wind strengths it follows that there should actually be several different load models each covering some critical situation. The types of load acting on a rig can be classified in five ways;

(1) loads imparted by the sails. Sails act as membranes which cannot support bending moments nor compressive loads, so that the loads imparted to the mast equate in magnitude and direction to the tensile membrane stresses in the adjacent sail. These stresses in turn depend upon sail shape, the stiffness of the sail cloth, the nature of the edge restraints (sliding, fixed or free) and the aerodynamic pressure distribution.

(2) loads from the sail halyards. These appear as extra compressive loads on the mast.

(3) loads from pretensioning the rigging. These are prescribed as fixed extensions, as in the case of rigging screws, or as a fixed force as in the case of running backstays, (where the amount of cordage pulled through is not relevant).

(4) indirect sail loads. The main boom and spinnaker pole impart large loads to the mast.

(5) inertial loads. This component is not included in the analysis developed so far, but given typical yacht motion and the element masses, inertial loading could be added.

Loads in the first category are by far the most important, and to some extent the other loadings can be calculated from the membrane stresses (the halyard loads, for example). The first step towards a load model is therefore a membrane model of the sail itself. Rather oddly this basic step is ignored by most recognised works (e.g. Kinney 1973). The most common approach is to apply an assumed aerodynamic force distribution directly to the mast - this is normally approximately triangular in shape as shown in Figure 3 and implies a rigid sail able to support bending moments. Some analysts have tried to overcome the grossly inaccurate results that this produces by inventing unlikely pressure distributions on the sails. An example is due to A.B.S. (1982) where an 'upside down' triangular distribution is described with maximum lift produced at the top of the sail and zero lift at the bottom, as shown in Figure 4. In fairness, the American Bureau of Shipping were not recommending the method but were asking for other submissions.

The membrane stress model used by the authors essentially consists of a strip of high stress running from the top of the sail to the end of the boom (along the leech). In most respects it corresponds quite well with the reinforcing and structural design sailmakers have been using in sail design for a number of years. It is not based on a detailed structural analysis of the sail, but rather on a simple model which at least shows understanding of the way in which loads are carried and transferred by the sails. The stress magnitudes are determined by estimates of the mainsheet loads, and by consideration of the righting moments, centre of pressure and sail shapes.

The load model applies discrete forces to the mast throughout its length. The resultants of these forces resolve to the lateral direction that the sail cloth exits the luff track. The net moment of the sideways components of the forces multiplied by their respective heights above the centre of lateral resistance equals the yacht's righting moment. A number of other logical constraints are also observed, and when all combined they provide an accurate picture of the mast loading.

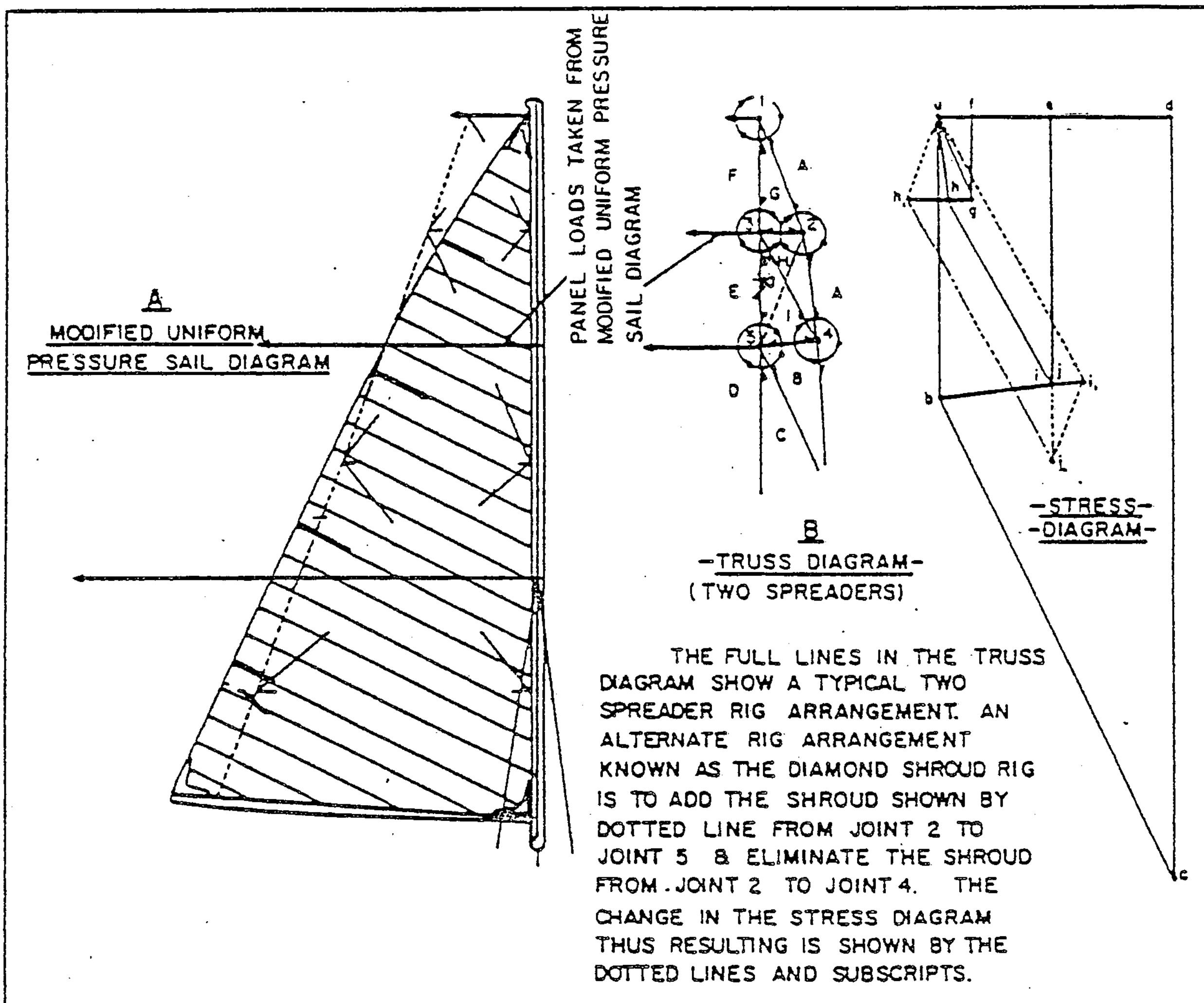


FIGURE 48. Rigging and mast stress calculation, truss diagram.

Figure 3 Example of Pressure Distribution applied to mast (Henry, 1965)

FIG. 1

F_{1m}, F_{2m} & F_{3m} are main sail loads on A_{1m}, A_2M and A_3M areas of main sail

V_1, V_2 & V_3 are compressive loads on mast panels

D_1, D_2, D_3, T_2 & T_3 are tensile loads in shrouds

S_1 & S_2 are compressive loads in spreaders.

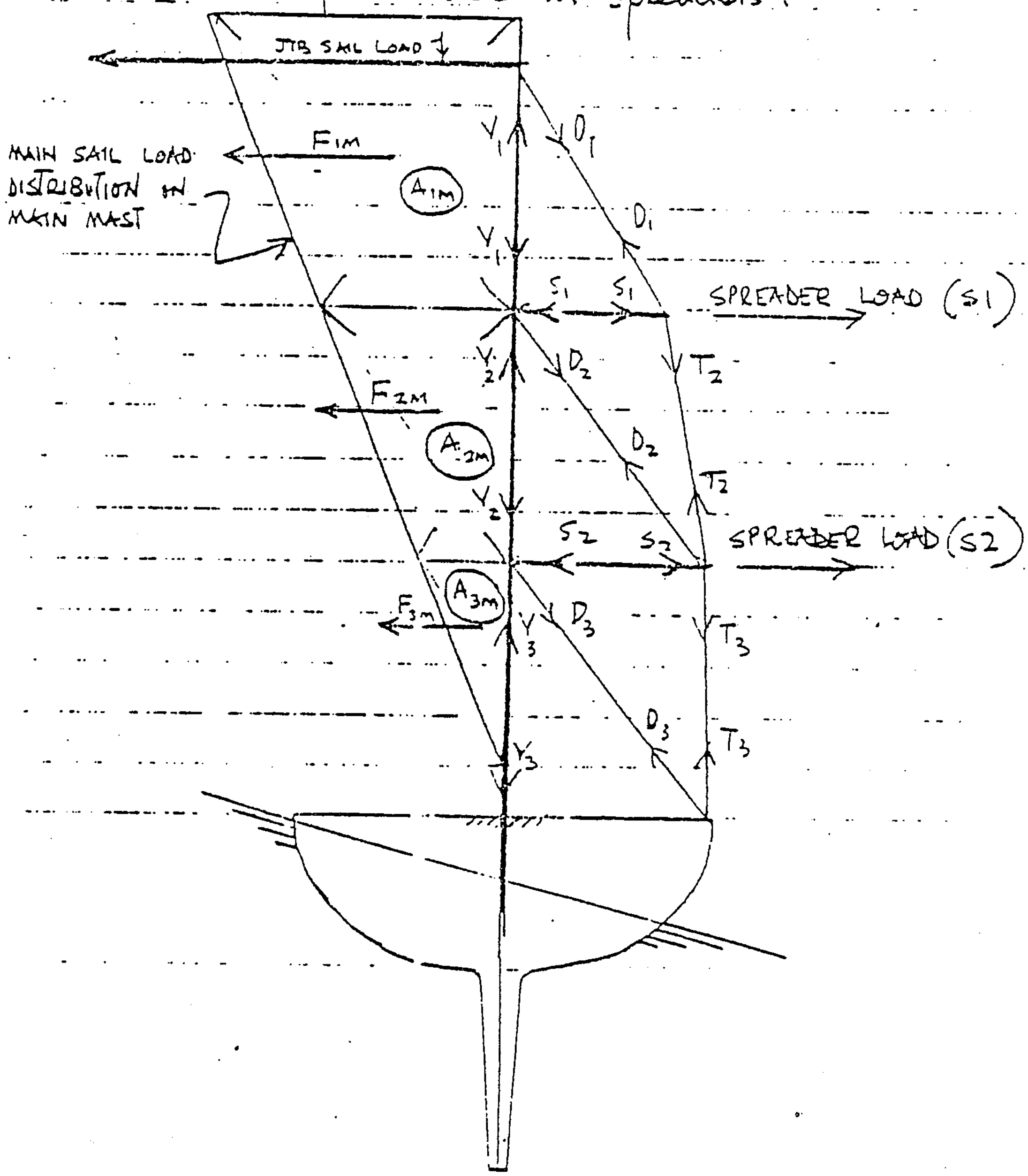


Figure 4 Example of an unlikely Pressure Distribution (A.B.S.1982)

4. Typical Results

The output of the programs is in a tabular form, conveying the deformed position of the rig and the resultant loads in its members. Separate outputs are obtained for each loading case ; that is, for close-hauled, reaching and running. The deformed rig shapes are also drawn by plotter. The plots are normally done with colour pens, the colours being used to indicate load levels. Colours are omitted in this paper.

Unfortunately, all of the America's Cup computer predictions and measured stay loads are confidential, but a typical analysis for a light displacement keel yacht is given below. The Ross40 was designed by Murray Ross (Auckland, New Zealand) who supplied design data to the authors for their use while developing the program. Figures 5 and 6 show two calculated views of the rig sailing to windward. Notice that the mast is bent, and that the leeward (left) running backstays are omitted. Table 1 compares the stay breaking strengths specified in the actual design, with the predicted stay loads at thirty degrees heel using a factor of safety of 3.0 (choosing an appropriate factor of safety is a topic beyond this paper). The terms 'V1' and 'D1' refer to vertical and diagonal rigging wires and should not be confused with the mast loads of Figure 4. Rigging nomenclature is noted on Figure 5 and 6.

Table 1 implies that the D2 member is overly strong. However, this analysis is under full sail, whereas in a fully reefed analysis the D2 load would be sharply higher. This illustrates that several analyses are required to gain a full picture of the rig loads. All the results were within the range of only one size graduation of rigging wire, but it is not known what the intended safety factor for the rig was. Many designers operate with factors less than 3.0.

Table 1

Rigging Element Name	Mitchell/Jackson Analysis Safety factor 3.0 kN	Actual Design Breaking Strengths kN (Navtec rod)
V1	47.	55. (Navtec #12)
D1	35.	46. (Navtec #10)
V2	42.	36. (Navtec #8)
D2	2.5	21. (Navtec #4)
D3	42.	36. (Navtec #8)

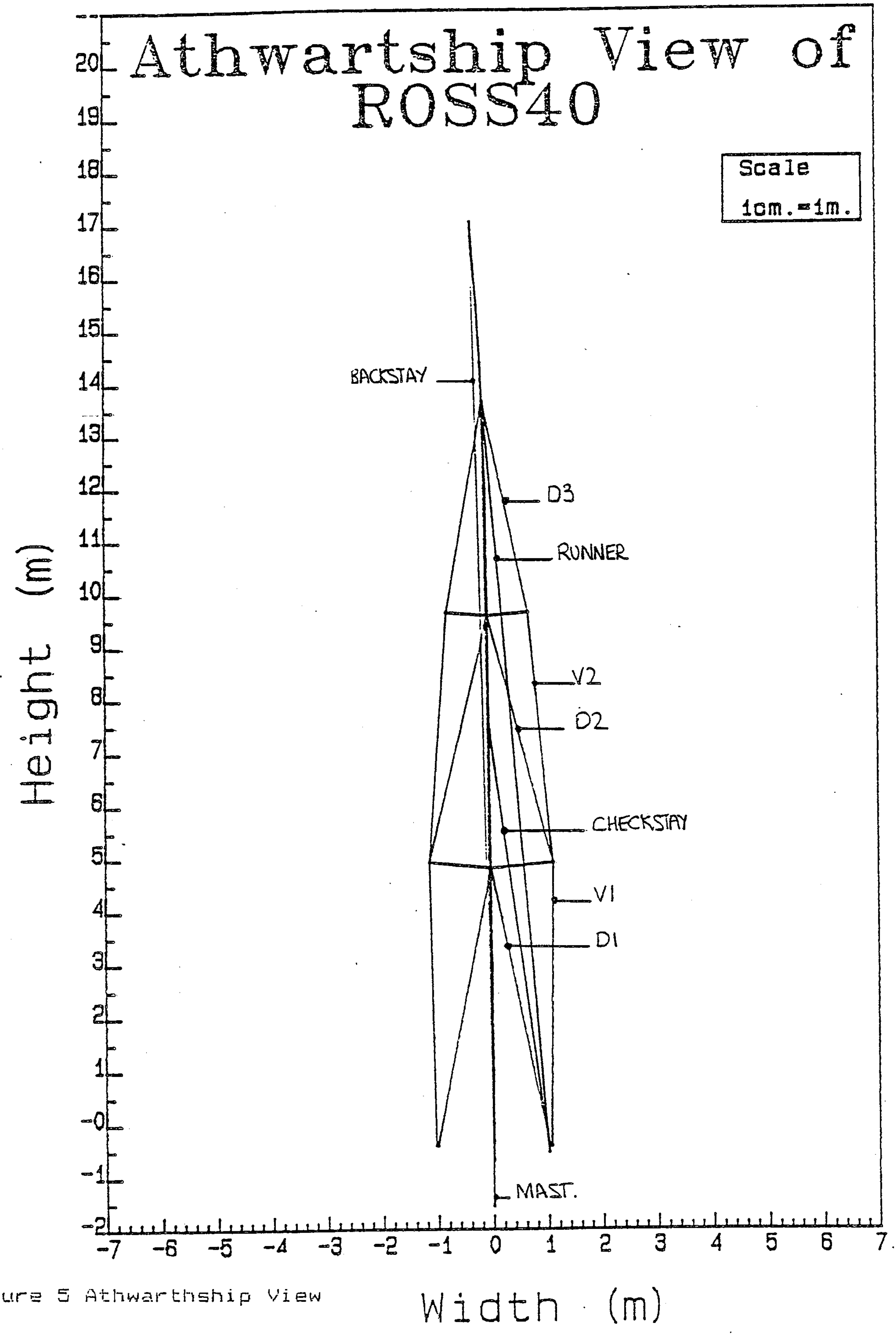


Figure 5 Athwarthship View

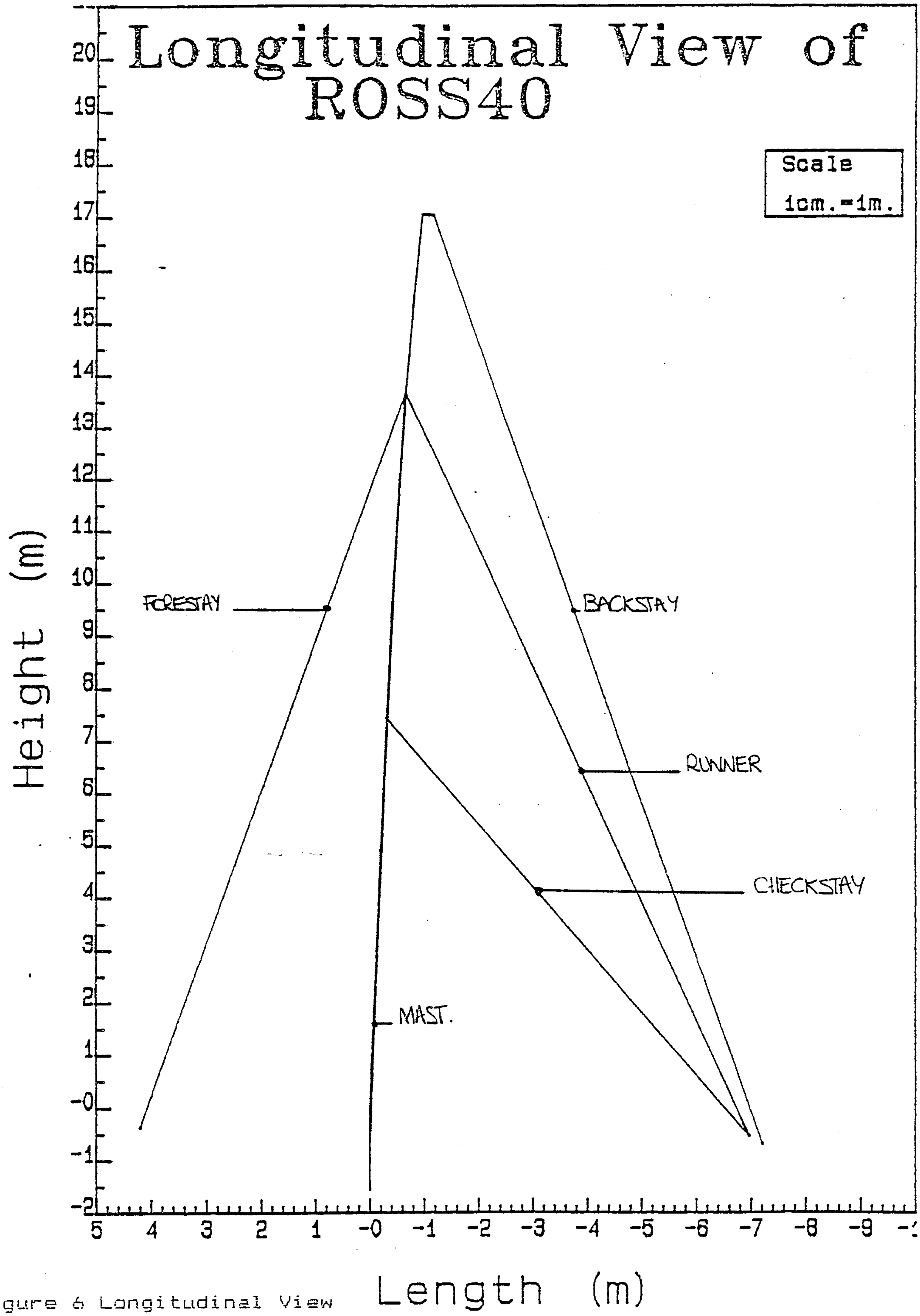


Figure 6 Longitudinal View

5. Conclusions

It has been shown that many traditional mast design techniques are inflexible and based on unsound principles. The new method presented here applies realistic loads from the sail and rigging, to the mast, and this is in itself a great improvement over earlier methods. The space-frame analysis allows properly for the nonlinear behaviour caused by large deflections and by extreme compression, and hence models realistically the structural response of the rig.

The authors have also found that the ability of the programs to quantify the displacements of a rig under different loads and supporting rigging of great assitance in focussing on the critical components of the structure. While there is still scope to improve the model described here, it does at least provide an accurate base from which rigs can be tuned, compared and improved, in an objective procedure, more suited to the engineering community.

6. References

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